These forms are exact, and have the advantage that only two Bessel function "look-ups" are required.

D Stator Only. When considering a stator only, no variation in stagnation enthalpy occurs, so equation (19) reduces to

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2} = -\frac{1}{2r}\frac{\partial}{\partial z}\frac{\partial (rv)^2}{\partial \psi}$$

In the example considered in the text, it was assumed that

$$(rv)^2 = (-k\psi + w_{-\infty}C_s)^2$$

It can be seen that the above equation for the radial velocity will remain as before, if a further term is added to give

$$(rv)^2 = (-k\psi + w_{-\infty}C_s)^2 + \delta w_{-\infty}\psi$$

Equation (18) then gives for the matching condition across the disk

$$\left[\frac{\partial u}{\partial z}\right]_0 = -\frac{k\Psi^2_0}{r} + \frac{k}{r}\left(C_s - \frac{\delta}{2k}\right)$$

Comparison with equation (25) then shows that the radial and axial velocity distributions for this case of a three parameter stator may be obtained directly from the two parameter stator only solutions, simply by replacing the previous C_s by $C_s - \frac{\delta}{2k}$. Obviously, the tangential velocity will change both directly from the addition of the term to the expression for $(rv)^2$ as given above, and through its effect on the axial velocity which appears when describing the stream function as in equation (42). The latter effect is accounted for by again replacing C_s by $C_s - \frac{\delta}{2k}$ in the expression for Z_n for use in the series, and in the expression for $\Delta V_\infty{}^S$ (equation (52)).

No calculations for the three parameter stator were carried out.

References

1 Marble, Frank E., "Three-Dimensional Flow in Turbomachines, *Aerodynamics of Turbines and Compressors, High Speed Aerodynamics and Jet Propulsion,* Vol. X, Section C, Princeton, 1964.

2 Marble, Frank E., "The Flow of a Perfect Fluid Through an Axial Turbomachine with Prescribed Blade Loading," *Journal of Aeronautical Science*, Vol. 15, 1948, pp. 473–485.

3 Horlock, J. H., and Lewis, R. I., "Non-Uniform Three Dimensional and Swirling Flows Through Diverging Ducts and Turbomachines," International Journal of Mechanical Science, Vol. 3, No. 3, 1961, pp. 170–196.

4 Hawthorne, W. R., and Horlock, J. H., "Actuator Disc Theory of the Incompressible Flow in Axial Compressors," *Proceedings of the Institute of Mechanical Engineers*, Vol. 176, No. 30, 1962, pp. 789–814.

5 Bettner, J. L., "Experimental Investigation in an Annular Cascade Sector of Highly Loaded Turbine Stator Blading," NASA Contractor Report, No. CR-1219, Vol. I, 1968.

6 Bettner, J. L., "Experimental Investigation in an Annular Cascade Sector of Highly Loaded Turbine Stator Blading," NASA Contractor Report, No. CR-1323, Vol. II, 1969.

7 Bettner, J. L., "Experimental Investigation in an Annular Cascade Sector of Highly Loaded Turbine Stator Blading," NASA Contractor Report, No. CR-1341, Vol. III, 1969.

8 Bragg, Stephen L., and Hawthorne, William R., "Some Exact Solutions of the Flow Through Annular Cascade Actuator Discs," *Journal of Aeronautical Science*, Vol. 17, No. 4, 1950, pp. 243–249.

9 Oates, G. C., "Theory of Throughflow in Axial Turbomachines with Variable Wall Geometry," California Institute of Technology, Jet Propulsion Center Technical Note No. 1, AFOSR TN 59-680, Aug. 1959.

10 Heiser, W., "Modern Turbine Aerodynamics," Lecture notes for University of Tennessee Space Institute, *Advanced Air-Breathing Propulsion Systems and Technology Symposium*, December 1969.

11 Jahnke, E., and Emde, F., *Tables of Functions*, Dover, 1945.

12 Gray, A., and Mathews, G. B., *A Treatise on Bessel Functions and Their Applications to Physics*, Dover, 1966.

DISCUSSION

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It is a curious and often overlooked fact that there are very few analytical solutions available to guide students and designers of turbo-machinery. In this paper, Dr. Oates has shown considerable ingenuity in extending actuator disk theory to highly rotational, nonconstant work flows. Since this is precisely the direction being taken by industrial design teams in order to increase stage loading and efficiency (e.g., "The Application of Controlled Vortex Aerodynamics to Advanced Axial Flow Turbines," by T. E. Dorman, H. Welna, and R. W. Lindlauf, *Journal of Engineering for Power*, July 1968), his work is directly applicable to their efforts. His writing reflects an awareness of this fact, and his results are evidently arranged to shed light on such problems.

One would expect, therefore, that these highly transparent analytical solutions will replace considerable amounts of high speed machine computation for engineers who wish to develop an intuitive understanding of turbomachine flows. One would also hope that Dr. Oates will continue to explore these methods in order to further explain and clarify the behavior of a very important class of flows.

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