tion. The method, initially introduced for solving problems in structural and continuum mechanics, has been applied to steady fluid flow analyses by, for example, Argyris, Mareczek, and Scharpf [16].

Application of Inviscid Analysis. The importance of viscous effects was determined from the full Navier-Stokes equations for incompressible liquid. Some problems are pressure-dominated, such as the expansion of a high pressure submerged bubble. Other problems are velocity-dominated, like prescribed outflow of liquid at some reference velocity from a tank. Still other problems like the dam break or liquid sloshing in a tank are acceleration-dominated. It was shown that inviscid analysis applies if:

$$\frac{\mu}{L}\sqrt{\frac{1}{P_0\rho g_c}} \ll 1.0 \qquad \text{Pressure-dominated} \qquad (11)$$

$$\frac{\mu}{\rho V_0 L} \ll 1.0 \qquad \text{Velocity-dominated} \qquad (12)$$

$$\frac{\mu}{\rho\sqrt{a_0 L^3}} \ll 1.0 \qquad \text{Acceleration-dominated} \qquad (13)$$

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References

1 Emmons, H. W., Chang, C. T., and Watson, B. C., "Taylor Instability of Finite Surface Waves," *Journal of Fluid Mechanics*, Vol. 7, No. 2, 1960, p. 177.

2 Laitone, E. V., "The Second Approximation to Cnoidal and Solitary Waves," *Journal of Fluid Mechanics*, Vol. 9, No. 3, 1960, p. 430.

3 Perko, L. M., "Large-Amplitude Motions of a Liquid-Vapor Interface in an Accelerating Container," *Journal of Fluid Mechanics*, Vol. 35, No. 1, 1969, p. 77.

4 Harlow, F. H., and Welch, J. E., "Numerical Calculation of Time-Dependent Viscous Incompressible Flows With Free Surface," *The Physics of Fluids*, Vol. 8, 1965, pp. 2182–2189.

5 Harlow, F. H., and Welch, J. E., "Numerical Study of Large-Amplitude Free Surface Motions," *The Physics of Fluids*, Vol. 9, 1966, pp. 842–859.

6 Chan, R. K. C., and Street, R. L., "A Computer Study of Finite-Amplitude Water Waves," *Journal of Computational Physics*, Vol. 6, No. 1, 1970.

7 Chan, R. K. C., "SUMMAC—A Numerical Model for Water Waves," PhD thesis, Stanford University, 1970; available from University Microfilms, 300 N. Zeeb, Ann Arbor, Mich., 48106; Order Number 71-12,874; 168 pp.

8 Milne-Thomson, L. M., *Hydrodynamics*, 4th ed., The Macmillan Company, 1960.

9 Lamb, Sir H., *Hydrodynamics*, 6th ed., Dover, 1932.

10 Stoker, J. J., *Water Waves*, Interscience Publishers, Inc., 1957.

11 Moody, F. J., *Prediction of Fluid Free Surface Motions Under the Influence of Body Force Acceleration and External Pressures*, PhD thesis, Stanford University, 1971; available from University Microfilms, 300 N. Zeeb, Ann Arbor, Mich., 48106; Order Number 71-19,730; 215 pp.

12 McCracken, D. D., and Dorn, W. S., *Numerical Methods and Fortran Programming*, Wiley, 1964.

13 Carre, B. A., "The Determination of the Optimum Accelerating Factor for Successive Over-Relaxation," *Computer Journal*, Vol. 4, No. 1, 1961, p. 73.

14 Reynolds, W. C., and Satterlee, H. M., "Liquid Propellant Behavior at Low and Zero g," *The Dynamic Behavior of Liquids in Moving Containers*, Edited by Abramson, H. N., NASA SP-106, 1966.

15 Zienkiewicz, O. C., and Cheung, N. K., *The Finite Element Method in Structural and Continuum Mechanics*, McGraw-Hill, 1967.

16 Argyris, J. H., Mareczek, G., and Schrapf, D. W., "Two and Three Dimensional Flow Analysis Using Finite Elements," *Nuclear Engineering Design*, Vol. 11, No. 2, 1970, p. 230.

DISCUSSION

J. A. Adams[2]

Now that the computer is providing the solutions to a plethora of useful and interesting problems, professional journals and societies should insist upon the careful nondimensionalization of the mathematical model and identification of the governing nondimensional parameters *before* any numerical solution is attempted. This is the only logical and efficient method for carrying out a thorough parametric analysis. This paper clearly defines dimensionless variables in equations (1), although the dual interpretation possible from the sentence immediately following equations (1) is a little disturbing. I would also like to see the important nondimensional parameters which are necessary to classify the problem placed in their rightful place at the beginning of the paper, rather than as an afterthought at the end.

Once the decision is made to use dimensionless variables, the results should be presented in these terms. This makes them useful in a wide variety of applications. This paper presents a mixture of dimensional results (Figs. 3, 5, and 6), and nondimensional results (Figs. 4 and 7). The solution to Pharaoh's problem in Fig. 8 indicates both a dimensional and a nondimensional variable. If a member of Pharaoh's Egyptian army looked up and saw the impending inundation, he would not know whether to nondimensionalize the length or dimensionalize the time.

Methods such as an optimum relaxation factor were used in the integration scheme to improve the accuracy and reduce the computation time for obtaining the solution to the Laplace equations which led to the velocity potential. However, it appears that the time advance calculations for the next free surface markers and surface potentials are based solely on current values. This explicit scheme has stringent stability requirements which limit the allowable step size in the time increment. The authors accounted for this limitation but it is suggested that an implicit scheme (or a combined explicit-implicit scheme) might save a significant amount of computation time. This would be especially helpful for sloshing in a tank where solutions over a lengthy time interval are of interest.

The authors state the necessity of using a sufficient number of mesh points and a sufficiently small time increment to control computational errors. What are the implications of these restrictions? How expensive, in terms of computer time and storage, is the devised computational tool? What advantage does it have over the (MAC) method which includes viscous effects? It would also be interesting to have the authors comment on the best way for an engineer to obtain the initial free surface velocity potentials needed to start the iteration process.

E. Kordyban[3]

Motion of liquid with a free surface presents an important, but difficult problem which has occupied many researchers for

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years, and any technique which expands the field of solutions is very welcome.

Unfortunately, I have received a copy of this paper somewhat late, and, therefore, I could not familiarize myself with the background material, and the paper is condensed to the extent that it is difficult to judge the flexibility and the extent of the proposed method.

In particular, the boundary conditions on the free surface are not described in detail. Can the external pressure imposed on the free surface be any function of time and position? If so, I would be particularly interested to have the authors comment on the possibility of applying this method to the problem of surface waves being acted upon by pressure fluctuations in a gas flowing over the surface.

I have been working on the problem of formation of slugs in horizontal two-phase flow, which seems to be due to the presence of low aerodynamic pressure at the wave crests. The possibility of the application of the presented technique to this case is very attractive.

Authors' Closure

We appreciate the discussions of Professor Adams and Professor Kordyban.

Professor Adams prefers nondimensional representation, and so do we, because it makes results applicable to numerous situations. The only reason for including both dimensional and nondimensional results in this paper was to demonstrate flexibility of the FREESURF program. With regard to FREESURF itself, the program is linked, requiring 30K on the Honeywell 6070 computer. Process times are between 0.1 and 0.2 hours, and problems with less than 100 mesh points require about 0.03 hours. The main restriction on number of mesh points is that straight line connections between boundary grid intersections (black dots in Fig. 2-A) should give accurate boundary resolution. The main advantages of FREESURF are simplicity in its formulation with velocity potential for inviscid flows, its ability to handle more than one free surface with constant pressure prescribed on each, and its relatively short process time. Initial free surface velocity potentials for a stationary liquid are specified as constant, usually zero. If a liquid surface initially is moving (e.g., a solitary wave), starting values of velocity potential generally can be obtained from other analyses. (See reference [11] for further discussion.)

In response to Professor Kordyban, the present version of FREESURF requires constant pressure on all free surfaces, although modification could be made to accomodate time or space variable pressure. The problem of two adjacent fluids in relative motion could be handled so long as interface shear is negligible and density of one fluid is small relative to the other.