Energy Spectra of High Energy Neutrinos and Cosmic Rays from Pulsars

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(Received June 4, 1979)

High energy neutrinos of $10^{12} \sim 10^{16}$ eV may be generated during a few months after the birth of a pulsar, through the nuclear collisions of high energy particles generated at the pulsar with dense envelope matter. The permissible maximum flux of such neutrinos is estimated phenomenologically under a constraint that the flux of the particles ejected into the Galactic pool does not exceed the observed flux of the Galactic cosmic rays. For reasonable choices of the parameters, the neutrino flux is too small to be detectable by the DUMAND project.

§ 1. Introduction

The DUMAND (Deep Underwater Muon And Neutrino Detector) project aims to detect high energy neutrinos using the sea water of cubic kilometers at 6 km below the surface of the ocean. It was first discussed by Berezinsky that such high energy neutrinos could be generated at the birth time of a pulsar through the nuclear collisions of high energy particles with dense envelope matter. Several authors have further discussed this process, but their estimates of the neutrino flux do not well agree with each other.

The reason for these diverse flux estimates comes from lack of a reliable model concerning high energy particle generation in a pulsar. In this paper, therefore, we try to derive the permissible maximum flux of high energy neutrinos under a phenomenological constraint that the flux of the particles which are ejected into the Galactic pool in the transparent phase of the expanding envelope does not exceed the observed flux of the Galactic cosmic rays in all regions of energy spectrum. Even if we put such a constraint, the flux estimate still contains various undetermined parameters, among which the initial braking time of the pulsar rotation seriously affects the result. For reasonable choices of the parameters, the neutrino flux seems to be too small to be detectable by the DUMAND project.

In § 2, we introduce a phenomenological model concerning the generation of high energy particles in a pulsar, where the energy flux of high energy particles is assumed to be a constant fraction of the total energy flux. In § 3, we derive the energy spectrum of the particles ejected outside the supernova envelope, which should be compared with that of the Galactic cosmic rays. In § 4, using the phenomenological constraint stated above, we estimate the high energy neutrino
In Appendices A and B, we discuss some related problems such as the energy dependent confinement time of cosmic rays and the spectral power index of the ejected particles.

§ 2. Generation of high energy particles by a pulsar

In order to discuss a relation between the cosmic rays and the high energy neutrinos which may be generated during a few months after supernova explosion, we need a model about the generation of high energy particles by a pulsar. Since the aim of this paper is to give a constraint on the flux of these high energy neutrinos phenomenologically, we use a rather crude model containing many unfixed parameters which are explained in the following:

(i) The energy loss rate of a pulsar

The loss rate of rotational energy of a pulsar, $L(t)$, is supposed to have a form

$$L(t) = \frac{d}{dt} \left( \frac{1}{2} I \Omega^2 \right) = a \Omega^{k+1}, \quad (2.1)$$

where $a$ and $k$ are constant parameters, $I$ and $\Omega$ being the moment of inertia and the angular velocity of pulsars, respectively. Equation (2.1) gives

$$\Omega(t) = \frac{\Omega(0)}{(1 + t/\tau)^{(k-1)/k}}, \quad (2.2)$$

$$L(t) = \frac{L(0)}{(1 + t/\tau)^{(k+1)/(k-1)}}, \quad (2.3)$$

where $t=0$ is the birth time of a pulsar and $\tau$ is the braking time of rotation. According to the theory, $k$ and $\tau$ are given for magnetic dipole radiation

$k=3$, \[ \tau_m \approx 1.3 \times 10^7 \left( I / 10^{45} \text{ g cm}^2 \right) \left( m_\perp / 4 \times 10^{30} \text{ G cm}^3 \right)^{-2} (\Omega(0) / 10^4 \text{ sec}^{-1})^{-2} \text{ sec}, \quad (2.4a) \]

and for gravitational quadrupole radiation

$k=5$, \[ \tau_\perp \approx 1.2 \times 10^8 \left( I / 10^{45} \text{ g cm}^2 \right) \left( D_\perp / 2 \times 10^{30} \text{ g cm}^3 \right)^{-2} (\Omega(0) / 10^4 \text{ sec}^{-1})^{-4} \text{ sec}, \quad (2.4b) \]

where $m_\perp$ and $D_\perp$ are the magnetic dipole moment and the mass quadrupole moment perpendicular to the rotation axis, respectively. Ostriker and Gunn took $\tau_\perp \approx 1.2 \times 10^8 \text{ sec}$ in order to explain the age of the Crab pulsar. From the observational data of $\Omega$ and $\dot{\Omega}$ for many pulsars, the best fit values are $k=3.4$ and $\tau \approx 0.3 \text{ years}$ for $\Omega(0) = 10^4 \text{ sec}^{-1}$.\footnote{In the case of the Crab pulsar, however, the data of $\Omega$, $\dot{\Omega}$ and $\ddot{\Omega}$ gives $k \approx 2.6$.}
The energy spectrum of the accelerated particles, which we call the injection spectrum, is assumed in a form 

\[ Q(E, t) dE = \begin{cases} A E^{-\gamma} dE & \text{for } E_i < E < E_u \\ 0 & \text{otherwise} \end{cases}, \tag{2.5} \]

where \( Q(E, t) dE \) is the number of particles injected in unit time and in the energy range between \( E \) and \( E + dE \). The lower and upper cutoff energies are assumed to change as

\[ E_u(t) = E_u(0) \left( \frac{\Phi(t)}{\Phi(0)} \right)^{\delta} \tag{2.6} \]

and

\[ E_i(t) = E_i(0) / E_i(0). \]

Such a power law dependence of \( E(t) \) on \( \Phi(t) \) will be reasonable according to the models of acceleration by a pulsar; for example,

\[ E \approx 10^{19} (\Phi/10^4 \text{ sec}^{-1})^{1/2} B_{12} \text{ eV}, \tag{2.7a} \]

in the Goldreich-Julian model and

\[ E \approx 2 \times 10^{19} (\Phi/10^4 \text{ sec}^{-1})^{1/2} B_{12}^{2/3} \text{ eV}, \tag{2.7b} \]

in the Gunn-Ostriker model, where \( B_{12} = B/10^{12} \) gauss is the magnetic field strength on the surface of a neutron star. For other models, similar dependences also hold.

Constant efficiency of the particle acceleration in \( L(t) \)

We assume a constant ratio \( \varepsilon \) of the energy flux of accelerated particles \( L_i(t) \) to \( L(t) \) and put

\[ \varepsilon L(t) = L_i(t) = \int_{E_i(t)}^{E_u(t)} Q(E, t) E dE. \tag{2.8} \]

From Eqs. (2.5), (2.6) and (2.8), we get

\[ A = (\gamma - 1) \varepsilon L(t) E_i(t) \gamma^{-1} / (1 - (E_i(0) / E_u(0))^{\gamma^{-1}}). \tag{2.9} \]

The injection number flux \( Q(t) \) from a pulsar is

\[ Q(t) = \int_{E_i(t)}^{E_u(t)} Q(E, t) dE. \]

In Appendix B we will discuss the power index in comparison with the argument in Ref. 12).
§ 3. Spectrum of high energy particles from pulsars

Here, we discuss the energy spectrum of the particles which have been ejected from a pulsar and accumulated in the galactic pool. The number of particles, in the energy range between $E$ and $E + dE$, ejected during the whole life of the pulsar is given

$$Q(E) dE = \int_{t_1}^{t_2} Q(E, t) dt dE,$$  \hspace{1cm} (3.1)

where $t_1$ and $t_2$ are defined from Eqs. (2.2) and (2.6) by

$$E = E_i(t_1) = E_i(0) / (1 + t_1/\tau)^{\beta/(k-1)},$$  \hspace{1cm} (3.2)

and

$$E = E_u(t_2) = E_u(0) / (1 + t_2/\tau)^{\beta/(k-1)}.$$  \hspace{1cm} (3.3)

Then $Q(E)$ is easily calculated as

$$Q(E) = \frac{(\gamma-1)}{(1+\beta(\gamma-1)/2)} \frac{\varepsilon W}{(E_i(0))^2} \cdot g(E)$$  \hspace{1cm} (3.4)

and

$$g(E) = \begin{cases} (E/E_i(0))^{(q+1)-1} [1 - (E_i(0)/E_u(0))^{(q+1)/(q-1)}] [1 - (E_i(0)/E_u(0))^{\gamma-1}]^{-1} & \text{for } E < E_i(0), \\ (E/E_i(0))^{-(q+1)} [1 - (E/E_u(0))^{(q+1)/(q-1)}] [1 - (E_i(0)/E_u(0))^{\gamma-1}]^{-1} & \text{for } E_i(0) < E < E_u(0), \\ 0 & \text{for } E > E_u(0), \end{cases}$$  \hspace{1cm} (3.5a)

where $W$ is the total energy output from a pulsar and is given by

$$W = (k-1) L(0) \tau/2.$$  \hspace{1cm} (3.6)

The behavior of $Q(E, t)$ and $Q(E)$ is schematically depicted in Fig. 1. The spectrum given by (3.5a) increases toward the lower energy if $\beta > 1$. However, we should remember that the spectrum has the lower cutoff of order of

$$E_i(t_i) \approx E_i(0) \left( \frac{\tau}{t_i} \right)^{\beta/(k-1)},$$

$t_i$ being the lifetime of an active pulsar. For $t_i \sim 10^6$ years, $\tau \sim 1$ year, $k = 3$ and $\beta = 4/3$, $E_i(t_i) \approx 10^{-4} E_i(0)$.

Considering the steady balance of supply and loss of the high energy particles in the Galactic pool, the particle flux, $\phi(E)$, is given as

$$\phi(E) dE = \frac{1}{4\pi} \frac{Q(E)}{V_G} \left( \frac{\tau_{CR}}{\tau_{SN}} \right) c dE,$$  \hspace{1cm} (3.7)
Fig. 1. Time variations of the injection spectrum $Q(E,t)$ and the spectrum integrated during the pulsar's life $Q(E)$ are schematically depicted.

Fig. 2. Number fluxes of high energy particles are displayed for the parameter $\tau=2, E_{\nu}(0)/E_{\nu}(t_0)=10^6, \delta=8$ and $\varepsilon=10^{-6}$. The observed flux of cosmic rays is also depicted which is adopted by Margolis et al.\cite{9} and is given as\cite{9,10,11}

\[
\phi_{\text{CR}}(E) = \begin{cases} 
2.6 \times 10^{-7} E^{-2.75} & (E<10^6 \text{ GeV}) \\
4.4 \times 10^{-5} E^{-3.11} & (10^6 < E < 10^9 \text{ GeV}) \\
5.1 \times 10^5 E^{-1} & (E>10^9 \text{ GeV}).
\end{cases}
\]

where $V_G$, $\tau_{\text{CR}}$ and $\tau_{\text{SN}}$ are the volume of the Galaxy, the confinement time of the particles and the rate of supernova explosion in the Galaxy, respectively. $\tau_{\text{CR}}$ in high energy part is simply assumed to be constant in energy and is taken as $\tau_{\text{CR}}=10^6$ years, though it has been checked as $\tau_{\text{CR}}=10^{6-7}$ years only in low energies, i.e., $10^8 \sim 10^{11}$ eV.\cite{10} Taking $V_G=10^6 \text{ cm}^3$ and $\tau_{\text{SN}}=10^9$ sec, we get the following expressions:

\[
\phi(E) = 4.5 \times 10^{-38} \varepsilon (W/10^{22} \text{ erg}) (\tau_{\text{CR}}/10^6 \text{ years}) (E_0(0)/10^6 \text{ GeV})^{-2} g(E) \quad (\text{GeV} \cdot \text{cm}^2 \cdot \text{sec} \cdot \text{st})^{-1}, \tag{3.8}
\]

where we normalize $E_0(0)$ by $10^6$ GeV and put $\gamma=1$. In Eq. (3.8), we have neglected a distribution of the parameter's values among sources, though this distribution may be effective.\cite{14} As an example, we show $\phi(E)$ for $\delta=8$ and $\varepsilon=10^{-6}$ in Fig. 2. The observed cosmic rays flux $\phi_{\text{CR}}(E)$ is also depicted in Fig. 2.
If we impose the constraint that the flux $\phi(E)$ should be less than that of Galactic cosmic rays $\phi_{\text{CR}}(E)$, we can derive the condition on $\varepsilon$ for each range of $\delta$ as follows:

$$
\varepsilon \leq \begin{cases} 
4.4 \times 10^{-1.75} \alpha & \text{for } \delta \leq 4, \\
10^{-1.59} \alpha & \text{for } 4 \leq \delta \leq 6, \\
10^{3-1.18} \alpha & \text{for } 6 \leq \delta ,
\end{cases}
$$

(3.9)

where

$$
\alpha^{-1} \equiv (W/10^{38} \text{ erg}) (\tau_{\text{CR}}/10^{6} \text{ years}) (V_{G}/10^{24} \text{ cm}^{3})^{-1}
$$

is a parameter of order of unity. This is a more elaborated version of the argument done in Ref. 3), where $\varepsilon$ was written as $A$.

An interesting point in Eq. (3.5a) is that the power index of the energy spectrum cannot become below $-2$, since it is natural to assume $\beta \geq 0$, $(2/\beta) - 2 \geq -2$. Relating to this point, the condition on the energy dependence of the confinement time will be discussed in Appendix A.

§ 4. High-energy neutrino spectrum from pulsars

The generation of high energy neutrinos at the birth time of a pulsar has been discussed by several authors. However, the flux estimate contains many uncertainties and the energy spectrum has not been studied in details. In this section, we estimate the neutrino flux under the phenomenological constraint that the flux of high energy particles does not exceed the observed flux of cosmic rays, using the model mentioned above.

As the pulsar is surrounded by a dense envelope for a while, the accelerated particles cannot escape to the outer space until $t_{c}$ after the explosion, where $t_{c}$ is given as

$$
t_{c} = (3M\delta/4\pi u^{2} m_{p})^{1/2} = 3.4 \times 10^{4} (M/M_{\odot})^{1/2} (u/10^{9} \text{ cm s}^{-1})
\times (\sigma/4 \times 10^{-26} \text{ cm}^{2}) \cdot \text{sec},
$$

(4.1)

$\sigma$, $u$ and $M$ being the cross section of proton-nucleon interactions, the expansion velocity and the total mass of the envelope, respectively. Before $t_{c}$, the accelerated particles collide with the envelope matter to produce neutrinos through meson decays or by direct lepton production.

Here we simply assume that the high energy particle of energy $E$ is converted into the neutrinos of energy $\varepsilon = \xi E/m$ with multiplicity $m$ of pion production in $p$-Nucleon collisions, $\xi$ being a fraction of the energy transferred into neutrinos. Thus the energy spectrum of neutrinos $Q_{\nu}(\varepsilon)$ is given as

$$
Q_{\nu}(\varepsilon) = \int (\bar{Q}_{p}(E) m \delta (\varepsilon - \xi E/m) dE.
$$

(4.2)
$Q_p(E)$ is the total number of particles ejected between $t=0$ and $t=t_0$;

$$Q_p(E) = \int_0^{t_0} Q(E, t) \, dt = \left[ (\tau - 1) \left( 1 + \frac{\beta}{2} (\tau - 1) \right) \right] \times \varepsilon W f(E) / E_1(0)^{2},$$

where

$$f(E) = (E/E_1(0))^{(\alpha - \beta - 1)} \left[ 1 - (E_c(t_c) / E_1) \right]^{(\alpha / \beta - 1 - 1)} \left[ 1 - (E_c(0) / E_1(0))^{(\alpha / \beta - 1)} \right]^{-1},$$

for $E_1(t_c) < E < \operatorname{Min}(E_1(0), E_c(t_c))$,

$$= (E/E_1(0))^{-1} \left[ 1 - (E_c(t_c) / E_1(0)) \right]^{(\alpha / \beta - 1)} \left[ 1 - (E_c(0) / E_1(0))^{(\alpha / \beta - 1)} \right]^{-1},$$

for $E_1(0) < E < E_c(t_c)$ in the case $\operatorname{Min}(E_1(0), E_c(t_c)) = E_1(0)$,

$$= g(E) \text{ for } E < E_c(t_c). \quad (4.3)$$

If we denote the multiplicity by $m = \overline{m} E^{\frac{\mu}{2}}$, the neutrino flux from each pulsar is given by

$$\bar{Q}_d(E) = \frac{\bar{Q}_d \left( \left( \frac{\overline{m}}{\xi} E \right)^{1/(1-\mu)} \right)}{(1 - \mu) \xi} \left( \frac{\overline{m}}{x} E \right)^{2/(1-\mu)}. \quad (4.4)$$

In Fig. 3, we schemetically display $Q_p(E)$ and $Q_d(E)$, where $Q_p(E)$ is the number of high energy particles ejected into the outer space after the time $t_c$, that is,

$$Q_p(0) = \bar{Q}_d(E) - \bar{Q}_p(E). \quad (4.5)$$

The flux of the neutrinos ejected during the cosmic time $1/H$ is given as

$$\phi_d(E) dE = \frac{Q_d(E)}{4\pi} \delta_T \left( c / H \right) dE$$

$$= 5.8 \times 10^{-34} \times \frac{\xi}{\xi} \left( 10^{52} \text{ erg} \right) \left( E_1(0) / 10^{5} \text{ GeV} \right)^2$$

$$\times \left( \frac{\delta_T}{2 \times 10^{-27} \text{ yr}^{-1} \text{ cm}^{-1}} \right) \left( \frac{H}{50 \text{ km sec}^{-1} \text{ Mpc}^{-1}} \right)^{-1} dE \text{ cm}^{-2} \text{ sec}^{-1} \text{ ster}^{-1}, \quad (4.6)$$

Fig. 3. The energy spectra $Q_p(E)$ and $Q_d(E)$ are displayed, normalizing at the high energy part of $Q_d(E)$. The parameters adopted are $\tau = 2$, $E_\alpha(0) / E_c(0) = 10^4$, $E_c(0) = 10^4 \text{ GeV}$, $\xi = 0.1$, $\mu = 0$ and $m = 1$, where we adopt the leading pion model in $p-N$ collision. And these parameters are also adopted in Figs. 4~6. The case $\operatorname{Min}(E_c(t_c), E_\alpha(t_c)) = E_c(t_c)$ is depicted, where we put $E_c(t_c) / E_\alpha(0) = 10^{-5} \cdot 1$ and $\epsilon = 10^{-4}$. 
where $\delta_T$ and $H$ are the supernova rate per unit volume and Hubble constant respectively. In Eq. (4.6), we neglect the cosmological evolutionary effects. In Figs. 4 and 5, we show how the neutrino flux $\phi_p(E)$ is restricted by the accompanying particle flux $\phi_p(E)$, which should be bounded by the Galactic cosmic rays. In Fig. 6, the maximum neutrino fluxes under the above constraint are given for various choices of the parameters, which are given in the figure captions. The ratios $t_c/\tau$ and $E_i(0)/E_i(t_c) = (1 + t_c/\tau)^{\beta/(k-1)}$ are the most sensitive parameters for the estimate of $\phi_p(E)$. When $t_c/\tau \approx 1$ and $E_i(0)/E_i(t_c) \approx 1$, $\phi_p(E)$ is very small compared with $\phi_{CR}(E)$. But, when $t_c/\tau \gg 1$ and $E_i(0)/E_i(t_c) \gg 1$, $\phi_p(E)$ becomes comparable with or greater than $\phi_{CR}(E)$, as shown in Fig. 6.

However, for a probable case such as $t/\tau \leq 1$, $\beta = 2$ and $k = 3$, $\phi_p(E)$ is very small and the detection of these neutrinos by the DUMAND project seems very difficult as seen in Fig. 7. In Fig. 7, the neutrino flux estimate derived by
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Fig. 6. The neutrino flux \( \phi \) is displayed for the following cases:

1. \( k=3, \beta=2/3, t_e/\tau=1 \) (Fig. 4)
2. \( k=3, \beta=2, t_e/\tau=1 \)
3. \( k=3, \beta=2/3, t_e/\tau=10^4 \)
4. \( k=3, \beta=2, t_e/\tau=10^4 \)
5. \( k=3, \beta=4, t_e/\tau=10^4 \)
6. \( k=2, \beta=2, t_e/\tau=10^8 \) (Fig. 5)

Margolis et al.\(^a\) is also shown.

We can write the ratio of the total neutrino energy output \( \Delta \varepsilon \) to the total energy output \( \Delta \varepsilon_p \) of high energy particles from a pulsar as

\[
\frac{(\Delta \varepsilon)}{(\Delta \varepsilon_p)} = \xi \left( \frac{E_t(t_e)}{E_t(0)} \right)^{-\gamma/\beta}
\]

\[
= \begin{cases} 
\frac{\xi}{\tau} & \text{for } t_e/\tau \lesssim 1 \\
\xi \left(1 + \frac{t_e}{\tau} \right)^{\gamma/(k-1)} & \text{for } t_e/\tau \gtrsim 1
\end{cases}
\]

Thus, the ratio \( E_t(t_e)/E_t(0) \) and/or \( t_e/\tau \) characterize the \( \phi/\phi_{CR} \) ratio in Figs. 6
and 7. The $\phi_e/\phi_p$ ratio is given from Eqs. (3·7) and (4·6) as

$$\frac{\phi_e(E)}{\phi_p(E)} = \left[ \frac{Q_e(E)}{4\pi} \frac{\delta_\tau}{H/} \right] \left[ \frac{Q_p(E)}{4\pi V_G \tau_{SN'}} \frac{\tau_{CR}}{c} \right]^{-1}$$

$$\simeq 1.2 \times 10^{-3} \frac{Q_e(E)}{Q_p(E)} \left( \frac{10^9 \text{ years}}{\tau_{CR}} \right).$$

(4·7)

This ratio depends on the confinement time of cosmic rays in the Galaxy.$^{35,13}$ If the confinement time becomes shorter with $E$, the neutrino flux could become much greater.

Through Figs. 3~7, we have assumed $E_t(0) = 10^8 \text{ GeV}$ or $\delta = 8$ referring to the theoretical estimate such as Eq. (2·7). If we take different values of $\delta$, the flux $\phi_e$ varies following Eq. (4·6).

Acknowledgement

The authors would like to express their thanks to Professor C. Hayashi for his continuous encouragement and interest.

Appendix A

——The Energy Dependent Confinement Time——

The confinement time of cosmic rays in the Galaxy is poorly known except for the low energy region like $10^8$~$10^{11}$ eV. It has been expected that $\tau_{CR}$ becomes shorter for the high energy region as $\tau_{CR} \propto E^{-\gamma},^{13,19}$ where $\gamma$ is a positive parameter. If this is true, we might be able to explain the power index of the cosmic rays, $\gamma_{CR}$, in such a way as

$$\gamma_{CR}(E) = \frac{2}{\beta} - 2 - \gamma(E).$$

Referring to the observed spectrum given in the caption of Fig. 2, this equation gives the condition for $\gamma$ as

$$\gamma \geq 0.35 \quad \text{for} \quad 10^4 < E < 10^6,$$

$$\gamma \geq 1.1 \quad \text{for} \quad 10^6 \text{ GeV} < E.$$ 

And if confinement time becomes as small as $10^3$ years at $10^{18}$ eV,$^{18}$ the expected neutrino flux could increase by $10^3$, since the $\phi_e/\phi_p$ ratio depends on $\tau$ as in Eq. (4·7).

Appendix B

——Spectral Power Index——

Ostriker$^{10}$ derived the cosmic rays spectrum on the assumptions

$$n_t = A \Omega^\alpha, \quad E = B \Omega^\beta, \quad \dot{Q} = C \Omega^\delta,$$

(B·1)
where $n_i$ is the injection rate. Since the number of particles emitted in a given energy interval is related to the corresponding time interval as

$$N(E) dE = n_i(t) \frac{dE}{d\Omega}^{-1} dE,$$

we have

$$N(E) dE = DE^\alpha dE$$

and

$$\phi = (\alpha + 1 - \beta - k)/\beta.$$  \hspace{1cm} (B.2)

using the relations (B.1). He derived $\phi = -2.5$ assuming $\alpha = 2$, $\beta = (4/3)$ and $k = 5$; $\beta = (4/3)$ was derived from his acceleration theory and $k = 5$ was adopted assuming the braking by the gravitational radiation.

In Eq. (2·8), we have assumed the constant efficiency of the particle energy flux in the total flux, that is,

$$n_iE = \text{const} \times \Omega \dot{\Omega}.$$  

Therefore we have put such a restriction as $\alpha + \beta = k + 1$. So we obtain $\phi$ as

$$\phi = (\alpha + 1 - \beta - k)/\beta = \frac{2}{\beta} - 2 \geq -2,$$  \hspace{1cm} (B.3)

which is the same power index with that of Eq. (3·5a).

In the very early stage, the braking of rotation may be mainly due to the gravitational radiation. In such a case, however, the particle energy flux will not be proportional to the total energy flux but will be proportional to the magnetic dipole radiation flux $L_m$. Since $L_m \propto \Omega^4$, we have that $n_iE \propto \Omega^4$ or $\alpha + \beta = 4$ in spite of $\dot{\Omega} \propto \Omega^2$ for the gravitational radiation. Thus we have the power index in this case as

$$\phi = \frac{\alpha + 1 - \beta - k}{\beta} = -2.$$

If we incorporate the energy dependent confinement time, we might be able to modify this to the observed power index $\gamma_{CR} = 2.5^{+0.2}_{-0.19, 0.20}$.

References

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