This is only 16 percent lower than the resonant frequency of the comparable flat diaphragm.

Conclusions

The investigation provides results which demonstrate that a machined diaphragm of simple contour can be designed which is an improvement over conventional uniform thickness diaphragms in that, if made by modifying an existing uniform thickness diaphragm, at the same pressure:

(*a*) it more than doubles center deflection,
(*b*) it develops only a small increase in design stress level which is somewhat compensated by an improved strain gradient at the diaphragm boundary,
(*c*) it results in only a small decrease in first mode natural frequency, and
(*d*) its thickness variation can be produced for a nominal cost.

Further, using equations (34) through (38) and Fig. 7, the designer can rapidly modify an existing instrument design to (*a*) increase diaphragm center deflection while maintaining linearity and stress level, or (*b*) maintain center deflection while reducing stress level- and consequent hysteresis and linearity problems, or (*c*) reduce diaphragm diameter while maintaining stress and deflection levels.

Acknowledgment

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DISCUSSION

A. N. Palazotto[2]

The author should be congratulated on his fine addition to the practical use of engineering know-how. The writer has several comments which, he believes, can lead to clarification of certain aspects within the paper. The assumptions, among others, usually made for this type of analysis are: (*a*) shear deflection is negligible and (*b*) membrane stress and strain may be ignored. The writer feels the assumptions considered should be stated and discussed where needed. Assumption (*b*) in particular, is a very important simplification with physical accuracy dependent upon its actual realization. Reference [3] has indicated certain experimental data which show that membrane strains occur for circular lates rigidly suppported in a fashion assumed by the paper. The realization of this membrane effect can only be evaluated mathematically through the use of large deflection theory which is, of course, quite lengthy. It is felt, though, that some comments should be made by the author with reference to this point since it can greatly affect any final conclusions. The same point can be made about assumption (*a*) stated in the foregoing. Because of the geometry, large thickness theory may be necessary to use in analysis but the author should at least comment upon the possibility of errors appearing in his results if the basic assumptions are not met. Of course, one way of defending the use of assumptions is through some experimental verifications.

A second point this writer would like to make is with regard to the statement related to the economical surfaces. It would be interesting to compare the economics of fabrication for an ellipsoidal surface and spherical surface recognizing the fact that the ellipse could better approximate the thickness exponential variation. This whole area of economics would be a good point for the author's comment.

A third point of discussion is related to the application of the finite difference method. The author stated that findings using the finite difference approximation for a model suggested in [3] led to results which disagree with known solutions found in [1] and [2]. It is important that the author expands upon this disagreement indicating the whys and wherefores of its occurrence.

The final comment is concerned with the first mode frequency expression, equation (31). It was pointed out that for the self loading pressure, p can be taken constant for an annular plate element of width Δx. Some idea should be included, by the author, about the size of Δx not only for this equation but for all the finite difference equations. Furthermore, what type of convergence, within the finite series approximation to the first mode natural frequency, occurred (see equation (33)).

These points of discussion are not meant to distract in any way from this paper. It is a very good addition to engineering literature.

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Authors' Closure

The first point made by the discusser is well taken, but it must be kept in mind that one purpose of this investigation was to provide the designer with analytical tools which permit rapid and accurate prediction of actual diaphragm behavior. Clearly, the tools must be simple and, probably, they must have certain limitations. The simple relations obtained in this study do assume shear deflection and membrane strain are negligible. However, the range limitation imposed on the parameter a/h_a justifies these assumptions, a fact which has been verified by limited experiments.

The question of economics of the ellipsoidal surface versus the spherical surface is the same as that posed by the exponential-spherical comparison. With, for example, a numerically controlled machine, each of these surfaces would have equal cost. However, the spherical surface is the only one which can be produced in either large or small quantity with modest tools, thus meeting the last of the optimization criteria.

Dr. Palazotto's third point is excellent. The finite element approach in [3] uses a uniform thickness element for which the constant coefficient form of equation (4) is valid, and the method satisfies equilibrium and compatibility between elements. If finite differences are used on the constant coefficient form of equation (4), it is inherently assumed that the equation is valid for any three adjacent incremental elements (which is not true because of the thickness variation), thus equilibrium is not satisfied across these three elements. Use of the variable coefficient form resolves this difficulty since it is, in fact, valid across any three adjacent elements.

Thirty increments were sufficient to obtain convergence to the significant figures in the natural frequency equation. Although fewer increments were required to obtain the other numerical coefficients and the data for curve plotting, thirty were used for convenience.