the outer stop. Thus the absorber does not operate at speeds which might allow resonance at the two natural frequencies.

Design of Absorber

An equation useful in proportioning the absorber is obtained from Equation [9] when $\omega = \omega_n$

$$2m\,R\,\Omega r_o \omega_n = T_o \quad [13]$$

The angular velocity Ω, the natural frequency ω_n, and the torque T_o will be known The mass m and radius R must be chosen so that the amplitude r_o will be reasonable. The stops will be located to eliminate the natural frequencies of Equation [12]. The spring constant is found from Equation [11]. The initial force in the spring must balance the centrifugal force on the mass at the speed at which the absorber is to begin operating.

The details of construction will vary with the application, but a possible design is shown in Fig. 3. The mass is mounted on a cantilever spring, which transmits the absorber torque to the vibrating body. An alternative construction would have the mass pivot on antifriction bearings. Adjustable stops are above and below the mass. The spring is loaded in compression, the initial load being adjustable. The distance between the center of gravity of the mass and the line of action of the spring force is also adjustable, allowing the spring constant to be varied slightly. The type of mounting depends on the shape of the vibrating body. For greater effectiveness, two or more absorbers may be mounted radially about the center of rotation.

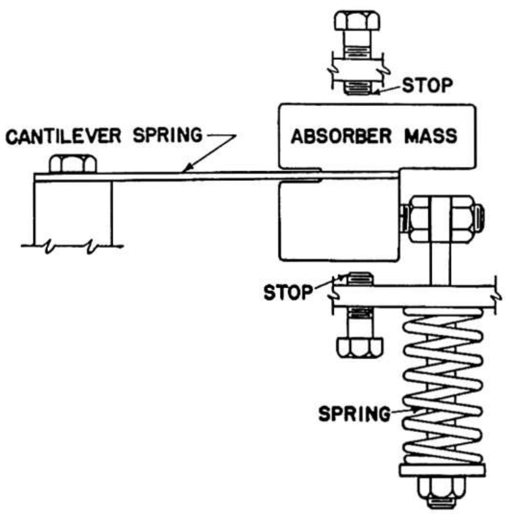

Fig. 3 A Possible Absorber Construction

Comparisons

Fig. 3 emphasizes the fact that, if desired, the absorber may be constructed so that its characteristics are somewhat variable. This would facilitate exact tuning to correct for manufacturing tolerances, approximations in calculations, or wear. Other types of dynamic absorbers, particularly the rotating-pendulum absorber, are often difficult to tune exactly.[3]

An important comparison may be made by writing the equation for either the rotating-pendulum absorber or the Frahm absorber which corresponds to Equation [13]. This equation is

$$m\,R\,r_o\,\omega_n^2 = T_o' \quad [14]$$

Equation [14] states that a rotating-pendulum absorber of a certain mass, radius, amplitude, and frequency will balance a certain exciting torque T_o'. Equation [13] states that the absorber under discussion of the same mass, radius, amplitude, and frequency will balance a different exciting torque T_o. For the sake of comparison, let the theoretical relative effectiveness E_r be the ratio of T_o to T_o'. Then

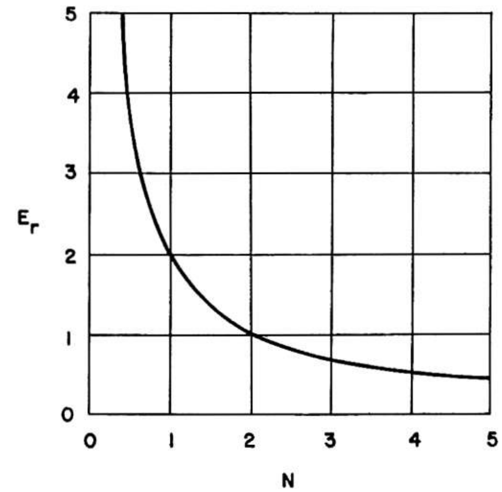

Fig. 4 Absorber Compared With Rotating-Pendulum Absorber

$$E_r = \frac{2m\,R\,\Omega\,r_o\,\omega_n}{m\,R\,r_o\omega_n^2} = \frac{2\Omega}{\omega_n} = \frac{2}{N} \quad [15]$$

This result is visualized in Fig. 4. For second-order vibration both absorbers are equally effective; in other words, for a given application, the absorbers would be about the same size. For higher orders, the rotating-pendulum absorber would be preferred on the basis of Equation [15]. On the other hand, for orders of vibration lower than the second, the absorber under discussion would be the most effective.

Discussion

R. J. Harker.[4] This paper on the use of a radially oscillating mass to suppress torsional vibrations in rotating systems is novel and appears to present some interesting possibilities, particularly with respect to low orders of excitation. The author is to be congratulated for his ingenious proposal and for his contribution to the theory of the tuned dynamic vibration absorber.

Equations [8] and [9] of the paper may be simplified by defining the main system as a single mass at a radius of gyration equal to the radius R, or $I = MR^2$, and by using the following notation

$f = (\omega_n/\Omega_n)$ = absorber tuning factor

$g = (\omega/\Omega_n)$ = forced-frequency ratio

$\mu = (m/M)$ = mass ratio

Then Equations [8] and [9] become

$$\theta_0 = \frac{(T_o/K_t)\,[1 - g^2]}{[1 - g^2]\left[1 - \left(\frac{g}{f}\right)^2\right] - 4\,\mu\left(\frac{g}{f}\right)^2\left(\frac{g^2}{N^2+1}\right)}$$

$$r_0 = \frac{\left(\frac{T_0R}{K_t}\right)\left[\frac{2}{N}\left(\frac{g}{f}\right)^2\right]}{[1 - g^2]\left[1 - \left(\frac{g}{f}\right)^2\right] - 4\,\mu\left(\frac{g}{f}\right)^2\left(\frac{g^2}{N^2+1}\right)}$$

[3] "Practical Solution of Torsional Vibration Problems," by W. K. Wilson, John Wiley & Sons, Inc., New York, N. Y., second edition, vol. 2, 1941, p. 570.

[4] Associate Professor of Mechanical Engineering, University of Wisconsin, Madison, Wis. Mem. ASME.

If the tuning of the absorber is correct, $f = 1$, and these equations reduce to

$$\theta_o = \frac{\left(\frac{T_0}{K_T}\right)[1 - g^2]}{[1 - g^2]^2 - \frac{4\,\mu g^4}{N^2 + 1}}$$

$$r_o = \frac{\left(\frac{(T_0 R)}{K_T}\right)\left[\frac{2g^2}{N}\right]}{[1 - g^2]^2 - \frac{4\,\mu g^4}{N^2 + 1}}$$

The resulting frequency equation for the combined system with exact absorber tuning is then

$$g^4\left[1 - \frac{4\mu}{N^2 + 1}\right] - 2g^2 + 1 = 0$$

with the solution

$$g^2 = \frac{1 \pm \sqrt{\frac{4\,\mu}{N^2 + 1}}}{1 - \frac{4\,\mu}{N^2 + 1}}$$

The exciting frequencies corresponding to infinite amplitudes of the system are seen to be a function of the mass ratio and the order of excitation. It is apparent that for high mass ratios and low orders of excitation the factor $\left(\frac{4\,\mu}{N^2 + 1}\right)$ may be greater than unity. Since g^2 can only be positive, this condition apparently would result in a single resonant frequency. In general, however, there would be two resonant frequencies as indicated.

Author's Closure

The author wishes to thank Professor Harker for his comments. The simplification of Equations [8] and [9] and subsequent derivation of an expression for the resonant frequencies added by the absorber is worth while in view of the fact that Equation [12] given in the paper for this purpose is rather tedious to solve.

To avoid confusion in the use of these equations it is necessary to realize that they apply only if the original main system has but a single degree of freedom. Also, the manner in which the exciting torque acts on the system must be defined. If the excitation is of a definite order, its frequency varies with angular velocity. The natural frequency of the absorber system and radius of the absorber mass, which vary with angular velocity, are then functions of exciting frequency. Equation [12] applies to such a case. On the other hand, the excitation may not be of a definite order, but may vary in frequency at constant angular velocity. Then the natural frequency of the absorber system and radius of the absorber mass remain unchanged. In this instance the absorber tuning factor f will equal unity at all times as assumed in Professor Harker's derivation. Inspection of the equations indicates that there is not too much difference in resonant-frequency values obtained by proceeding upon either assumption, and values obtained in either manner will be approximately correct for the other case.