Proceedings of the Institution of Mechanical Engineers, vol. 170, 1956, p. 95.

8 D. F. Denny, "An Experimental Study Air-Entraining Vortices in Pump Sumps," *Proceedings of the Institution of Mechanical Engineers*, vol. 170, 1956, p. 106.

9 H. B. Squire and K. G. Winter, "The Secondary Flow in a Cascade of Aerofoils in a Nonuniform Stream," *Journal of the Aeronautical Sciences*, vol. 18, 1951, p. 271.

10 W. R. Hawthorne, "Secondary Circulation in Fluid Flow," *Proceedings of the Royal Society of London*, vol. 206, no. 1086, 1951, p. 374.

11 J. H. Preston, "A Simple Approach to the Theory of Secondary Flow," *Aeronautical Quarterly*, vol. 5, part 3, 1954, p. 218.

12 A. W. Marris, "The Generation of Secondary Vorticity in an Incompressible Fluid," *Journal of Applied Mechanics*, vol. 30, TRANS. ASME, vol. 85, Series E, 1963, p. 525.

DISCUSSION

A. W. Marris[3]

The author is to be complimented on an interesting paper.

This case of a swirling boundary flow in which the "draw off" streamlines were essentially parallel to the tank base plate was the case to which the writer's analysis may apply.[4] The author's experimental verification that the streamlines are of this form, Figs. 10 and 11, is gratifying.

The situation considered by the author would seem to be manifestly different from the case of tank drainage when the water elevation is relatively deep in the tank; in the latter case, in the author's words, the flow is similar to potential flow into a point sink. In this respect one refers to the perceptive physical discussion of Kelly, Martin, and Taylor [4], who point out that the central orifice is "selective" in that it tends to give priority of discharge to the fluid particles with the smaller angular momenta. At first, the fluid particles discharged will be those at small radii but as the discharge proceeds, fluid from increasing large radii will come to the orifice, and finally more and more fluid will come from the boundary layer on the bottom of the tank.

In the case of discharge when the water elevation in the tank is high, and particularly in the initial stages of such a discharge, the vertical z-direction component of the vertical velocity gradient will be effective in generating vertical z-vorticity, according to the equation

$$\frac{\partial \Omega_z}{\partial t} = (\Omega_z + 2\omega_z)\frac{\partial v_z}{\partial z} \qquad (26)$$

A first approximation of a constant $\partial v_z/\partial z$ gives an exponential growth of vorticity Ω_z with time, both from amplification of residual vorticity and from the earth's rotation.[5] The writer would like to ask the author if he obtained an indication of an exponential growth of vorticity with time during his experiments.

Kelly, Martin, and Taylor [4] also report that reversal of rotation could be generated as a result of shock to their apparatus, the tendency to reversal being more pronounced if the initial swirl were clockwise, i.e., in the opposite sense to that expected from the earth's rotation in the Northern Hemisphere. One wonders if any correlation of vorticity changes with vibration was observed in the author's experiments.

The author is to be commended on his paper.

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[4] A. W. Marris, "Secondary Flows in an Incompressible Fluid of Varying Density in a Rotating Reference Frame," JOURNAL OF BASIC ENGINEERING, TRANS. ASME, Series D, vol. 88, pp. 533–538.

[5] A. W. Marris, "Theory of the Bathtub Vortex," *Journal of Applied Mechanics*, vol. 34, TRANS. ASME, Series E, vol. 89, 1967, pp. 11–15.

Author's Closure

The author wishes to thank Dr. Marris for his constructive remarks and additions to this paper.

It is not easy to measure the vorticity when the depth of water is kept constant during the measurement, because a continuous inflow does not give a definite initial condition of the stream. The author has investigated simply the maximum vorticity of the bathtub vortex at the water surface.

When the drainage suction head is variable, the streamlines near the drain hole are not similar, owing to the variation of strength of sink. The greater part of the streamline has similar form to each other in spite of different strength of sink, as shown in Fig. 10. The relation of continuity is obtained by an assumption of a kind of "selective" discharge that fluid at small radii is discharged at first. Thus equation (20) is derived.

The vorticity growth and reversal are given by Sibulkin [3] and the author did not want to put any experimental data upon it.

The effect of vibration on the vortex reversal is not ascertained, and it should be investigated by a separate series of experiments considering the effect of amplitude and direction of vibration. It is expected that further investigations of the bathtub vortexes will be carried out with regard to the foregoing.