Fundamentals of quantitative risk analysis

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ABSTRACT

The scientific analysis of water distribution networks is considered an important and essential field in the design and implementation of water delivery networks for cities and populated areas around the globe. This field is enhanced by the development of computational analysis resulting from major advances in digital computers in recent decades. This allowed for some critical changes in the way consultancies, companies and businesses conduct their design, implementation and maintenance of large delivery systems that are supposed to last, reliably functional, for tens of operational years.

This paper presents some of the new developments conducted in the derivation of fundamental principles for quantitative risk analysis and the techniques used to estimate the risk level factors that water networks face during all stages of their life span. The present analysis will also concentrate on the effect of simulated probabilistic pipe fractures on the performance of networks to deliver demand with a simulation period relative to the life span of the pipes.

Results of risk analysis simulations for typical network scenarios showed that risk levels can be calculated in advance for any network. Those risk levels are found to increase with time, as expected. They have different pattern trends for different network connections, i.e. they can only be compared relative to some logistic changes inflicted on the same network under consideration, but have no meaning when comparisons are made between two completely different connectivity arrangements. The results also showed characteristic curves consistent with their theoretical evaluation derived, based on local segments, of star and delta arrangements of pipe connections. Those curves were seen to follow an exponential increase in risk values with time.

Key words | consequence, demand, mathematical modeling, probability, risk analysis, supply water network

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NOMENCLATURE

Latin symbols

b	Specific constant (see Equation (4))
C_{event}	Consequences of risk related event [–]
i, j	Subscript indexes
L	Length of pipe [m]
N	Break rate per year for a specific pipe
n_0	Break rate per year per unit length at time t_0 for a specific pipe
P_{event}	Probability of risk related event [–]
R_{event}	Risk of an event [–]
t	Time [years]
t_0	Starting time [years]
t_s	Ending time [years]

Greek symbols

η	Variable defined by Equation (5) [–]
$\prod$	Multiplication operator (see Equation (2)).

INTRODUCTION

The water cycle in nature forms one of the most essential processes in sustaining life on Earth. In life's most complex form, the human body converts food and water into wastes. Over the millions of years of the Earth's existence, life has depended on the same cycle of water, which nature recycles through the dual processes of evaporation and condensation, in balance with the twin forces of the Sun and Earth's gravity.

Water makes up about 70% of the human body. It covers 80% of the Earth's surface. Of the world's water, 97% is salty, 2% is frozen and only 1% is from a freshwater source, much of which is polluted. Over half the people in the Third World do not have clean water to drink and three-quarters have no sanitation. Water is the most important resource on this Earth. We catch it, store it, filter and purify it, distribute it, use it, collect it, treat it and then return it to the sea. It is necessary to maintain life in all its different forms. This necessitates that a well-designed distribution network be installed in areas where it is necessary to maintain life's activities. Such networks are also supposed to function properly under different operational conditions. The basic components of such water distribution networks are several pipes and segments of different lengths connected together to form a network of pipelines distributed between the collection rivers and dams and the demand points. The water pressure inside the network, required to deliver water at a prescribed pressure head, is maintained by several primary and secondary pumps distributed all over the area under consideration. Additional components are used, like pressure valves, tanks, etc., to assist in the operation, delivery and control of the whole water network functionality. The American Water Works Association, AWWA (AWWA 1986), is a good reference to a thorough description of the practical details of what a water distribution network consists of. This reference also describes the environmental and public relations issues pertaining to this matter from an engineering point of view.

RISK ANALYSIS RESEARCH BACKGROUND

Two different approaches are categorized in the field of network reliability analysis: risk analysis and risk assessment. Risk analysis may be defined as the systematic use of available information to determine how often specified events might occur and the magnitude of their consequences, as described by Australian/NZ Standard Risk Management (1995). On the other hand, risk assessment is defined as the process used to determine risk management priorities by evaluating and comparing levels of risk against predetermined standards, target risk levels or other criteria, as described by the same reference. Earlier, published research concentrated on conducting risk analysis for piping systems in the oil and gas distribution networks. However, studies in the water industry concentrated mainly on conducting such analyses in a qualitative approach.

The reliability of any water distribution network may be conducted through qualitative and quantitative risk analysis and assessment of impact generating events. In recent years, piping reliability analysis used several methods to generate information about piping service data. Those may be summarized in the following points:

- analytical methods such as probabilistic fractures,
- expert judgment,
- statistical analysis of piping history data,
- combined methods of the above.

Recent advances in the statistical analysis of actual measurements made it possible to study this subject thoroughly. Thomas (1981) used the combined approach of these methods to address the reliability of pipes in nuclear reactors. Lydell (2000) revisited Thomas's work and used statistical methods to generate long-term analysis of the pipes' ages under different fatigue causes and criteria. He demonstrated that the actual measurements show that the rate of failure of the pipe due to rupture and leak is dependent on time and diameter. He used a time reference of 10 years and determined that the larger the pipe diameter the lower the rate of frequency of rupture. (It is the opposite case for times above the 10 years reference.) That rate of rupture frequency tends to become constant and stay that way for all types of pipe diameters at an age of around 12–14 years.

In general, looped networks have a higher reliability over branched line systems. Kansal *et al.* (1995) used a probability approach to investigate the reliability of networks according to this principle. The reliability of large networks was investigated by Tanyimboh *et al.* (1999) under cases of component failure and growing demands, and by Xu & Goulter (1998) using probability distribution functions of nodal demands, pipe roughness and reservoir/tank levels.

Break rates in a given system can fluctuate by a factor of ten from one year to the next, so an average of the previous several years is the most reliable indication of the current break rate upon which to base estimates of future break rates. Discussing breaks in terms of total breaks in a system can be somewhat misleading, since the number of breaks will depend on the size of the system and the time period under consideration. Determining the rate of change of the break rate involves extrapolating trends in the break rate. Shamir & Howard (1979) proposed an exponential equation relating pipe break rates at some year to a base break rate at some starting base year. Kleiner *et al.* (2001) studied the problem of pipe deterioration in networks due to breakages. Using the Shamir and Howard pipe break exponential formula, they implemented an economical optimization approach to making decisions on the rehabilitation of water networks. Through developing a pipe cost function, they managed to give an economically efficient computation on when it becomes justified to replace pipes in any given network versus repairing them. However, their analysis was limited to small networks of up to 15–20 pipe links.

For any given set of circumstances, the level of risk may be calculated as the multiplication of the probability of an event or adverse outcome (chance/likelihood/frequency, expressed numerically as occurrences per unit time) and a measure of the consequences of that event (damage/detriment/severity, expressed as a specific value measure). This will generate qualitative levels of risk, accordingly. This is true for any network risk analysis. It has to be stressed that the above definition is not sufficient in itself to fully describe the real risk of real situations. However, for a given situation in which the terms may be specified with reasonable accuracy, it provides an adequate basis for comparing risks or making resource decisions, as was described by Helm (1996). Another study by Basson *et al.* (1994) introduced the time factor, by the recurrence interval of failure, which is the reciprocal of the annual risk of failure. Some definitions of the risk of failure of the supply of water resource system are also described and its relation to the reliability criterion.

It is generally accepted that, when analyzing water networks under damaged situations, the problem arising from eliminating negative (gauge) pressures appears during the iterative procedure. Some of those analyses are based on artificial damage scenarios simulating the probability of occurrence of damage in each pipe segment. Pressure-driven simulations of the effects of failure of major system components were made by Tanyimboh *et al.* (1999). One common indicator of the deterioration of water distribution systems is the number of water mains breaks. Sægrov *et al.* (1999) discussed effective methods of the managerial rehabilitation of water systems. They indicated the effect of climate changes on the rate of pipe bursts and reported that the average lifetime of water mains in fully developed networks lies in the range from 100–200 years.

The research survey shows a need to identify the base components of factors determining risk levels in networks. This should be based on trends in break rates, as this provides useful information on the causes of breaks and their possible related remedial actions. Data collected for water main bursts and breaks are especially useful in identifying broader trends in water mains deterioration. In recent years, the advancement in computational facilities made it possible to generate complex analyses for large systems. Risk analysis falls into such a category, and applying that to water distribution networks is no exception. Earlier, only qualitative approaches could be made. Recently, more and more analyses are becoming quantitative in nature, where measures and comparisons can be made to such large systems, and in all different operational states. The present study will try to establish and calculate the risk levels for water distribution networks under normal operation and make those values a reference for subsequent system states to be compared with. This will generate a way to enhance the decision-making process of which system state to choose from in the process of designing and/or rehabilitating the water distribution networks. Such an analysis requires that a mathematical measuring formula be established for calculating the various levels of risk. Such a formula will be based on the aforementioned Shamir and Howard empirical pipes' break rates (Shamir & Howard 1979) to define risk-related event probabilities.

MEASURING QUANTITATIVE RISKS OF EVENTS

Mathematical modeling of risk levels for water supply distribution networks is useful in determining the reliability of networks to meet predetermined water demand levels. Quantitative risk assessment combines three key ideas:

1. the chance of something going wrong,
2. the consequences if it does and
3. the context within which the situation is set.

In symbolic terms, the following equation for estimating levels of risk of an event can be written:

$$R_{event} = P_{event} C_{event}. \quad (1)$$

Since the objective is to establish the total risk values for a water network under a specific operational state, therefore, definitions should be given for the total probability of collective risk-causing events and the total consequences resulting from those. Those definitions were found to be convenient for this study, although variations may be made in this respect, and as appropriate.

The probability of an event is an indication of how frequent that that event occurs. It can either be found through actual statistical data, or be calculated from a suitable simulation process. It can be represented by a numerical value, customarily between 0 and 1, inclusive (0 being an impossible event and 1 for a definitely occurring event). The above principle can be applied to water distribution networks by calculating the resulting probability of a network undergoing probable occurrences of failure in some of its components over a specific period of time. An effort is made here to simulate, as accurately as possible, the trend of failure in network components. Taking, for example, the failure rates of pipe segments (due to breakage, leaks and scheduled maintenance), it can be said that the probability of failure of a specific pipe in a network is a time-dependent criterion, being at its lower value when the pipe is new and deteriorating with time to a higher value at which point the pipe is considered irreparable and should be replaced. If it is assumed that a pipe segment has the probability trend described above, then its probability function can be simulated by the curve shown in Figure 1.

Figure 1 represents the probability of failure for one pipe in a network, where the damage effect is assumed to last only for a specific period of time and the pipe is repaired by the end of that period. A suitable formula will be used here to simulate the trend shown in the above curve. The same formula will be applied to each and every pipe segment of the water network under discussion. All resulting failure events are considered "independent" (i.e. a failure event happening for any pipe has no effect in changing the probability of failure for the other pipes). Therefore, the total probability of failure for all mutually exclusive pipe failure events at any one time can be calculated by

$$P_{tot} = \prod_i P_i = P_1 P_2 P_3 \cdots P_n. \quad (2)$$

The index i here represents the index of all pipes in the network. Since probability values are figures less than 1, therefore, P_{tot} will have a value much lower than each of the individual probability figures involved, which is reasonable since the probability of some two events happening at the same time is definitely lower than the probability of each of these events occurring separately. The same previous figure shows the concept of the life span of a pipe. This figure is not constant but changes from pipe to pipe even for a single network. Several factors affect such variation, such as pipe diameter and type, i.e. the physical characteristics of the pipes in general, operating conditions, location in the network and acidity of the soil, to name just a few.

The problem of measuring the consequences of events endured during the operation of water networks is closely related to economics. It is far more difficult to calculate the consequences of any risk-related events in water networks than to try to determine the probability of such events. Of course, deciding what the consequences are is part of the job. To name a few they are reduction in demand, reduction in pressure, quality of water, dissatisfaction of clients,

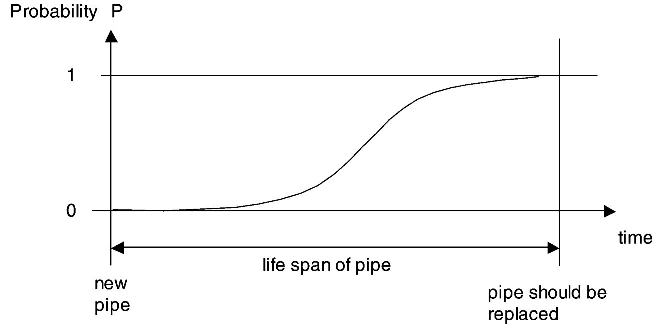

Figure 1 | Probability of failure of a pipe versus time.

damage to the environment, etc. All the above have economical values and weights attached and associated with them. These, of course, differ in perspective from person to person, group to group, society to society and time to time as well. So determining a universal yardstick will be more complicated and related to the general cultural society at any given time. Specifically speaking, the major objective of every water distribution network is to maintain the delivery of potable water quantities and to meet the requested demand at the adequate pressure set. This specific objective is used here in the calculation of risk levels for a water network by giving an indication of the consequences of any failure event that may be caused.

If it is assumed that such an objective represents the ultimate case where the network is in its fully operational water delivery state, then any reduction in that water delivery level, due to some piping failure event, can be counted as a measure of the damaging consequence of that event. To put it in a mathematical form, the following equation can be written:

$$C_{event} = \frac{\text{reduction in water delivery}}{\text{total demand to be met}}. \tag{3}$$

The reduction in water delivered to customers is actually the difference between the total demand requested and the actual water quantities delivered. Therefore, the consequences of any failure related event would be calculated from

$$C_{event} = 1 - \frac{\text{actual demand met}}{\text{total demand requested}}. \tag{3a}$$

According to this definition, values of C_{event} will be in the range between 0–1, inclusive. Low values indicate a low level of reduction in the water delivered to meet demand, while high values, on the other hand, indicate a large shortage in water delivery. The calculation of levels of risk will be based upon the above-mentioned definitions of the probabilities of failure and the consequences of failure. Equations (1), (2) and (3a) were used to generate the risk levels presented in this paper. A typical reference network is used in the current analysis, the proposed connectivity of which is shown in Figure 2.

THEORETICAL MODELING OF NETWORK RISK RATES USING PROBABILITY ANALYSIS

It is the objective of this section to link the probability of the breaking rates for pipes established from some empirical trends to the principles of risk estimation mentioned earlier. This will theoretically predict and model the variations of risk levels over time due to pipe breaking events. In doing so, some assumptions are made which are as reasonable as possible to keep the model simple yet realistic and, most

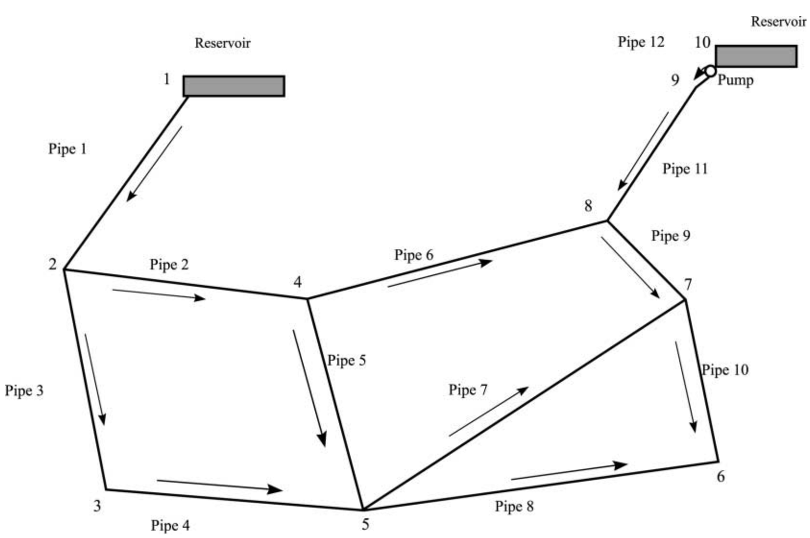

Figure 2 | Schematic diagram of a simple network used as a reference network for the analyses conducted in the present work.

importantly, reliable in giving the predicted risk values for specific segments of water networks.

In their research work, Shamir & Howard (1979) proposed an empirical formula to calculate future expected yearly rates of breaking occasions for pipe segments of networks based on actual yearly measurements of the breakage rates for pipes. Their proposed formula may be written as

$$N = n_0 L e^{b(t-t_0)}. \tag{4}$$

For presentation purposes and after a suitable selective choice of constants, the above equation would give a variation trend with changing time similar to the curve shown in Figure 3, which shows that the break rate N increases exponentially with time t as the pipe gets older.

Such a trend is almost never allowed to go to infinity in actual operational networks since at some stage in time a decision has to be made to replace that specific pipe either partially or completely by a suitable new one as it seems, from a logical point of view, that this increase in breaking rate is one of the factors giving rise to risk levels for that particular pipe segment and for the water network as a whole.

At once it can be seen that, following the Shamir and Howard equation mentioned earlier, the probability of the breaking of a pipe in a network varies with time and has some reliance relation with the value of breaking rate per year, N. As it seems, there is a direct link between this breaking rate increase and the increase in the probability of pipe breakages with time. As defined, the probability is predicted to have a change rate resembling the curve shown in Figure 1. It starts from zero or, realistically speaking, somewhere slightly above zero, and converges to unity as the pipe ages with time, indicating the certain breakage during operation. The relational link of P to N values is circumstantial. It depends on several environmental and operational factors. In this research work, several equations and formulae have been proposed to model and simulate this increasing behavior of P with time following the increasing trend of P with N. Some of those assumptions were exponential or even higher functions in nature. However, it was found that the easiest and most direct way of mathematically modeling this is to normalize N values between the lowest and highest expected values, as is explained below. Upon assuming that N varies between its value at t_0 (the starting time) and t_s (the expected life span of the pipe), the following normalizing factor may be written:

$$\eta = \frac{N - N_0}{N_s - N_0} \tag{5}$$

or

$$\eta = \frac{n_0 L e^{b(t-t_0)} - n_0 L e^{b(t_0-t_0)}}{n_0 L e^{b(t_s-t_0)} - n_0 L e^{b(t_0-t_0)}} \tag{5a}$$

which results in

$$\eta = \frac{e^{bt} - e^{bt_0}}{e^{bt_s} - e^{bt_0}}. \tag{5b}$$

For a selection of $t_0 = 0$ and $t_s = 25$ years, then this last equation is conveniently drawn as shown in Figure 4. It shows that η takes values between zero and unity in the time range between t_0 and t_s. Therefore, equating η values to P in the same time range can act as a good way of simulating P values since P is low at times close to t_0 and increases to a high of unity as time approaches t_s. It is worth mentioning that, by this normalizing process, the effect of the pipe length L has reduced from probability values as seen by Equation (5b). However, the model still incorporates the length variable in the consequences generated from the hydrodynamic calculations of the network. Consequences of pipe breaking events really depend on people's perception of what are considered losses. To keep the subject in focus, we will concentrate only on how much reduction in water delivery is caused by a direct consequence of a pipe

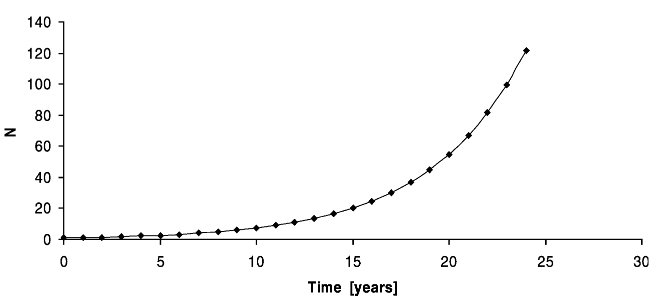

Figure 3 | Variation of the breaking rate of a pipe along the years, according to Shamir & Howard (1979), Equation (4).

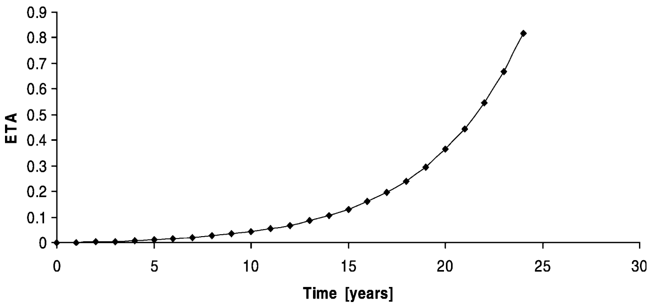

Figure 4 | Representation of the η values with respect to time t, as in Equation (5b).

breakage situation. As defined earlier in this chapter, the consequence C can be calculated from the definition in Equation (3a). This definition will be used to estimate the consequences of pipe breakage events.

FUNDAMENTAL NETWORK COMPONENTS SCENARIOS

Partial segments of network components will be isolated and investigated following the above mathematical and probability analysis. Several design criteria will be used to show the effect of changing network component design on risk factors and the network performance in general. Three network piping combinations have been selected, namely:

- the star connection,
- the delta connection and
- the modified delta connection.

All these connections share that they are composed of multiple pipe segments joined together. However, they differ in the way they function, and therefore each will have different characteristic performances which contribute to the total operation of the network, and therefore it is expected that the three scenario connections will have the same risk variation trends with time. However, they are expected to differ in risk value levels, as will be seen later. Those three connection variations will be analyzed and explained comprehensively below.

The star connection is generated in network design from the intersection of pipes to form a nodal point called the junction. For the current analysis, we assume an intersection of four different pipes (pipes 1–4) joined together at one nodal point A, as shown in Figure 5.

From the intersection point A of the four pipes, demand d is assumed to be supplied. From such a connection of pipes, it is concluded that the four-pipe network combination delivers a net water flowrate to node A at an assumed rate of d, and that a breaking situation in any of those pipes will have some direct effect on the capability of this network segment to deliver the quantity of water flowrate requested at node A. Thus, water delivery will be the objective of any risk analysis conducted here and its values will be calculated accordingly, as explained earlier. For such a network connection segment, and at any one time, there are several possibilities for breaking of the four pipes. To be exact, there are theoretically sixteen different possibilities, listed in Table 1.

Their estimated probability values are also shown in the same figure. Such breaking scenarios will have a direct effect on the delivered flowrate to point A. This is hydrodynamically determined by flowrate patterns and the specific network design. But before going into enhanced detail, thet resulting effect will be estimated here, according to the breaking possibilities mentioned previously to estimate the risk levels generated from these possible scenario situations. Such an estimation will be presented for three different assumption models, where the objective will be to calculate the rate of change of risk levels with time for each assumption model, and then to compare risk values between them. Such an analysis will provide some clarification on the effectiveness of design patterns for real networks. Those three assumption models for the star connection arrangement are based on the following assumptions:

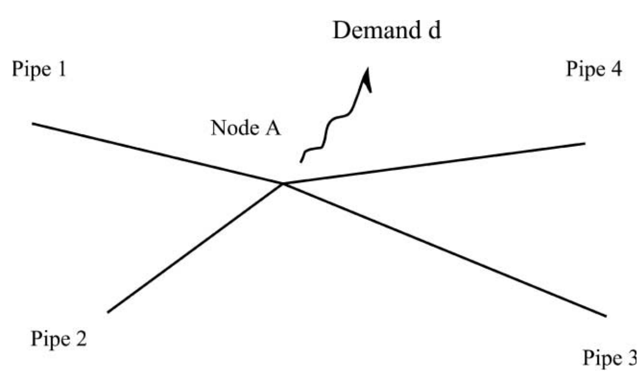

Figure 5 | The star connection.

Table 1 | Possible combinations of breakages of pipes of the star connection shown in Figure 5 and their respective probabilities according to Equation (5b)

Possibility no.	Number of broken pipes	Specific broken pipes	Probability
1	None	None	Zero
2	1	P_1	η_1
3	1	P_2	η_2
4	1	P_3	η_3
5	1	P_4	η_4
6	2	P_1 P_2	$\eta_1\eta_2$
7	2	P_1 P_3	$\eta_1\eta_3$
8	2	P_1 P_4	$\eta_1\eta_4$
9	2	P_2 P_3	$\eta_2\eta_3$
10	2	P_2 P_4	$\eta_2\eta_4$
11	2	P_3 P_4	$\eta_3\eta_4$
12	3	P_1 P_2 P_3	$\eta_1\eta_2\eta_3$
13	3	P_1 P_2 P_4	$\eta_1\eta_2\eta_4$
14	3	P_1 P_3 P_4	$\eta_1\eta_3\eta_4$
15	3	P_2 P_3 P_4	$\eta_2\eta_3\eta_4$
16	4	P_1 P_2 P_3 P_4	$\eta_1\eta_2\eta_3\eta_4$

- Assumption model A states that any broken pipe will reduce the water delivered to node A by one quarter.
- Assumption model B states that breaking of one pipe will not have any effect on water delivery, whereas two broken pipes will result in a one-third reduction and three broken pipes will result in a two-thirds reduction.
- Assumption model C states that breaking of any one or even two pipes will have no effect, while three broken pipes will result in a one-half reduction in the water delivered to point A.

Logically speaking, assumption model A will somehow have higher risk levels than the other scenarios, and a reduction in risk values results when changing the design from A to B or C. Using those scenario assumptions with the probability table, the following Table 2 for the three models is produced.

Based on the tables, those three scenarios can be drawn as three probability distribution curves, as shown in Figure 6.

Figure 6 shows that the areas under each of these curves describe probability distribution curves that are skewed in their values. A reduction in the mean consequence value results when changing the situation from A to B or even to C, indicating that the scenario represented by assumption model C is the best design scenario of the three. Consequence values can be written for each breakage possibility mentioned in Table 1 for the star connection categorized for each of the three assumption models. This will result in the values listed in Table 3.

Risk values can be calculated for the star connection from the two tables (1 and 3) by multiplying the values of

Table 2 | Table relating the reduction in water delivery for node A in Figure 5 to the number of occurrences for each assumption model for the star connection

Consequence (reduction in water delivery)	Number of event possibilities
Assumption model A	
0.000	1
0.250	4
0.500	6
0.750	4
1.000	1
Assumption model B	
0.000	5
0.333	6
0.667	4
1.000	1
Assumption model C	
0.000	11
0.500	4
1.000	1

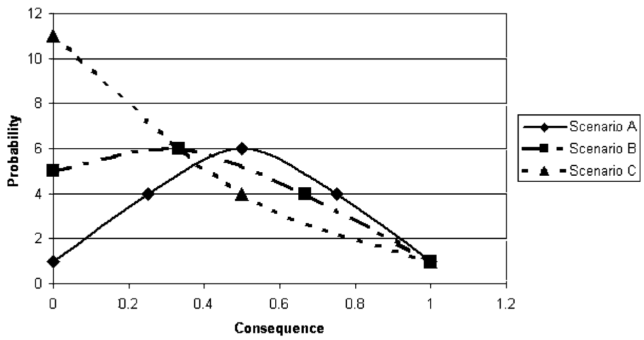

Figure 6 | Probability distribution curves for the three assumption models assumed for the star connection.

Table 3 | Consequence values written for each breakage possibility mentioned in Table 1 for the star connection categorized for each of the three assumption models A, B and C

	Consequences of assumption model scenarios		
Possibility	**A**	**B**	**C**
1	0.000	0.000	0.000
2	0.250	0.000	0.000
3	0.250	0.000	0.000
4	0.250	0.000	0.000
5	0.250	0.000	0.000
6	0.500	0.333	0.000
7	0.500	0.333	0.000
8	0.500	0.333	0.000
9	0.500	0.333	0.000
10	0.500	0.333	0.000
11	0.750	0.667	0.500
12	0.750	0.667	0.500
13	0.750	0.667	0.500
14	0.750	0.667	0.500
15	0.750	0.667	0.500
16	1.000	1.000	1.000

the probabilities by the respective consequences values. When drawn as graphs for each assumption model, these results may be presented as Figures 7, 8 and 9, plotting the risk values calculated for the three star scenarios.

Although there are sixteen possible situations that may occur for each model scenario, nevertheless only one can take place at any one specific time since part of the network will be malfunctioning and will require some sort of immediate repairing. At the beginning of the simulation time the chances are that no pipe or only one may break down. As it goes through time, several pipes may break, which is justified by the fact that pipes age, as indicated by the three figures. That is why the scenario lines representing the risk rate of change take the route of multiple curves, as shown in the figures. Such multiple curves are seen in any segmental part connection of networks. It is dependent on

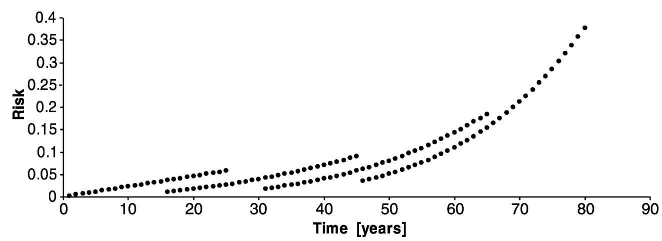

Figure 7 | Analytical representation of risk levels versus time for assumption model A for the star connection shown in Figure 5.

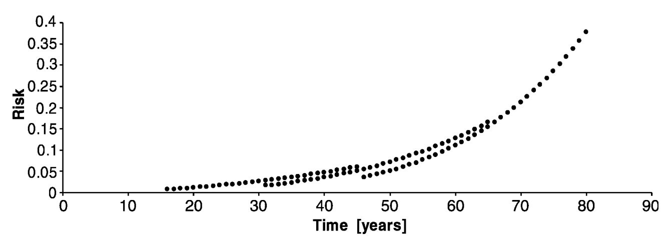

Figure 8 | Analytical representation of risk levels versus time for assumption model B for the star connection shown in Figure 5.

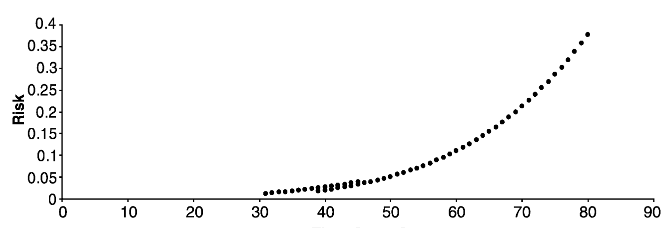

Figure 9 | Analytical representation of risk levels versus time for assumption model C for the star connection shown in Figure 5.

many variables, including environmental ones, and is a characteristic fingerprint of the hydrodynamic activities for the water network in general. The theoretical simulation of the three scenarios represented by the figures clearly shows a reduction in risk values as the design is changed from scenario A to B, or to C. This behavior is expected and is a direct reflection of the change in design and the performance of the network under each of these model scenarios. A last word can be said that similar trends are seen but in a somewhat more complex form when generating the hydrodynamic solution of actual networks, as will be seen later in the next chapter.

Another variation of network connectivity is the delta connection, as shown in Figure 10. The delta connection is generated in network design from the intersection of three pipes (P_2, P_3 and P_4), joined together and connected to a source through pipe P_1 to form a network segment that delivers water through a loop formation.

From such a connection of pipes, it is concluded that the four-pipe network combination delivers a net water flowrate to nodes A, B and C at an assumed total rate of d, and that a breaking situation in any of those pipes will have some direct effect on the capability of this network segment to deliver the quantity of water flowrate requested at node A, B and C. Furthermore, each node is assumed to request one-third of the total demand delivered, (i.e. $d/3$). For such a network connection segment, and at any one time, there are several possibilities for the breaking of the four pipes. As was explained earlier for the star connection, there are theoretically sixteen different possibilities here, tabulated in Table 4.

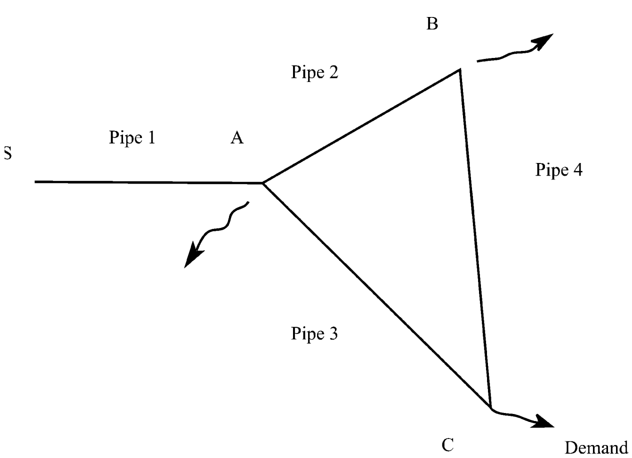

Figure 10 | The delta connection, showing four pipe connections, with one source S and the nodal demand points A, B and C.

Such breaking scenarios will have a direct effect on the delivered flowrate to points A, B and C. Based on such configurations and according to the possibilities tabulated above, Table 5 results.

As was conducted before for the star connection, the results of calculating the risk levels for such delta connections are represented by the plot in Figure 11. This figure shows clearly the same pattern of risk level rate of change. It increases with time for the delta connection indicating deterioration of the pipes as the network grows older. The only difference is the first line which shows relatively very

Table 4 | Possible combinations of the breakages of pipes in the delta connection shown in Figure 10 and their respective probabilities according to Equation (5b)

Possibility no.	Number of broken pipes	Specific broken pipes	Probability
1	None	None	0
2	1	P_1	η_1
3	1	P_2	η_2
4	1	P_3	η_3
5	1	P_4	η_4
6	2	P_1 P_2	$\eta_1\eta_2$
7	2	P_1 P_3	$\eta_1\eta_3$
8	2	P_1 P_4	$\eta_1\eta_4$
9	2	P_2 P_3	$\eta_2\eta_3$
10	2	P_2 P_4	$\eta_2\eta_4$
11	2	P_3 P_4	$\eta_3\eta_4$
12	3	P_1 P_2 P_3	$\eta_1\eta_2\eta_3$
13	3	P_1 P_2 P_4	$\eta_1\eta_2\eta_4$
14	3	P_1 P_3 P_4	$\eta_1\eta_3\eta_4$
15	3	P_2 P_3 P_4	$\eta_2\eta_3\eta_4$
16	4	P_1 P_2 P_3 P_4	$\eta_1\eta_2\eta_3\eta_4$

Table 5 | Consequence values calculated for each breakage possibility mentioned in Table 4 for the delta connection shown in Figure 10

Possibility no.	Consequence
1	0.000
2	1.000
3	0.000
4	0.000
5	0.000
6	1.000
7	1.000
8	1.000
9	0.667
10	0.333
11	0.333
12	1.000
13	1.000
14	1.000
15	0.667
16	1.000

high values for risk at the early stages of the simulation time compared with the rest of the curves. This high-value line is a direct effect of the vulnerability of the network segment represented by pipe 1 (see Figure 10). This pipe is the only supplier to the whole delta connection from the source S and therefore any fracture or break that may happen to this pipe, especially at the early stages, makes this network segment loose function and the consequences will be very high. For this reason, it can be said that the risk levels are, to some extent, an indication of the nature of the quality of the network design, and calculating it from the early stages of preparing the blueprints is essential. To prove this point further, a slight modification to the delta design is conducted, as is explained in the next section.

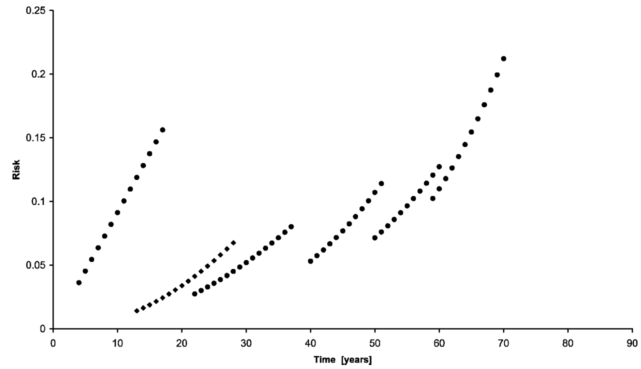

Figure 11 | Analytical representation of risk levels versus time for the delta connection shown in Figure 10.

The proposal for a modified delta connection is generated in the network design by supplying each nodal demand with several lines of water delivery in a looping design system that will eventually reduce the risks involved in the operation and maintenance of the water network. This looping is already used in actual water networks and in similar electrical wiring systems as well. This ensures the provision of more reliable working conditions for any design. However, how much looping is needed is not known unless comparative studies are conducted to investigate the effect of adding additional components to the network itself, and whether that will be economically justifiable or not.

In order to remedy the branching design of the delta connection from one source S_1, a new modified design is proposed here. This is shown in Figure 12, where another pipe P_2 is added. This new pipe is connected to another source S_2 (or even the same source for that matter) to ensure that the delta connection is supplied with water by two sources, as shown in the figure. The objective here is to prove that a reduction in the risk levels is accomplished by this modification, especially in the early stages of the simulation process.

From such a connection of pipes, it is concluded that the five-pipe network combination delivers a net water flowrate to nodes A, B and C at an assumed rate of d and that a breaking situation in any of those pipes will have some direct effect on the capability of this network segment to deliver the quantity of water flowrate requested at nodes A, B and C. Furthermore, each node is assumed to request one-third of the total demand requested (i.e. $d/3$). Since

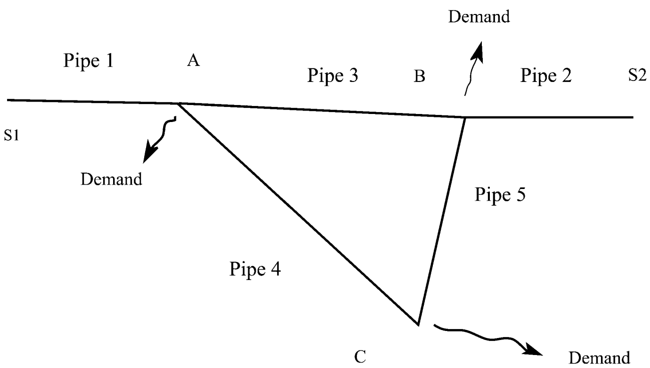

Figure 12 | The modified delta connection, showing two sources delivering water to the delta connection, as compared with the delta connection diagram shown in Figure 10.

there are now five pipes involved, for such a network connection segment, and at any one time, there are thirty two possibilities for the breaking of the five pipes. These possibilities are tabulated in Table 6.

Since for this modified delta connection design, two sources of supplied water are provided, the hydrodynamic performance is therefore enhanced to the extent that any one nodal demand will have different paths for water supply, and hence lower restrictions are imposed when assuming the hydrodynamic effects and their consequences. However, the following are directly concluded from the figure:

- Simultaneous breaking of pipes P_1 and P_2 will shut down water delivery totally since they are connected to the two sources.
- Simultaneous breaking of pipes P_4 and P_5 will shut down point C, as seen from the figure.

Based on such configurations, and according to the possibilities tabulated above, Table 7 lists the analytical consequences of the thirty two possibilities.

The results of calculating the risk levels for such a delta connection are represented by Figure 13. This figure shows a substantial decrease in the rate of change of the risk levels. Compared with the original delta curves of Figure 11, it shows the diminishing situations of possible risky operational points, especially in the early stages of the simulation time. This is a good indication of the enhancement and reliability of the looping system when incorporated in the design of water distribution networks and its

Table 6 | Possible combinations of breakages of pipes of the modified delta connection shown in Figure 12 and their respective probabilities according to Equation (5b)

Possibility no.	Number of broken pipes	Specific broken pipes	Probability
1	None	None	0
2	1	P_1	η_1
3	1	P_2	η_2
4	1	P_3	η_3
5	1	P_4	η_4
6	1	P_5	η_5
7	2	P_1 P_2	$\eta_1\eta_2$
8	2	P_1 P_3	$\eta_1\eta_3$
9	2	P_1 P_4	$\eta_1\eta_4$
10	2	P_1 P_5	$\eta_1\eta_5$
11	2	P_2 P_3	$\eta_2\eta_3$
12	2	P_2 P_4	$\eta_2\eta_4$
13	2	P_2 P_5	$\eta_2\eta_5$
14	2	P_3 P_4	$\eta_3\eta_4$
15	2	P_3 P_5	$\eta_3\eta_5$
16	2	P_4 P_5	$\eta_4\eta_5$
17	3	P_1 P_2 P_3	$\eta_1\eta_2\eta_3$
18	3	P_1 P_2 P_4	$\eta_1\eta_2\eta_4$
19	3	P_1 P_2 P_5	$\eta_1\eta_2\eta_5$
20	3	P_1 P_3 P_4	$\eta_1\eta_3\eta_4$
21	3	P_1 P_3 P_5	$\eta_1\eta_3\eta_5$
22	3	P_1 P_4 P_5	$\eta_1\eta_4\eta_5$
23	3	P_2 P_3 P_4	$\eta_2\eta_3\eta_4$
24	3	P_2 P_3 P_5	$\eta_2\eta_3\eta_5$
25	3	P_2 P_4 P_5	$\eta_2\eta_4\eta_5$
26	3	P_3 P_4 P_5	$\eta_3\eta_4\eta_5$

Table 6 | (*continued*)

Possibility no.	Number of broken pipes	Specific broken pipes	Probability
27	4	$P_1\ P_2\ P_3\ P_4$	$\eta_1\eta_2\eta_3\eta_4$
28	4	$P_1\ P_2\ P_3\ P_5$	$\eta_1\eta_2\eta_3\eta_5$
29	4	$P_1\ P_2\ P_4\ P_5$	$\eta_1\eta_2\eta_4\eta_5$
30	4	$P_1\ P_3\ P_4\ P_5$	$\eta_1\eta_3\eta_4\eta_5$
31	4	$P_2\ P_3\ P_4\ P_5$	$\eta_2\eta_3\eta_4\eta_5$
32	5	$P_1\ P_2\ P_3\ P_4\ P_5$	$\eta_1\eta_2\eta_3\eta_4\eta_5$

effect on reducing the risks involved from operating a water delivery system.

COMPUTATIONAL ANALYSIS

The simulation technique for calculating risk levels for a water distribution network relies on a technique developed for this purpose. It uses extended period analysis in that it requires updating the system parameters at every time step. Different random failure functions are assigned to each pipe segment and the resulting reduction in water delivery (if any) is calculated using a marching technique as shown by the structured computer code in Figure 14.

The resulting values of risk levels will be dependent upon the accompanied time values and they will be calculated for a dimensionless period of time set in advance. As the figure shows, for each time step a calculation loop is conducted. This loop will include calculating the total probability of all randomized failures for the piping network that were inflicted automatically on each pipe segment of the designated network, according to the distribution shown in Figure 1. This stage will result in the generation of the total probability of occurrence of all failures. The next stage is that the network is solved with those inflicted pipe failures. Estimation of the hydrodynamic quantities will be generated at this stage and an actual distribution of the calculated demand that was met will be developed, thus paving the way to calculating the consequences resulting from the pipe failure for the network as a whole. Those

Table 7 | Consequence values calculated for each breakage possibility mentioned in Table 6 for the modified delta connection shown in Figure 12

Possibility no.	Consequence
1	0.000
2	0.000
3	0.000
4	0.000
5	0.000
6	0.000
7	1.000
8	0.000
9	0.000
10	0.000
11	0.000
12	0.000
13	0.000
14	0.000
15	0.000
16	0.333
17	1.000
18	1.000
19	1.000
20	0.333
21	0.667
22	0.333
23	0.667
24	0.333
25	0.333
26	0.333
27	1.000

Table 7 | (*continued*)

Possibility no.	Consequence
28	1.000
29	1.000
30	0.667
31	0.667
32	1.000

consequences are calculated next and according to Equation (3a). The risk level for that time step is calculated next and the time loop is incremented for the next loop step. A table results from these calculations giving the variation of risk level values of that particular network with time.

RISK ANALYSIS RESULTS

The procedures and techniques used to obtain the results of the risk analysis relied on a C++ software program designed for this purpose. The network that is chosen for this purpose is shown in Figure 2. The risk analysis is based on successive computation of steady states having different damage conditions. To give a better view of the computational procedures, let us assume that for the network in Figure 2, pipes 3 and 4 are temporarily damaged and isolated, i.e. they are out of service. Then, the steady state solution can be generated to have the hydrodynamic profile shown in Figure 15, which shows that demand at node 3 is not met. The other nodes are receiving their normal share of water as in normal steady state operation. However, it should also be noted that the total system has a different pipe flowrate distribution and a different pressure profile, as indicated by the change in coloring of the pipes involved. For such an event, the probability of the simultaneous occurrence of two damaged pipes can be calculated using Equation (2) by multiplying together the probability for each pipe if they are assumed mutually exclusive events, i.e. each pipe damage occurs irrespective of the other. On the other hand, Equation (3a) is used to calculate the consequence resulting from the reduction in demand involved. Multiplying those two calculated figures, as in Equation (1), will give the level of risk (risk index) for that specific event of two damaged pipes. The same can be repeated day after day of network operation, after assuming that any damage lasts for one day only and is repaired by the end of each day.

By following the above technique, risk levels can be obtained for a long period of time. That period, when normalized, is taken in this study as unity of operation,

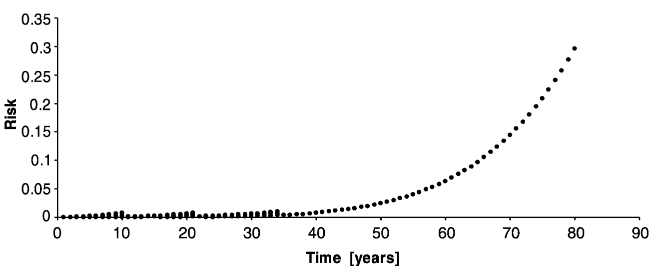

Figure 13 | Analytical representation of risk levels versus time for the modified delta connection shown in Figure 12.

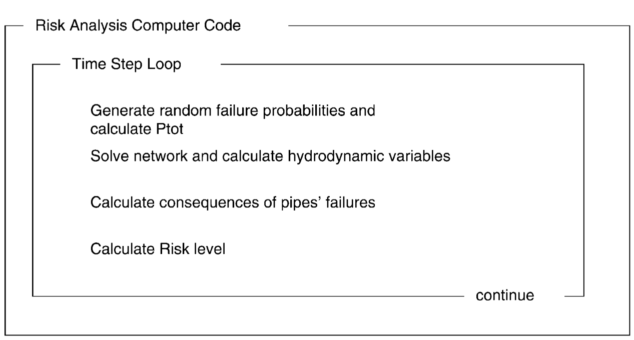

Figure 14 | Structured computer code of the technique used in the risk analysis calculations.

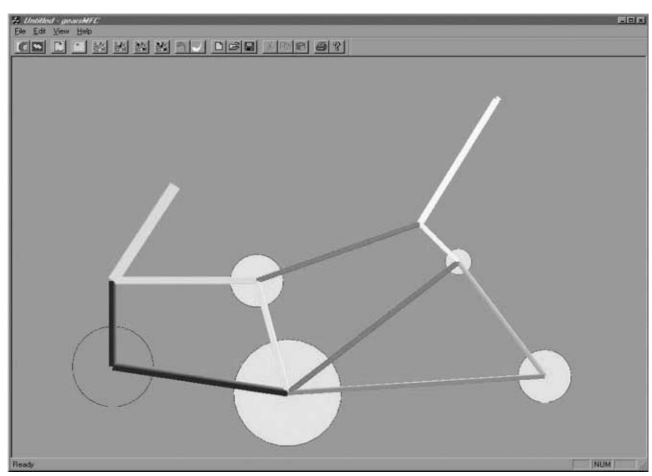

Figure 15 | Steady state solution of the hypothetical network with pipes 3 and 4 damaged.

which means that it can be applied to any period. The reason for this selection is that the intention is to concentrate on the variation of risk over time and to take all other values with respect to that, to keep the analysis as general as possible. The damage is separately calculated for each pipe through a function having a random distribution, similar to that shown in Figure 1.

A typical run for such a function is shown in Figure 16, where it may be seen that a total damage of 100% will be exerted on each pipe. The whole computation process took about 6 hours to conclude and the resulting risk levels generated using Equations (1), (3a) and (5b) for the hypothetical network under normal operation are shown in Figure 17. From this figure the following points may be observed:

- There are several lines of risk levels, called here patterns or characteristic curves of risk levels. The number and distribution of these patterns are dependent on the network characteristics. These trend lines are similar to the trend pattern derived earlier in conducting the analytical solution of the risk analysis for network connection (the star, delta and modified delta).
- For each line in those patterns visualized separately, as expected the level of risk increases with time.
- There is a specific line determining the highest risk level the network might incur. This line is the upper imaginary line connecting points of high risk, as shown in Figure 17.

The calculations were also conducted for two other operational scenarios: one with pipe 7 permanently damaged and the other with pipe 5 permanently damaged. Results of their highest risk levels are calculated and compared together with their counterpart of normal operation mentioned earlier in Figure 18. This figure shows clearly that eliminating pipe 7 increased the risk at the staring point by about threefold, while eliminating pipe 5 increased the starting highest risk levels by about fivefold.

The results of the three scenarios are drawn in semi-logarithmic form to stress showing the actual formation of these risk level patterns. These are shown in Figures 19, 20 and 21, respectively. For the three scenarios similar types of risk patterns are seen, with levels of risk being different. The results show that comparisons can be made from risk results for the analyzed simulation period of the life span of the pipes. The levels of risk have increased in each of the aforementioned cases as compared to their values when the network is in normal operation, with the last being the worst of those three cases.

As a conclusion, it is believed that the more the number of pattern curves a network reflects in the total risk calculations, the better is its looping design structure capability in meeting demand requirements even under a

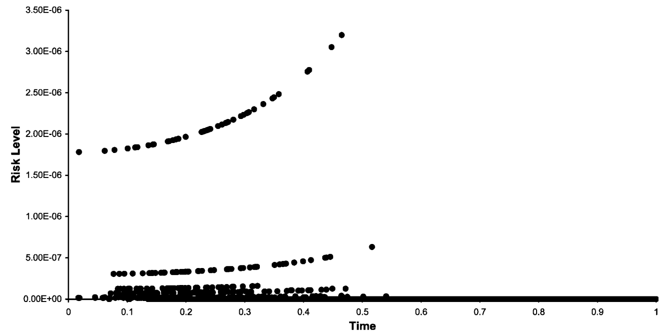

Figure 17 | Calculated risk levels over time for the network shown in Figure 2 under the normal operational situation.

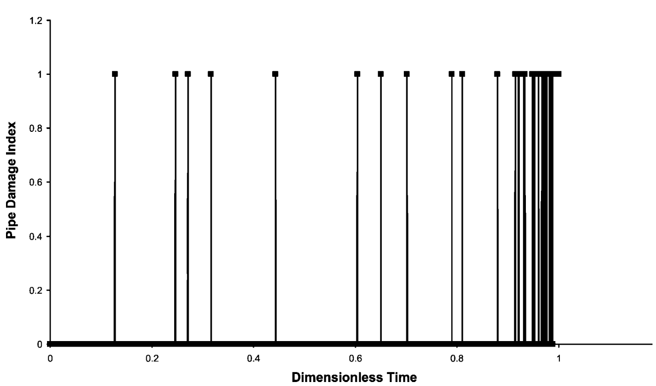

Figure 16 | Typical random function of damage exerted on each pipe of the network for the life span of the pipes' simulation period.

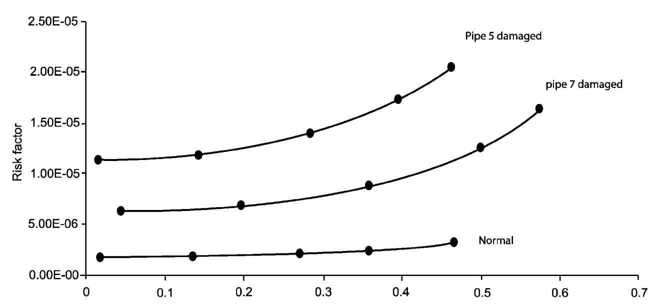

Figure 18 | Risk levels comparisons for three operational scenarios of the network shown in Figure 2.

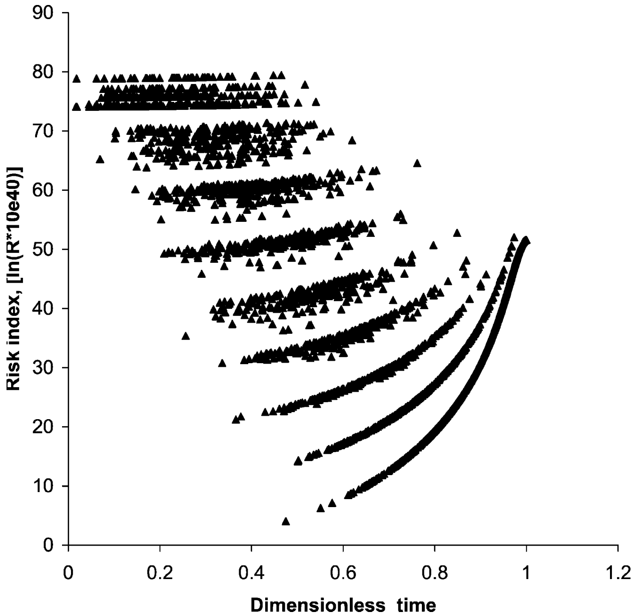

Figure 19 | Risk analysis results for the hypothetical network for the life span of the pipes' simulation period in normal operation.

burst pipe situation. All pattern curves resulting from the simulations were seen to follow Shamir and Howard's empirical formula (Shamir & Howard 1979) with an exponential increase in risk values with time.

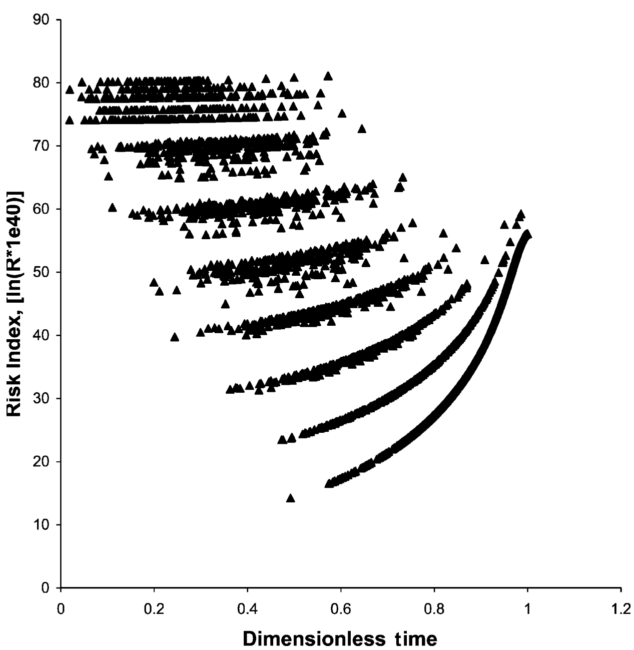

Figure 20 | Risk analysis results for the hypothetical network for the life span of the pipes' simulation period of operation, with pipe 7 permanently damaged.

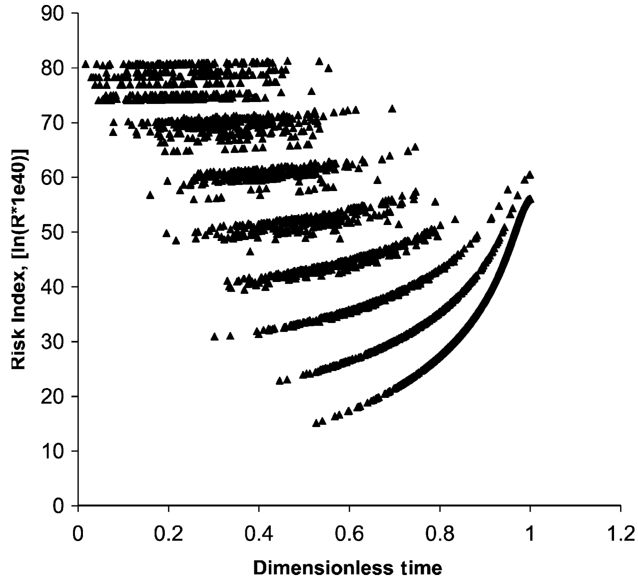

Figure 21 | Risk analysis results for the hypothetical network for the life span of the pipes' simulation period of operation, with pipe 5 permanently damaged.

CONCLUSIONS

In this work, risk analysis fundamentals and risk quantitative values were established for segment connections and for typically selected networks. The risk analysis results generated in this work gave pattern variations of quantitative values of operating network risks with time. This was based on the probability estimation of randomly selected events, specifically speaking pipe damage events, and their consequences in meeting flowrate demand requirements. For segment connections, several star and delta arrangements were investigated. Fundamental principle analysis showed patterns of discontinuous curves of risk values and their variation with time. These pattern curves were found to be characteristic of the specific network arrangements and their behavior under specific pipe damage scenarios. Such observed patterns were clearly seen when calculating risk values for a complete network under randomly generated damage scenarios for long periods of time. Each pattern curve, as expected, was seen to increase its value with time, indicating that risk values are increasing with the increasing frequency of pipe damage events with time. However, the results also showed that, depending on the design, some pipe bursts have no effect in depriving demand-nodal-points from delivered water, except for changing the flowrate distribution across the whole network. As a consequence, it is believed that the greater the number of pattern curves a network reflects in the total risk calculations, the better is its looping design structure capability in meeting demand requirements even under burst pipe situations. All trend characteristic curves resulting from the simulations were seen to follow the Shamir and Howard empirical formula (Shamir & Howard 1979) with an exponential increase in risk values with time. Support for damage and risk analyses is recommended in any designed water distribution network analysis package. Results from the theoretical simulation extracted for each of the specific damage scenarios in the risk analysis were greatly enhanced by the visualization styles adopted.

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