An Efficient Low-Rank Separable T-Matrix Formalism and Its Application to the Reid Soft Core Potential

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We will show that an extremely low-rank separable t-matrix can reproduce the $^1S_0$ and the $^1S_1-^3D_1$-state phase shifts and the mixing parameter of the nucleon-nucleon interaction. The full off-shell scattering amplitudes are compared with current work by Adhikari and Sloan. Our results in rank 2 are comparable with their rank 3 results. In Fig. 1, the off-shell t-matrices at $E=0$ MeV are compared for the off-shell momentum $p'=1.4$ fm$^{-1}$. Figures 2 and 3 illustrate to $^1S_1-^3D_1$ phase shifts and the corresponding mixing parameter, where dashed lines show our rank 1 approximation, dash-dotted lines are our rank 2 approximation by means of Eq. (3·1) of Ref. 2 and solid lines are our rank 3 approximations and the GSE exact rank 1 values. Furthermore, the GSE* exact rank 1 results are better than Adhikari-Sloan rank 2~4 calculations for the $^1S_1-^3D_1$ phase shifts and the mixing parameter.

Our formalism has been described in Refs. 2) and 3), so in this paper we would like to point out the following: The Kowalski-Noyes method then has the defect of a pinching singularity in the negative energy region. This defect does not occur in our formalism. If one extends the Kowalski-Noyes (K-N) method to the negative energy region in the case of the Yukawa potential:

$$V(p,p') = \frac{C}{pp'} Q(\frac{p^2+p'^2+\mu^2}{2pp'}) , \quad (1)$$

then a pinching singularity will appear in the $k$-plane ($k=\sqrt{mE}$: on-the-energy-shell momentum) at $p=0$ and $k=i\mu$.

**Proof**
The logarithmic cuts of Eq. (1) are given by

$$p'^2+p'^2+\mu^2=2pp't \cdot (\frac{1}{t}) \quad (2)$$

Since the K-N method takes

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* Abbreviation to generalized separable expansion.
as the separable potential, the cuts of $V(p, k)$ are given by using Eq. (2) by

$$V(p, k) V(k, p') = V(k, k)$$

(3)

This is illustrated in Fig. 4 by the solid lines $\overline{PQ}$ (for $p=0$), $\bar{B}_1C_1$ (for $p=\mu/2$), $\bar{B}_2C_2$ ($p=\mu$), $\bar{B}_3C_3$ ($p=3\mu/2$) and so on. Furthermore, $V(k, k)$ has the so-called left hand cuts (shown in Fig. 4 by cross-hatched lines) from $i\mu/2$ to $i\infty$ and from $-i\mu/2$ to $-i\infty$. Therefore, the logarithmic cuts $\overline{PQ}$ pinch the left hand cuts at $k=i\mu$ and $p=0$. In the three-body case, this difficulty becomes clearer and is unavoidable. The two dashed lines $s\to0\to a$ and $s\to0\to b$ illustrate conventional integral contours in the three-body Faddeev calcula-
lation. Each suffers from a pinching or a cut.

In our formalism, however, \( k \) in Eq. (3) is replaced by a parameter \( k_1 \geq 0 \), and no difficulty appears. For convenience, the first \( (k_1) \), the second \( (k_2) \) and the third \( (k_3) \) parameters are given in the Table for the \( ^1S_0 \)-state of the Reid soft core potential.

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