

Application of an optimal predictive controller for a small water distribution network in Luxembourg

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ABSTRACT

This paper deals with predictive control applied to the management of water storage in a small water distribution network. This online optimisation-based strategy is computed iteratively by solving a set of mathematical equations which describe the operative goals, in a given time horizon, and uses a representative model for the network dynamics as well as a demand forecast. The approach has been tested on a simulator developed for a four-reservoir water network of a commune of Luxembourg. Mathematical optimisation of the water distribution network is defined to account for all the requirements put forward by the authorities (both the commune and the regional drinking water provider as well as the national water agency), without ignoring the operating and physical constraints of the network. Based on realistic consumption scenarios, the starting situation (tanks always completely filled) and different control strategies (proportional integral derivative or PID level regulation or global predictive control) have been compared and the results are discussed. Moreover, the question of whether to integrate natural ventilation has been explored. Global predictive control leads to improved management in comparison to PID and mechanical control. Further work is needed to evaluate performance when goals, such as tank aeration, are to be met.

Key words | optimisation, predictive control, storage, ventilation, water distribution network

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INTRODUCTION

Because of the full cost recovery mandate in the EU-Water Framework Directive (CEC 2000), the price of drinking water in Luxembourg increased on 1 January 2011 and communes were encouraged to find ways to manage water in a way that would minimise costs. Wormeldange, a small Luxembourg wine-producing commune, took the initiative to modernise both the management of water storage and its water distribution network by evaluating advanced control strategies.

The present work focuses on a global predictive control (GPC) approach applied to real-time control of a water distribution network run by a commune, where tanks are connected only by the use of the gravity principle. The proposed GPC is an optimisation-based strategy inspired by model predictive control (MPC) theory. In this paper, the GPC method is also compared with a widely used proportional integral derivative (PID) control algorithm. In fact, optimisation and predictive control techniques provide

an important contribution to strategy computation in urban water network management: both sewer network control and management of drinking water supply and water distribution networks (WDN; Cembrano *et al.* 2004; Pleau *et al.* 2005; Brdys *et al.* 2008; Fiorelli & Schutz 2009; Ocampo-Martinez *et al.* 2009).

Let us put our work into perspective. GPC has been shown to be an efficient technique to deal with online management of WDN (Duzinkiewicz *et al.* 2005; Ocampo-Martinez *et al.* 2009, 2012; Skworcow *et al.* 2009). However, in most WDN, the electricity energy cost due to water pumping is the major component of the operation costs of the network. The optimisation problem is therefore equivalent to optimising the scheduling of pumps by using operational storage to shift pumping to times of low energy cost. The water quality is the other major preoccupation for drinking water operators, generally by

controlling chlorine concentration throughout the distribution network.

Naturally, integrated approaches to control quantity and quality in WDN have been proposed (e.g. Duzinkiewicz et al. 2005) which consist of optimising the operational cost, meeting demand for water of desired quality and maintaining the constraints. The leakage issue has been also integrated in the optimal operation of WDN, either by minimising the overall network pressure for leak prevention purposes or minimising the water loss when the network suffers leakage. Skworcow et al. (2009) reported an approach to combine energy and leakage optimisation and Wang & Brdys (2006) developed a strategy of softly switching between cost control, pressure control and leak control. A recent successful application of MPC to WDN has been realised in the city of Barcelona where the management criteria are: (1) minimising water production and transport costs; maintaining a stored volume in each tank around a given safety value; and (2) smoothing the control actions (Ocampo-Martinez et al. 2009, 2012).

The WDN of Wormeldange, the case study of this paper, is a gravity distribution network that allows the water to be transported without the need for energy. The drinking water comes from a regional water supplier and no 'raw' water taken from a communal spring is used so the chlorine concentration control is not managed by the commune. In this specific case study, only the stored volume and the distribution of water into the network have to be optimised. We also propose a simple but very powerful approach based on water fluctuation in order to ensure a sufficient water quality.

The paper is organised as follows. The following section describes the studied water distribution network as well as the requirements given by the operators. We then deal with the mathematical formulation and the methods to solve the optimal control problem. Numerical results are given, and we then address the integration of natural ventilation. Finally, some conclusions and outlooks are drawn.

CASE STUDY: THE WATER DISTRIBUTION NETWORK OF WORMELDANGE

Wormeldange is a small Luxembourg wine-producing commune located in the southeast part of Luxembourg on the

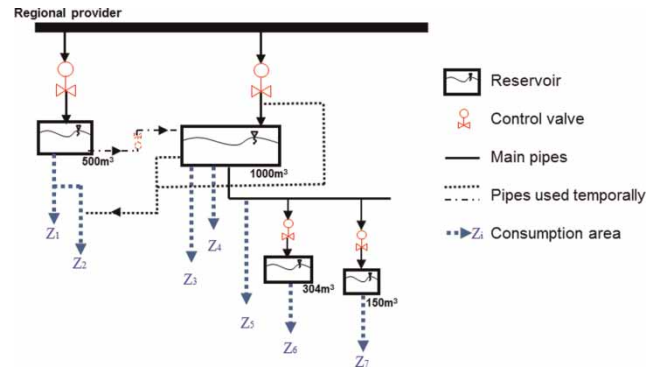


Figure 1 | WDN of Wormeldange, June 2011.

banks of the River Moselle. Its current network (Figure 1) is composed of four storage tanks that feed its customers with a total daily consumption averaging around 450 m³. However, the daily consumption can rise to more than 700 m³ during summer or the grape harvest season. From 2010, the consumed water volume was measured and recorded every 15 minutes for each consumption area Z_i ($i = 1, \dots, 7$).

A capacity reservation contract links the commune to the regional water supplier. If the commune exceeds a fixed daily water volume supply limit, defined for the case study at 690 m³, it is charged a high penalty. Avoiding this cost acts as the most important operation goal. However, other objectives desired by both the commune and the regional provider should also be taken into account in the control system design. The required operating objectives are:

1. Avoid exceeding the volume of the daily reserved capacity.
2. Maintain constant flow rates into the reservoirs.
3. Limit the residence time of the water in each tank.
4. Ensure natural (zero energy) ventilation in a tank, by using water level fluctuations.

Moreover, the control should satisfy the constraints:

5. Meet customer demand.
6. Always have a minimum water volume in each tank for fighting fire.
7. Do not overload capacity of pumps, valves, pipes and tanks.

Goal 2 tends to stabilise the control actions as valves should operate smoothly in order to avoid big flow changes

in the pipes that can lead to damage. Above all, having nearly constant flow rates makes the supply of water by the regional provider easier. Residence time has an impact on microbial activity and the taste of the water. Goal 3 integrates this aspect into the control problem. The longer time that the water is in the distribution system, the more likely it is to become contaminated. Directive DVGW-W 400 (2004) stipulates that the residence time in the global distribution system (from the water production plant to the consumer) should be limited to 7 days. The Luxembourg Water Agency recommends that communes limit residence time to around 3 days in their network. As in our case study there is no forced ventilation (ventilator) to prevent mould formation, and with this water contamination from condensed water, ventilation of the tanks is realised through water level fluctuations (Goal 4). The Luxembourg Water Agency advises the communes to have a daily air volume exchange in each tank close to 10% of the water volume.

In the mathematical formulation of the problem (next section), requirements 2 and 3 above are considered as cost objectives while goals 5–7 are integrated as constraints. Due to the high importance placed on cost, goal 1 is implemented as a constraint. Requirement 4 will be the subject of a special section in the section on ‘Natural ventilation issue’ below.

PROBLEM FORMULATION

The original water storage control in the Wormeldange network was rudimentary. A pure mechanical ballcock controlled the water volume $V(t)$ in each tank and the tanks were always completely filled, i.e. $V(t) = V_{\text{MAX}}$. Although this control ensures the maximum water volume in stock, it does not use the buffer capacity of the network since the consumption peaks impact directly on the tank’s inflows. The commune therefore sustained financial losses when the global daily consumption exceeded the daily reserved capacity.

In order to regulate the volume $V(t)$ of water storage in each tank, we replaced the float valves with automatically controlled valves. A daily recomputed setpoint $V_{\text{sp}}(d)$ was introduced as a function of both V_{ff} , the minimal volume reserved for fire fighting, and $\hat{C}(d)$, the average of the

consumption of the n previous days $C(d)$, defined:

$$\hat{C}(d) = \frac{1}{n} \sum_{k=d-n}^{d-1} C(k) \quad (1)$$

and

$$V_{\text{sp}}(d) = V_{\text{ff}} + K_{\text{sp}} \hat{C}(d) \quad (2)$$

where K_{sp} is a coefficient that indicates the mean duration in days of the tank to work in an autonomous way (i.e. without connection with the main supply pipe). Throughout this paper, we set $K_{\text{sp}} = 2$.

In order to control the water storage, we compared two approaches: one based on a PID controller, and the other based on a MPC technique which we call global predictive control (GPC), as it takes into account global objectives for the complete WDN.

In the PID method, the inflow rate of the tank $\text{In}(t)$ is adjusted depending on the magnitude, duration and rate of change of the error $\varepsilon(t)$ between the setpoint and the current measured volume:

$$\text{In}(t) = K_P \varepsilon(t) + K_I \sum \varepsilon(t) + K_D \Delta \varepsilon(t) \quad (3)$$

where $\varepsilon(t) = V_{\text{sp}}(d) - V(t)$ and K_P , K_I and K_D are the coefficients for magnitude, duration and rate of change of the error, respectively.

While PID control is based solely on local arguments and is inadequate to cope with efficient coordination of possible actions of all actuators over the network, GPC deals with an online global optimisation strategy. In other words, it iteratively minimises an objective function over a prediction horizon, subject to constraints on the system variables (Rawlings & Mayne 2009). The future behaviour is computed according to a model of the system, starting at the current time over a prediction horizon.

The reservoir operation is modelled using the conservation of mass:

$$V(t) = V(t-1) + \text{In}(t) - \text{Out}(t) \quad (4)$$

where $\text{In}(t)$ and $\text{Out}(t)$ are the water volume draining into and out of the tank during the time interval $\Delta t = [t-1, t]$ ($\Delta t = 15$ minutes in this case study). For simplicity, pipes and nodes are modelled as $\text{In}(t) = \text{Out}(t)$.

The model is intentionally chosen to be as simple as possible in order to keep the optimisation problem convex

linear-quadratic, which typically yields a unique global solution. An advantage of this approach is that it allows the direct integration of other objectives into the controller. The following sub-goals are taken into account in the optimisation problem:

1. Reach the setpoint $V_{sp}(d)$ at the end of the day.
2. Draining water coming from the main supply pipe homogeneously into the reservoirs of the network, i.e. the filling or emptying ratio, defined:

$$\frac{V(t) - V_0(d)}{V_{sp}(d) - V_0(d)} \quad (5)$$

should be the same for all reservoirs.

3. Avoid having a storage volume less than the minimal volume required for fire fighting.
4. Keep the water volume between the volume at the beginning of the day $V_0(d)$ and the setpoint $V_{sp}(d)$.

The first objective is obvious. Reaching the setpoint as fast as possible is not a priority. Conversely, keeping the inflow rate constant (second operative goal), it is better to reach the setpoint at the end of the prediction horizon (i.e. the end of the day). The second objective above is used to have an equitable distribution of the daily reserved capacity between the tanks according to their consumption. This objective is needed particularly in the case where the setpoints cannot be reached for every tank due to large consumption and/or insufficient daily reserved capacity. The third objective allows penalisation when the storage volume comes close to the minimal volume, and the last objective (4) avoids wrong behaviour when some tanks should empty out whereas others should fill up. Subgoals 3 and 4 are programmed as soft constraint:

$$\text{minimise } \varepsilon_s \text{ subject to } [V(t) + \varepsilon_s] \geq C_s \quad (6)$$

for $\varepsilon_s \geq 0$, $s = 1, 2, 3$ and $C_s = \{V_{ff}, V_0(d), V_{sp}(d)\}$ where a penalty term comprises the slack variables ε_s representing violations of the constraints C_s .

The hard constraints are linked to the technical characteristics and indicate that volume in reservoirs and flow rates in pipes are limited by a maximum capacity and can only be positive. Moreover, the daily reserved capacity is also modelled as a hard constraint (first operative goal)

since, as soon as the volume bought by the commune over a day reaches this limit, no more water drains into the network unless minimal volume for fire fighting is reached. We combine all objectives (1–4) above as well as constraints (previous section) in an optimisation problem which is formulated in standard form as:

$$\text{minimise } \|Ax - b\|_2 \text{ subject to } l \leq Dx \leq u \quad (7)$$

where x is a vector which consists of the inflow rate In and the slack variable ε_s ($s = 1, 2, 3$) for each tank.

Problem (7) is solved by using the Matlab-based modelling system for convex optimisation CVX (Grant & Boyd 2008) and/or its Python version CVXOPT (Dahl & Vandenberghe 2008). The development and the tests of the different control approaches have been made with the Matlab version. However, in order to provide to the commune an open-source solution, the Python version will be used for the implementation.

Solving the optimisation problem results in an inflow rate for each tank over the prediction horizon (i.e. up to the end of the day). The obtained result of the first control step (for the next $\Delta t = 15$ min) is applied, that is, until the next GPC loop. Due to the way the reserved daily capacity is specified (from 00:00 to 24:00), the prediction horizon is limited to the end of the current day. The length of the prediction horizon is therefore variable and decreasing by Δt at each iteration until the end of the day, which is not common in control theory.

Over the prediction horizon, the consumption is assumed to follow a typical pattern with a daily volume equalling $\hat{C}(d)$ (see Equation (1)), and $In(t)$ is assumed to be a constant (according to the second operating goal). Based on these assumptions, the expected trajectory of the water volume should fluctuate around a virtual linear trajectory from the current state to the setpoint at the end of the day (Figure 2). Such water level variations are important for natural ventilation of a tank.

RESULTS AND DISCUSSION

A simulator of the network was created using Matlab/Simulink environment. Simulations were performed based on

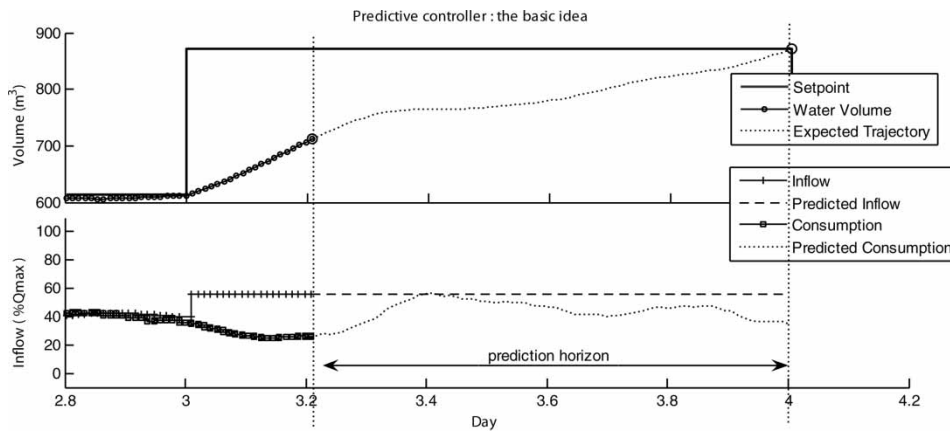


Figure 2 | Predictive control: the basic idea.

realistic consumption scenarios. In this paper, we focus only on the storage volume distribution over the drinking water network. We therefore do not need any specific software devoted to simulating a drinking water network such as EPANET or WATERCAD. In addition to the control algorithms, only the mass conservation principle (subgoal 4 in the previous section) was integrated into the Matlab/Simulink simulator. To illustrate the controller performances in this paper, a synthetic consumption scenario (see Figure 3) is used, representing a period of 16 days with 2 days of large consumption. Such a situation may result from the grape harvest or the filling of the municipal pool. Figures 4–6 depict the comparison of: (a) mechanical control; (b) PID control; and (c) GPC approach with respect to daily water volume brought

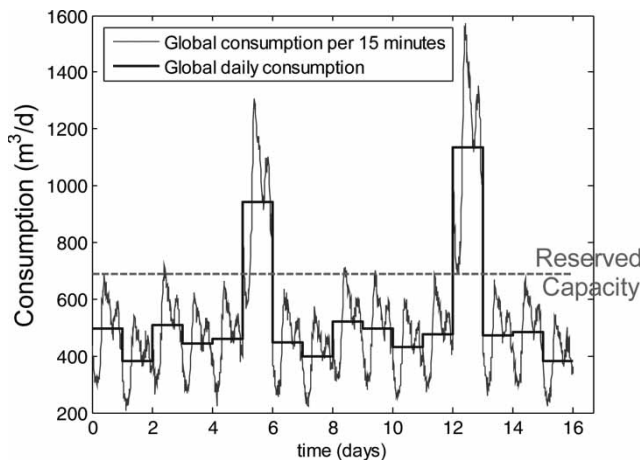


Figure 3 | Scenario consumption.

into the network, inflow rates to the tanks and residence time of the water in the tanks.

Mechanical control implies that commune consumption directly impacts on water brought into the commune's WDN, as there is no delay by using the tank volume to satisfy consumer demand (Figure 4(a)).

The peaks of consumption are attenuated when using the PID control approach, but the daily water volume brought into the commune can exceed the daily reserved capacity in some cases (Figure 4(b), e.g. day 13). Indeed, with this kind of regulation, all the daily capacity will be used during the first hours of the day, leaving no possibility for action after this period (see day 13, Figure 5(b)). Due to the concept of the PID controller, the larger the gap between current water volume and the setpoint, the more important it is that the valve be opened. Moreover, the request of maximum inflow of all the tanks at the same time cannot be realised by the regional provider. Such a distribution cannot anticipate local unexpected large consumptions for a reservoir, however. This could result in reaching the minimal limit, thus forcing the commune to bring in extra water despite high volumes stored in the other reservoirs.

Contrary to the PID control approach, which represents a set of local controllers, the GPC approach always acts depending on the global WDN situation. Using this control approach, the daily water volume brought into the WDN will never exceed the daily reserved capacity unless all the reservoirs reach their minimal fire fighting level. Additionally, the inflow to each tank stays stable compared with the other control approaches (Figure 5(c)). Nevertheless,

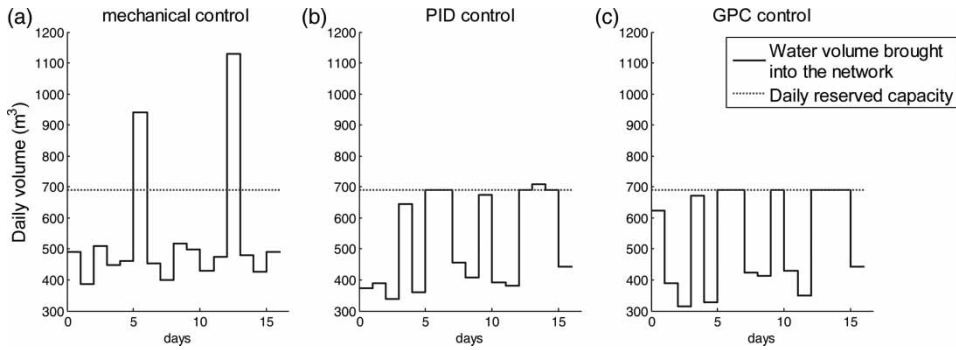


Figure 4 | Comparison of daily volume brought into the network.

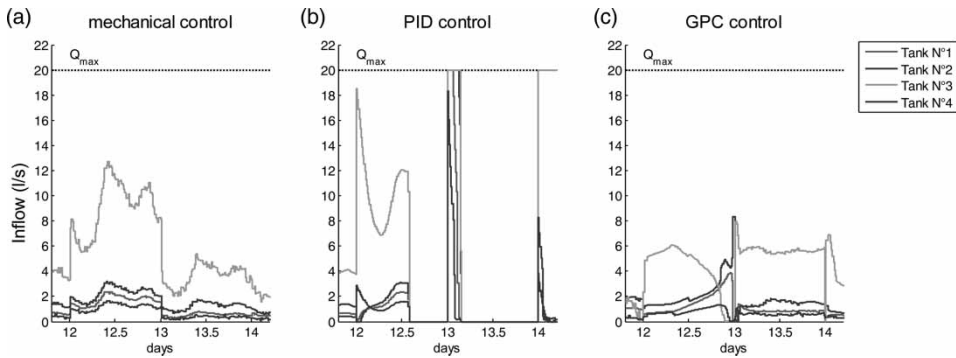


Figure 5 | Comparison of inflow into each tank (zoom on day 12 and 13).

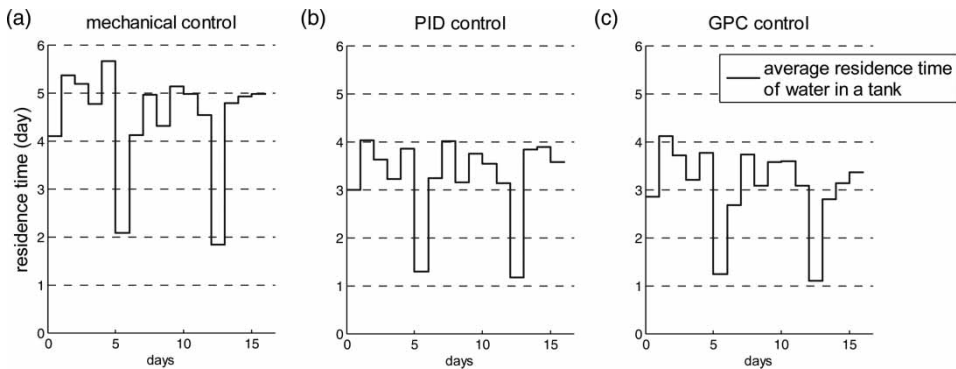


Figure 6 | Comparison of average residence time of water in tank.

large variations of the inflow towards the end of the day can be observed. This is due to the variable length of the prediction horizon. If the prediction horizon becomes small, the control system will be more sensitive to model error and forecast inaccuracies (Rawlings & Mayne 2009).

Finally, the use of a variable setpoint reduces the residence time of water in a tank compared with the

situation where the setpoint is fixed at the maximum capacity of the reservoir (Figure 6). This improvement depends on the value of $K_{sp}(=2)$ used to compute the setpoint value.

Summarising the results of the simulations, the following advantages of the GPC approach can be highlighted.

1. Days when volume of the daily reserved capacity is exceeded, even while reducing this limit by 15%, are completely eliminated. For example, based on the consumption of 2010, a reserved capacity of 570 m³ (reduction of 17.5%) could be adopted without any exceedance using the GPC approach. With mechanical control, there would have been 32 days where reserved capacity would have been exceeded (by 10–130 m³ or 60 m³ in average). Finally, there would have been one day with 70 m³ missing for the PID control. Note that the penalty the commune will have to pay if exceeding the daily limit is fixed according to the exceeded volume and is independent of the number of days a higher capacity is required.
2. The variations of the tank inflow rates are reduced by 20–40%.
3. Introduction of a variable setpoint allows us to significantly reduce the residence time of water in each tank (about 1 day).

Natural ventilation issue

As mentioned in the second section ('Case study'), a ventilation request was added by the commune in order to reduce relative air humidity in tanks and to prevent mould formation and water condensation. Avoiding additional operational costs (by using a fan or other energy-consuming systems), such ventilation can be realised in a natural way via water level fluctuations:

$$\text{Ventilation} = \frac{\sum |\Delta V| - |\sum \Delta V|}{2} \quad (8)$$

where $\Delta V(t) = V(t) - V(t-1)$

The amount of ventilation during a day is the volume of air exchanged between the inside and the outside of the tank over a day. Note that until now this requirement was not explicitly integrated into the objective function; however, the causal water level variation consists of about 5% of the water volume instead of the desired 10%. Integrating Equation (8) into the optimisation problem (Equation (7)) directly yields to a non-convexity of the objective function. Non-convex optimisation algorithms depend on an initial guess and therefore different local minima may occur. Furthermore, tuning and interpretability of the control behaviour become more complex. In parallel to the non-convex problem formulation, an engineering approach is therefore proposed in this paper. In this approach, an intermediate setpoint is fixed at noon which is equal to 10% more (or less) than the initial volume. When the water volume has to decrease over the day, it is suitable to fill the tank in the morning because the consumption is globally more important in the second part of the day. In the opposite case, starting to empty the tank in case the water volume has to increase over the day produces fewer variations on the inlet valve and therefore requires less energy. These two situations are shown in Figure 7.

The three proposed GPC control strategies – (a) neglecting ventilation; (b) with ventilation as an additional objective; and (c) with an intermediate setpoint – have been tested on the consumption data collected in 2010. Figure 8 and Table 1 allow comparison of these strategies based on simulations using these real measurements. The performance indicators used in Table 1 are: (1) the amount of ventilation; (2) the error between the setpoint $V_{sp}(d)$ and

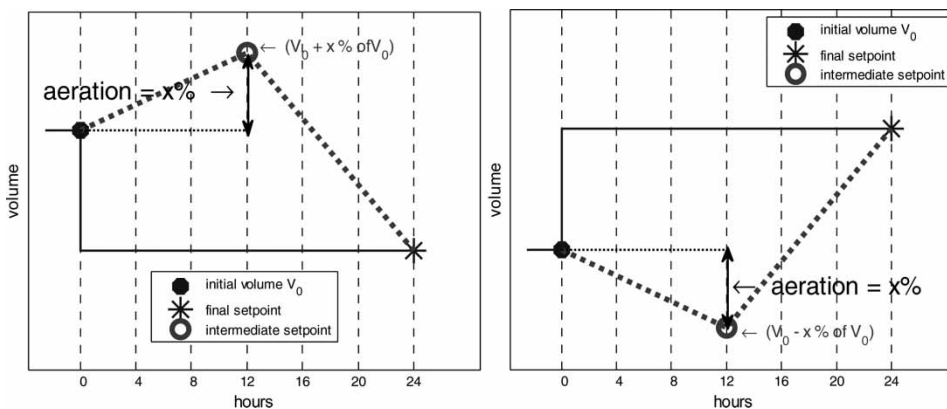


Figure 7 | Reference trajectory with an intermediate setpoint.

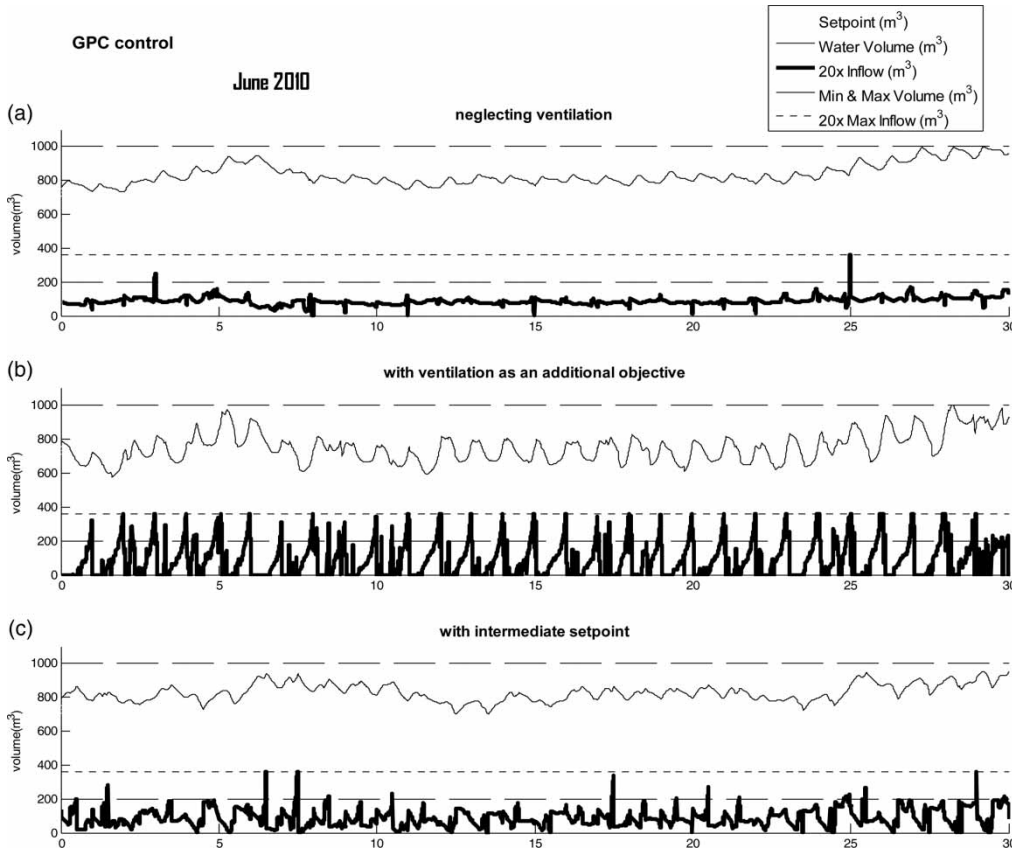


Figure 8 | Comparison of the three different approaches to create natural ventilation in a tank.

the water volume at the end of the day; (3) the control energy used to vary the inflow; and (4) the average residence time of water in the tank.

It is obvious that more volume fluctuation requires more variation of the tank inflow rate (see [Figure 8](#)). On the one hand, volume fluctuations in a tank are favourable for the aeration of this tank, e.g. a daily ventilation of about 12% ([Table 1](#), column (b)). In a general case (ventilation as an objective), it links to high inflow variation (dark line on [Figure 8\(b\)](#)). On the other hand however, high fluctuations

of inflow (fat line on [Figure 8\(b\)](#)) require high energy expenses ($82 \text{ m}^3 \text{ d}^{-1}$, [Table 1](#), column (b)). From this point of view, an intermediate setpoint approach seems to perform better: about 9% of daily ventilation with acceptable inflow fluctuations (dark line on [Figure 8\(c\)](#)) can be ensured. However, in this case ([Table 1](#), column (c)) a bigger error (5%) in the setpoint indicator is produced than for case (b), i.e. the consumption at the end of the day is not sufficient to reach the setpoint. The position of the intermediate setpoint should therefore be analysed in further work.

Table 1 | Comparison of the performances of the three different approaches to create natural ventilation in a tank (see text)

	(a)	(b)	(c)
Daily ventilation (% of water volume)	5	12	9
Error with setpoint at the end of the day (%)	1	1	5
Energy = $\sum \Delta V (\text{m}^3 \text{ d}^{-1})$	12	82	36
Residence time (d)	3.1	3.3	3.4

CONCLUSION

This paper provides insight into using instrumentation, control and automation on a small municipal water distribution network. It focuses on the control of a water distribution network and shows that the PID controller, the most widely used industrial controller, can deliver good results

but has significant limitations and additional drawbacks, particularly when objectives other than reaching a setpoint must be taken into account.

To take up the challenge of multi-goal control, the proposed global predictive control (GPC) approach is shown to be a valuable alternative. Moreover, this optimisation-based strategy is easier to adapt to changes in the network configuration (e.g. network extension and tanks maintenance) or to transfer to another water distribution network (WDN), as the general objectives and the type of constraints remains identical. However, notice that GPC (as a global control law) needs a great deal of data from the complete network; the design of such an optimal predictive control system may be complex and expensive to implement. In addition, the required level of operator training should not be underestimated.

This study should be regarded as pilot experimentation to demonstrate to stakeholders the significant cost reduction both on the reserved capacity and on the needed WDN infrastructure. The proposed optimal operation of the water distribution network without natural ventilation is currently being implemented.

To integrate the natural ventilation of the tank, the introduction of intermediate setpoints into the control law could be a viable solution. The pertinent number of intermediate setpoints and their positions are topics for further research. However, additional further analysis of a direct integration of an aeration goal as an objective into the cost function as well as suitable numerical methods (e.g. genetic algorithms) to solve this new non-convex optimisation problem should also be studied. Moreover, since the water volume fluctuations in the tank were requested to avoid condensation and mould formation, measurements of temperature and relative humidity both inside and outside the tank should be used to compute an adequate need for ventilation.

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