

## Turbulent Flow of Dilute Aqueous Polymer Solutions<sup>1</sup>

**A. G. FABULA.**<sup>2</sup> The authors are to be complimented on their contribution to the small but growing literature on the changes in turbulent flow structure that accompany friction reduction by polymer additives in liquids.

With respect to the mean velocity profile results, the possibility of non-negligible error in the pitot tube measurements does not seem to be necessarily eliminated by the authors' procedure of rejecting cases of over 2 percent volume flow error, since the error may be nonuniform over the pipe cross section. (The error is of viscoelastic origin and thus depends at least on the ambient velocity which controls the magnitude and time scale of the strain rate history in the pitot tube stagnation region.) Since several studies of pitot tube error are underway at present, it would be of interest if the authors gave the particulars of their measurements, such as tube diameter, tip configuration, manner of assigning static pressure, and whether any corrections were made for the effects of turbulence and mean shear.

In the authors' treatment of the mean velocity data, they fit straight lines in the semilogarithmic velocity defect plot in the region of  $0.04 < y/R < 0.25$ . (These straight lines would correspond to straight lines in the corresponding velocity plot in Fig. 13 for  $1.9 < \log yu_*/\nu < 2.8$ , but no fairings were fitted in that figure.) Thus the authors seem to suggest that there is in fact such straight-line behavior in what appears to be a greatly thickened transition zone. However, detailed inspection of the data points in Figs. 12 and 13 suggest that curves of monotonically changing slope, becoming parallel to the Newtonian line, would have provided a slightly better overall fit to the data.

The statement that "it is currently accepted that the preponderance of production occurs at low wave numbers"... may be questioned, since there is abundant evidence that turbulence production is concentrated near the wall where small eddies dominate.

The authors' speculations about the mechanism of the friction reduction have to be added to the many other tentative suggestions which have been advanced in recent years. The particular suggestion that their "second critical" Reynolds number may be correlated with polymer molecule size has been made (in effect) and verified in the extensive work of Virk.<sup>3</sup> Virk showed that the onset of friction reduction for a variety of molecules and pipe sizes can be correlated using the diameter of the unperturbed macromolecular coil, the fluid density and viscosity, and the wall shear stress. Thus, at least for the range of conditions studied by

Virk, pipe diameter is not relevant to the onset of friction reduction, and the introduction of a second critical Reynolds number based on pipe diameter seems undesirable.

**G. E. GADD.**<sup>4</sup> The paper by Dr. Goren and Professor Norbury is a most valuable contribution to the study of the puzzling phenomenon of drag reduction by polymer additives. It is no criticism of it to say that it does not answer all our questions. One unexplained peculiarity is the marked optimum concentration effect at 10 ppm. Although some other workers have found similar effects, this is not a universal finding. Thus, both White [1]<sup>5</sup> and Virk [2] find drag reductions monotonically increasing with concentration of Polyox WSR 301 up to much higher concentrations, despite the fact that their threshold shear-stress values are comparable with those of the present paper, so that presumably their solutions were not degraded in the premixing process.

Are the velocity profiles in the present paper all measured in the downstream test section? If any were obtained in the upstream section, did they agree with those downstream? It would be understandable if they did not agree, since recent work [3] on the pitot loss effect has suggested that the effect may arise from incomplete solution of the additive, and that it decreases with mixing. Thus, near the injection slot, the pitot losses might be bigger than further downstream.

The concentration distribution measurements neither confirm nor deny the interesting possibility suggested by Little [4] that in an extremely thin layer near the wall (much thinner than the diameter of any sampling probe) a greater equilibrium concentration may exist due to polymer adsorption on the wall.

The explanation of the threshold effect in terms of the first occurrence of sufficiently small eddies is interesting, but even so the molecular dimensions are of an order of magnitude smaller than the eddies, so it is difficult to see how the eddy structure is affected, unless molecular elongation is involved, as suggested by Tulin [5].

### References

- 1 White, A., "Turbulence and Drag Reduction With Polymer Additives," Hendon College of Technology Research Bulletin No. 4, Jan. 1967, p. 75.
- 2 Virk, R. S., "The Toms Phenomenon—Turbulent Pipe Flow of Dilute Polymer Solutions, M.I.T. thesis digest, Nov. 28, 1966.
- 3 Brennen, C., and Gadd, G. E., "Aging and Degradation in Dilute Polymer Solutions," *Nature*, Vol. 215, 1967, p. 1368.
- 4 Little, R. C., "Drag Reduction by Dilute Polymer Solutions in Turbulent Flow," NRL Report 6542, 1967.
- 5 Tulin, M. P., "Hydrodynamic Aspects of Macromolecular Solutions," *Hydrodynamics Technical Report 353-4*, 1967.

**BRUCE JOHNSON.**<sup>6</sup> The authors are to be complimented on their well-designed experiment. Investigations involving the injection

<sup>1</sup> By Y. Goren and J. F. Norbury, published in the December, 1967, issue of the JOURNAL OF BASIC ENGINEERING, TRANS. ASME, Series D, Vol. 89, No. 4, pp. 814-822.

<sup>2</sup> Department of the Navy, Naval Undersea Warfare Center, Pasadena, Calif.

<sup>3</sup> ScD thesis in Chemical Engineering, M.I.T., 1966.

<sup>4</sup> National Physical Laboratory, Middlesex, England.

<sup>5</sup> Numbers in brackets designate References at end of discussion.

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of polymer additives into a pipe or boundary-layer flow are especially valuable since most practical applications of the phenomenon of drag reduction will require injection techniques.

It is questionable, however, to attribute the threshold of friction reduction to a critical Reynolds number on the basis of experiments with one polymer and one pipe diameter. Elata<sup>6</sup> has shown that there is a threshold wall shear stress for each polymer relaxation time. This implies that there is a critical value of Reynolds number times the square root of the measured friction factor divided by the pipe diameter which should correlate the threshold stress, i.e.,

$$\frac{\text{Re} \sqrt{f}}{D} \Big|_{\text{crit}} = \frac{\sqrt{8}}{\nu} u_{\tau \text{crit}}$$

where  $u_{\tau \text{crit}}$  is a function of the solution relaxation time.

We have performed a series of experiments in the Naval Academy tow tank involving the injection of polymer solutions into the boundary layer of a towed body.<sup>7</sup> Our drag measurements indicate the same trend in the effect of polymer concentration as does Fig. 6. We found an optimum concentration followed by a decreased effectiveness of the polymer with increasing concentration. We had attributed this effect to the increased local viscosity near the injection point but the authors' data seem to invalidate this reasoning. This puzzling trend warrants further investigation. If the authors' apparatus is still set up, perhaps they could redo parts of the experiment with a test section near the injection point, sampling the flow to learn more about the influence of diffusion of the polymer away from the wall region. It is not surprising that the authors found a uniform concentration across the pipe diameter well downstream from the injection point.

The authors' speculations on the spectral distribution of eddy sizes agree with our hot-film measurements and our interpretation of these measurements. We suggest<sup>7</sup> that the mechanism of drag reduction is related to an inhibition of vortex stretching and turbulence production in the viscous sublayer and inner layer of the turbulent boundary layer. This hypothesis has also been suggested by Dr. Pfenninger<sup>8</sup> of the Northrop Corporation.

It appears that a mathematical explanation of this phenomenon will require the prediction of a shift in eddy size distribution to lower wave numbers with greatly reduced spectral density at high wave numbers. Previous attempts to explain the phenomenon in terms of effect on gross characteristics such as turbulence intensity and critical wall shear are inadequate except to correlate experimental data.

## Authors' Closure

The authors are greatly indebted to Drs. Gadd, Fabula, and Johnson for their thoughtful and constructive discussions of the paper. In reply, the first point to be made is that the main emphasis of the paper was intended to be placed on the experimental results and the explanation of the observed phenomena was speculative rather than conclusive. There is clearly a general agreement that many questions remain unanswered.

To answer Dr. Gadd's specific comments, the velocity profiles were all measured at the downstream measuring section. The purpose of measurements taken in the upstream measuring sec-

<sup>7</sup> Elata, C., Lehrer, J., and Kahanovitz, A., "Turbulent Shear Flow of Polymer Solutions," *Israel Journal of Technology*, Vol. 4, No. 1, 1966, pp. 87-95.

<sup>8</sup> Johnson, B., and Barchi, R. H., "The Effect of Drag-Reducing Additives on Boundary Layer Turbulence," Preprint 67-459 for AIAA 3rd Propulsion Joint Specialist Conference, July 1967.

<sup>9</sup> Pfenninger, W., "A Hypothesis of the Reduction of the Turbulent Friction Drag in Fluid Flows by Means of Additives," discussion presented at the 4th Winter Meeting of the Society of Rheology, Feb. 1967.

tion was to insure complete mixing of the injected polymer solution. Sampling showed that this was achieved quite rapidly and the present experiments throw little light on the possible effect of incomplete mixing on pitot tube error. It is true that the sampling probably was incapable of discerning concentration gradients in a very thin layer adjacent to the wall. It was found, however, that adsorption of the polymer to the pipe wall occurred principally when the solution was allowed to remain stationary in the pipe for some time, say overnight. In these circumstances a fairly thick layer could accumulate, and this effectively reduced the pipe diameter and increased the pressure drop. In order to avoid this difficulty, the pipe was flushed through with water at the conclusion of each series of experiments.

Dr. Fabula raises the question of the accuracy of the velocity measurements and while it is conceded that the pitot tube error may be nonuniform, it seems unlikely that it will be much different from that of the error in the measured volume flow. The pitot tubes used were of 0.08" outside diameter with hemispherical heads. The static pressure was obtained from a wall tapping located in the same plane as the pitot head. No corrections were applied for the effects of turbulence or mean shear.

Dr. Fabula is correct in suggesting that curves of monotonically changing slope would have provided a slightly better overall fit to the data presented in Figs. 12 and 13. However, the use of the straight line interpolation appeared to involve only small errors, and it allowed the simple derivation of the interesting results presented in Figures 14 and 15.

Both Dr. Fabula and Dr. Johnson make the point that the concept of a second critical Reynolds number is inadequate to describe the factors involved in determining the threshold of friction reduction, and this point is accepted. Dr. Johnson's results from the experiments with injected polymer solutions were interesting. When the authors' apparatus was set up, the principal object of the polymer injection arrangement was to make use of the high head available from the laboratory head tank without actually introducing polymer solution into the laboratory hydraulic system. The possibility of experiments on diffusion was envisaged as a secondary consideration, but in fact the injection was not sufficiently uniform to permit such experiments. The authors agree with Dr. Johnson on the desirability of diffusion experiments, but the present rig is not really suitable for these.

## Column Separation Accompanying Liquid Transients in Pipes<sup>1</sup>

**B. R. PARKIN.**<sup>2</sup> The encouraging agreement between experimental and calculated results indicates the author's keen appreciation of the governing physical factors in the present study. It remains to inquire as to the efficacy of various analytical refinements or experimental techniques which might narrow the differences between experiment and theory.

The author suggests as one source of disagreement between experiment and theory that energy losses may have been higher than anticipated. Were numerical calculations carried out for other values of the Darcy friction coefficient besides those obtained experimentally to see if variations in this dissipative factor produce the correct qualitative changes in the results?

Another possibility which could conceivably lead to differences is suggested by the author's observations of small cavitation bubbles ahead of the void and throughout the remainder of the

<sup>1</sup> By R. A. Baltzer, published in the December, 1967, issue of the *JOURNAL OF BASIC ENGINEERING*, TRANS. ASME, Series D, Vol. 89, No. 4, pp. 837-846.

<sup>2</sup> General Dynamics/Convair, San Diego, Calif.