A Model for Small \( P_T \) Inclusive Hadron Productions

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Small \( P_T \) inclusive spectra in fragmentation regions are studied from a quark fragmentation model and triple Regge model. It is shown that, if a composite model of quark proposed by the author is taken, we can nicely explain shapes versus \( x_F \), magnitudes and \( P_T \)-dependence of them by using quark decay functions consistent with large \( P_T \) hadron productions.

§ I. Introduction

In this paper we study small \( P_T \) inclusive hadron productions in fragmentation regions \((b \rightarrow c)\). Conventionally these processes have been described by the triple Regge model and many analyses based on the model have been performed for the data with \( P_T \approx 0(10 \text{ GeV}/c)\).\(^9\) These analyses have shown the unique feature of the model, i.e., the linear decrease of \( \alpha_{\text{eff}} \) with \(-t(E\sigma/dP^3 \sim G(t)(1-x_F)^{-1+2\alpha_{\text{eff}}})\). However, these analyses have also shown that the magnitudes of \( \alpha_{\text{eff}} \) obtained in inclusive processes are significantly smaller in many processes than those obtained in two-body reactions.\(^9\)

Recently high energy data with \( P_T = 100 \sim 2000 \text{ GeV}/c \) have been reported from ISR and FNAL. These data, especially those on \( p \rightarrow \pi, K \) by Singh et al.,\(^2\) have curious properties. The data show that \( E\sigma/dP^3 \) \((p \rightarrow \pi^+)\) behaves as \((1-x_F)^{1.6} \) for \( x_F = 0.6 \sim 0.8 \) in the region \( P_T = 0.55 \sim 0.95 \text{ GeV}/c \). The exponent of \( 1-x_F \) does not vary in this \( P_T \) region. In the triple Regge model the exponent should be larger by \( \sim 1.6 \) at \( P_T = 0.95 \text{ GeV}/c \) than at \( P_T = 0.55 \text{ GeV}/c \). On the contrary, this constant exponent is a characteristic feature of parton model. In the parton model \( E\sigma/dP^3 \) is expected to behave as \( G(P_T)(1-x_F)^m \) with constant \( m \).

The relevance of the parton model to inclusive productions has been made clearer by Ochs's observation.\(^3\) He has found that particle ratios in small \( P_T \) inclusive productions are similar to those in large \( P_T \) productions. This fact strongly suggests that small \( P_T \) inclusive reactions occur through a similar mechanism to large \( P_T \) productions. In fact, this similarity is automatically realized in the mechanism shown in Fig. 1 (quark fragmentation model), if we use the same quark distribution functions and quark decay functions as in large \( P_T \) productions. The quark fragmentation model (QFM) requires that the beam ratio is also common

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in small $P_T$ and large $P_T$ processes. In large $P_T$ processes, $pp\rightarrow \pi^0 X/\pi p\rightarrow \pi^0 X \sim (1-x_F)^{1.4}$. Therefore QFM predicts that $E\sigma/dP^3(\pi^+\rightarrow \pi^0) \sim (1-x_F)^3 \sim (1-x_F)^2$. The experiments by Barnes et al. show this behavior for $-t > 1.0$ GeV$^2$. Thus QFM can explain successfully the particle and beam ratios in small $P_T$ inclusive spectra. The experiments by Barnes et al., however, show also the triple Regge behavior at low $-t$. The experiments show that small $P_T$ inclusive spectra at fragmentation regions have both features of triple Regge model and of QFM.

As is well known this simple and interesting possibility (QFM) cannot be adopted in point like quark models, because $E\sigma/dP^3(\pi^+\rightarrow \pi^0) \sim (1-x_F)^2$ in these models and it gives only 1% of cross sections of $p\rightarrow \pi^+$. Inspired by this, other mechanism have been also proposed. They are quark exchange model by Brodsky and Gunion and quark recombination model by Das and Hwa. Their models are designed to reproduce $(1-x_F)^2$ behavior in $p\rightarrow \pi^+$. However, these models have some defects. In the quark exchange model, the factorization of projectile and target fragmentations is badly broken in contradiction to the experiments. In the recombination model, considerably larger sea components than that in deep inelastic scattering are necessary for the explanation of the $P\rightarrow M$ data. If we take account of the color and spin degrees of freedom, the discrepancy becomes huge (factor $\sim 100$). In addition, $E\sigma/dP^3(\pi^+\rightarrow \pi^0)$ is expected to behave as $(1-x_F)$ which is not supported by the data.

In recent investigations of small $P_T$ inclusive spectra, much attention has not been paid to their $P_T$-dependence. Although an exponent of $(1-x_F)$ is one of the most important quantities, the $P_T$-dependence is less important than it. In fact, the experiments show that the behaviors of $E\sigma/dP^3$ vary with $P_T$ in an interesting manner as shown typically by Barnes et al. It is impossible, we think, to find a correct theory for small $P_T$ inclusive processes without studying this point. There exist four types of processes, $B\rightarrow M$, $B'$ and $M\rightarrow M'$, $B$. Of course a correct theory is the one which can explain in a unified manner all these four types of processes. In this paper we shall study small $P_T$ inclusive spectra using QFM based on the model proposed by the author and challenge these problems. It will be shown that our model can explain shapes versus $x_F$, magnitudes and dependence on $P_T$ of various fragmentation processes. The essence of this paper has been already reported in short notes. In this paper we study in detail inclusive spectra.

In the next section we summarize our model to the extent relevant to the purpose of this paper and review briefly quark fragmentation model. In §§ 3~6, we analyze $p\rightarrow \pi, K, n, \bar{p}, \Lambda, \pi^+\rightarrow \pi^0, \pi^+, K^+, K^+\rightarrow \pi^-$ and $K^+\rightarrow \bar{K}^0$. In § 7, the consistency of our quark decay functions with large $P_T$ experiments is discussed. Section 8 is devoted to summary and remarks.
§ 2. Model

The process we study is a fragmentation of hadron $b$ into another hadron $c$ in hadron $a$-hadron $b$ collision at small $P_T$ and $x_F > 0.5$ region where $x_F$ is the usual Feynman variable. We denote this as $b \rightarrow a+c$. The relevant mechanism to this process is assumed to be a dissociation of $b$ via a pomeron exchange with a successive decay of quark into detected hadron $c$ (Fig. 1). We call this quark fragmentation model (QFM).

The contribution of QFM to the dimensionless quantity $F(x_F) = x_F / \sigma_T d\sigma / dx_F$ is expressed as follows, using the momentum distribution of quark in hadron $b$ ($f_q^b$) and the quark decay function into hadron $c$ ($g_q^c$),

$$F(x_F) = x_F \sum_i \int_{x_F}^{1} f^b_{q_i} (y) g^c_{q_i} \left( \frac{x_F}{y} \right) \frac{dy}{y},$$

where $\sigma_T$ is the total cross section of $a+b$. If diquark intermediate state is dominant, $q$ should be replaced by $(qq)$. If we assume a Gaussian $P_T$ distribution with slope $\beta$, we have the following form for the invariant cross section $E d\sigma / dP_T$ of QFM (we denote QFC)

$$\text{QFC} = \sigma_T (a+b) \frac{\beta}{\pi} e^{-\beta P^2_T} F(x_F).$$

In QFM, the transverse momentum of $c$ has two origins. One of them is the transverse momentum of quark in hadron $b$ and another that of hadron $c$ in quark. The experiments on large $P_T$ hadron productions show that the former is large ($\sim 0.8 \text{ GeV/c}$) and the latter is small ($\sim 0.3 \text{ GeV/c}$). Therefore the dominant source of $P_T$ of hadron $c$ is the former. This implies that slope parameter is a nearly common value $\beta_b$ in baryon fragmentation and another common value $\beta_M$ in meson fragmentation. $\beta_b$ should be roughly $\sim 2 \text{ GeV}^{-2}$ since $\langle k_T^2 \rangle = \beta^{-1}$.

In our model quark is not point-like but a three-point-like particle system and quark is responsible for the strong interaction of hadrons. The joint probability density function of $n$ point-like particle system is assumed to be as follows:

$$G_n(x_1, \ldots, x_n) \sim (x_1 x_2 \cdots x_n)^{-1/2}.$$  

From Eq. (3), we obtain $W(x) \sim \sqrt{x} (1-x)^3$ for valence parton in nucleon, which
is just a behavior required from the experiments. From Eq. (3), we have for the joint probability density of $k$ quark system due to $f_\pi^q \prod_i x_i^{-1/2} dx_i d(\sum x_i - x) \sim \sqrt{x}$,

$$G_q(x_1, \ldots, x_k) \sim (x_1 x_2 \cdots x_k)^{1/2}.$$  

(4)

From Eq. (4), we obtain for the momentum distribution of quark in baryon ($f_q^B$) and of quark in meson ($f_q^M$),

$$f_q^B(x) = \frac{105}{16} \sqrt{x} (1-x)^2$$  

(5)

and

$$f_q^M(x) = \frac{8}{\pi} \sqrt{x} (1-x)^{1/2}.$$  

(6)

For the momentum distribution of quark in $k$ quark system, we have,

$$f_q^{(k)} = B\left( \frac{3k}{2}, \frac{3k-3}{2} \right) \frac{1}{2} \sqrt{x} (1-x)^{(3k-6)/2}.$$  

(7)

For the momentum distribution of diquark in baryon ($f_{\pi}^B$), we have,

$$f_{\pi}^B(x) \sim \left( x_1 x_2 x_3 \right)^{1/2} \delta(x_1 + x_2 - x) \delta(x_1 + x_3 + x_4 - 1)$$

$$d x_1 d x_2 d x_3 = \frac{105}{16} x^2 (1-x)^{1/2}.$$  

(8)

As for quark decay functions $g_q(z)$, we assume the reciprocity relation for threshold behaviors. From Eq. (7), we obtain for threshold behaviors,

$$g_u^s, g_u^d, g_u^{s*}, g_u^{d*}, g_u^{s*}, g_u^{d*} \sim \sqrt{1-z},$$

$$g_u^s, g_u^d, g_u^{s*}, g_u^{d*} \sim (1-z)^{1/2},$$

and

$$g_u^s, g_u^d \sim (1-z)^{3/2} \text{ and } g_q^s, g_q^d \sim (1-z)^{3/2}.$$  

(9)

For a concrete form of $g_u^s$, we take for $z > 0.6$,

$$g_u^s = \frac{1}{4} \sqrt{1-z}.$$  

(10)

Equation (10) has a magnitude similar to the Field-Feynman parametrization as seen in Fig. (18). Other quark decay functions will be discussed in §§ 4-6.

As shown by the experiments on $\pi^+ \rightarrow \mu^+$, the unique behavior of the triple Regge model is well confirmed at low $-t$. What is a relation between the triple Regge model and QFM? In this paper, we adopt an idea that small $P_T$ inclusive cross sections are expressed as the sum of contributions of the triple Regge model.
with linear trajectories and of QFM. This is due to the following reasons.

(1) QFM itself works even at \( P_T = 0 \). There is no reason why QFM is excluded at \( P_T \) (or \( t \)) \( \sim 0 \).

(2) In inclusive processes, \( \alpha_{\text{eff}}(t) \) is significantly smaller at low \(-t\) than that expected from two-body reactions in many cases. For example, the data on \( \pi^+ \rightarrow \pi^0 \) show that \( \alpha_{\text{eff}} \) is significantly smaller at low \(-t\) than \( \alpha_P \). This implies that even at low \(-t\) other contributions besides triple Regge work.

(3) The triple Regge model has a feature different from QFM in an essential point. In the latter, a trigger interaction is a pomeron exchange between incoming hadrons. It has a similar property to elastic scattering. In the former it is a Reggeon exchange (not vacuum quantum number if \( b \neq c \)), the exclusive limit of which is a two-body inelastic scattering.

In this two-component model, only QFC survives at large \(-t\) (say \(-t = 1 \text{ GeV}^2\)) since the triple Regge contribution (TRC) becomes very small at large \(-t\) due to a large value of \( 1-2\alpha \). As an approximate expression of the two-component model, we take,

\[
E \frac{d\sigma}{dP_s} (b \rightarrow c) = \text{QFC} + \text{TRC}.
\]

The relation between \( t \) and \( P_T \) is,

\[
t = -\frac{P_T^2}{x_F} + M^2 (1 - x_F) + m^2 (1 - \frac{1}{x_F}),
\]

where \( M(m) \) is the mass of \( b(c) \).

The threshold behaviors of QFC are tabulated in Table I.

Resonance decay contributions are neglected as a first approximation in most cases. Since the parent-child relationship suppresses resonance decay contributions to inclusive spectra for \( x_F > 0.5 \), this is not a bad approximation. However we must take account of resonance decay contributions in the case that TRC does not exist and QFM is very small. Such processes are \( \pi^+ \rightarrow \pi^- \), \( K^+ \rightarrow \pi^- \) and so on.

\section{3. \( p \rightarrow \pi^+ \) and \( \pi^+ \rightarrow \pi^+ \) and \( \pi^- \rightarrow \pi^- \)}

According to the experiments, \( \pi^+ \rightarrow \pi^+ \) and \( \pi^- \rightarrow \pi^- \) behave as \((1-x_F)^{3.5}\) for \( x_F = 0.6-0.8 \) in the region \( P_T = 0.55-0.95 \text{ GeV}/c \). This implies that TRC is negligibly small in this process at least for \( x_F = 0.6-0.8 \). Therefore this process is a good candidate to test our QFC. Using Eqs. (5) and (10), we obtain,

\[
F(x_F) (p \rightarrow \pi^+) = 2x_F \int_{x_F}^1 g_{\pi^+} (x_F y) f_{\pi^+} (y) \frac{dy}{y} = \frac{1}{2} (1 - x_F)^{3.5}.
\]
The experimental data can be parametrized as follows:

\[ E \frac{d\sigma}{dP_T^2} = 16.4e^{-0.4P_T^2} (1 - x_F)^{1.4} \text{ mb/GeV}^2. \]  
(14)

From Eq. (14), we obtain using \( \sigma_T(pp) = 40 \) mb,

\[ \frac{d\sigma}{\sigma_T dx_F} = 0.54 (1 - x_F)^{1.4}. \]  
(15)

Comparing Eq. (13) with Eq. (15), we know that our model can explain not only the shapes versus \( x_F \) but also the magnitudes of \( pp \rightarrow \pi^+X \). Equation (10) cannot be changed so largely since it is constrained by large \( P_T \) hadron productions. In this sense we can say that Eq. (13) is derived directly from Eq. (5). This is a striking success of our model.

Next we consider \( pp \rightarrow n^+n^- \). Since \( n^0 = (uu-dd)/\sqrt{2} \) and \( n^+ = ud \), we take,

\[ g_u^* = g_d^* = \frac{1}{2} \frac{1}{8} \frac{1}{1 - \frac{1}{2}}. \]  
(16)

From Eqs. (6) and (16), we get,

\[ F(x_F) (\pi^+ \pi^0) = 2x_F \int \frac{1}{1} g_u^* \left( \frac{x_F}{y} \right) f_\pi^0 (y) \frac{dy}{y} = \frac{1}{4} (1 - x_F)^4. \]  
(17)

According to the experiments, \( \alpha_{\text{eff}} \) is in rough agreement with the linear \( \rho \) trajectory at low \(-t\) for \( x_F = 0.81 \sim 0.98 \), while for large \(-t > 1.0 \) GeV\(^2\), the value of \( \alpha_{\text{eff}} \) becomes a constant, approximately \(-0.5\) as shown in Fig. 2. This feature is expected in the two-component model. The experimental result that \( \alpha_{\text{eff}} = -0.5 \) is just our prediction. We can know QFC in \( \pi^0 \rightarrow n^0 \) from the data with \(-t > 1.0 \) GeV\(^2\). The data are parametrized for \(-t > 1.0 \) GeV\(^2\) and \( x_F = 0.81 \sim 0.98 \) as follows:

\[ \frac{d\sigma}{dt df_{\pi^0}} = 10e^{-t/4} (1 - x_F)^3 \text{ mb/GeV}^3. \]  
(18)

In order to compare Eq. (17) with the experiments, we must know \( t \)-dependence at low \(-t\) of the QFC. Here we assume that it is also \( e^{-t/4} \). In this case we obtain,

\[ F(x_F) (\pi^+ \pi^0) = \frac{x_F}{\sigma_T} \int \frac{d\sigma}{dx_F dt} = 0.27 x_F (1 - x_F)^4, \]  
(19)

where we take \( \sigma_T(ppp) = 25 \) mb. For \( x_F \sim 0.9 \), Eq. (19) behaves approximately as \( 0.24 (1 - x_F)^4 \). Thus our prediction is consistent with the data.

Next we attempt to explain the behaviors of \( \alpha_{\text{eff}} \) and \( G_{\text{eff}} \) of \( \pi^+ \pi^0 \) \( (d\sigma/dt dx_F) = G_{\text{eff}} (1 - x_F)^{1.2 \alpha_{\text{eff}}} \). Strictly speaking, we should parametrize QFC as \( A e^{-t/4} \) \( (1 - x_F)^4 \). However, since \( t \sim -P_T^2 \) near \( x_F = 1 \), we use Eq. (18) as QFC near \( x_F = 1 \). In the two-component model, we have for \( \pi^+ \pi^0 \),

\[ \text{Downloaded from https://academic.oup.com/ptp/article-abstract/63/2/552/1887917 by guest on 02 January 2019} \]
\[
\frac{d\sigma}{dtdx_p} = G_p(t) (1-x_p)^{1-\alpha_p(t)} + 10e^{14t} (1-x_p)^{1.5}.
\]  

(20)

Taking \(\alpha_p(t) = 0.5 + 0.8t\) and \(G_p(t) = 70e^{32} \alpha_p^2 \text{mb/GeV}^2\), we obtain \(\alpha_{\text{eff}}(t)\) and \(G_{\text{eff}}(t)\) for \(x_p \sim 0.9\) from Eq. (20) as shown in Figs. 2 and 3. The form \(G_p \propto \alpha_p^2\) is due to the wrong signature nonsense zero (WSNZ) at \(\alpha_\ell = 0\). The agreement is good. \(\alpha_{\text{eff}}\) is predicted to decrease with \(-t\) faster than \(\alpha_p(t)\) and to become a constant \(-0.5\) at \(-t > 1.5\text{ GeV}^2\). These are observed. The predicted \(\alpha_{\text{eff}}\) has a dip at the value of \(t\) where \(\alpha_\ell = 0\) and a bump at \(-t\) slightly larger than the dip position. This is due to the fact that TRC = 0 at \(\alpha_\ell = 0\). More precise experiments are needed to confirm this prediction. According to the experiments, \(G_{\text{eff}}\) has curious properties. It has a dip at the value of \(t\) \((-t = 0.4\text{ GeV}^2\) different from the WSNZ point \((-t = m_\pi^2 = 0.6\text{ GeV}^2\) and a bump near the WSNZ point. While it is difficult to understand these behaviors in the triple Regge model alone, they are just our prediction. Certainly \(G_p\) used in this analysis is tentative and can be changed. However, a slight change of \(G_p\) does not affect these predictions. Since a WSNZ does not exist at \(\alpha_{\text{WSNZ}} = 0\), \(\alpha_{\text{eff}}\) in \(\pi^+ \rightarrow \gamma (\alpha_{\text{eff}})\) behaves a little differently from the one in \(\pi^+ \rightarrow \pi^0\) in spite of the fact that \(\alpha_\ell = \alpha_{\text{eff}}\). Especially, \(\alpha_{\text{eff}}^2\) is slightly larger at low \(-t\) than \(\alpha_{\text{eff}}\). This is also observed. In Fig. 2, \(\alpha_{\text{eff}}^2\) is plotted in the case that \(G_{\text{eff}}(t) = 70/3e^{32}\) and \(QFC(\pi^+ \rightarrow \gamma) = \frac{1}{2} QFC(\pi^+ \rightarrow \pi^0)\).

From the above analysis, we know that slope parameters \(\beta\) are 2.4 GeV\(^{-1}\) in

![Fig. 2](https://example.com/fig2.png)  
Fig. 2. Predicted \(\alpha_{\text{eff}}\) in \(\pi^+ \rightarrow \pi^0\) for \(x_p = 0.81 \sim 0.98\) (solid line). The data are taken from Ref. 5). \(\alpha_{\text{eff}}\) is also plotted (broken line).

![Fig. 3](https://example.com/fig3.png)  
Fig. 3. Comparison of our prediction for \(G_{\text{eff}}\) in \(\pi^+ \rightarrow \pi^0\) with the data.
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proton fragmentation ($\beta_p$) and 1.5 GeV$^{-1}$ in pion fragmentation ($\beta_H$). As discussed in § 2, these should be roughly independent of a detected hadron. If the transverse momentum of quark in hadron is due to a Fermi motion, the following relation should hold:

$$\frac{\beta_p}{\beta_H} = \frac{\langle r^2 \rangle_p}{\langle r^2 \rangle_H}. \quad (21)$$

Equation (21) is satisfied well in the case that $\beta_H=2.4$ and $\beta_p=1.5$ GeV$^{-2}$. These slope parameters imply large quark $k_T$, $\sqrt{\langle k_T^2 \rangle}\approx0.8$ GeV in pion and $0.65$ GeV in proton, which is consistent with large $P_T$ experiments. Hereafter we take $e^{-2.4 F_p}$ for baryon (meson) fragmentation.

Our model predicts in $\pi^+ \rightarrow p\pi^0$ that the ratio of QFC to TRC increases as $x_F$ decreases and so $\alpha_{\text{eff}}$ decreases as $x_F$ does. The experiments suggest this tendency. In Fig. 4, the predicted $\alpha_{\text{eff}}$ for $x_F=0.7$ is plotted (using $e^{-4 F_p}$). QFC/TRC is 0.03 at $x_F=0.9$ and 0.23 at $x_F=0.7$ in $t=-0.2$ GeV$^2$. Therefore at such small $t$ TRC is dominant at $x_F=0.9$ and extra contributions ($\sim 20\%$) besides TRC exist at $x_F=0.7$. This is also observed in the experiments.

§ 4. Two-component model analysis of $p^+ \rightarrow \pi^+, K^+$

First we study $p^+ \rightarrow \pi^+$. From Eqs. (2) and (13) and $e^{-2.5 F_p^2}$ behavior, our model predicts for $p^+ \rightarrow \pi^+$,

$$\text{QFC} = 15.3 e^{-2.5 F_p^2} (1-x_F)^{0.5} \text{mb/GeV}^2, \quad (22)$$

which can explain nicely the data$^P$ for $x_F=0.6 \sim 0.8$ and $P_T=0.55 \sim 0.95$ GeV/c. In addition, our model predicts that the exponent of $(1-x_F)$ for $x_F>0.8$ is smaller than 3.5 at small $P_T$ and it is 3.5 even for $x_F>0.8$ at large $P_T$ (say 0.95 GeV/c) since TRC is large at small $P_T$ and $x_F\sim1$ and negligibly small at large $P_T$ due to a large value of $1-2\alpha$. Assuming nucleon trajectory dominance ($\alpha_Y = -0.35 + t$), we can obtain a good fit even for $x_F>0.8$ by the following choice for TRC as shown in Fig. 5,

$$\text{TRC} (p^+ \rightarrow \pi^+) = 30 e^{a F_p^2} (\alpha_Y + \frac{1}{2})^2 (1-x_F)^{1.7-2\alpha} \text{mb/GeV}^2. \quad (23)$$

The necessity of both QFC and TRC is not so visible at $P_T \geq 0.55$ GeV/c since TRC is not so large compared with QFC at such $P_T$ region. The necessity of
two components becomes clearer at $t=0$. In Fig. 6, compare our model (Eq. (22) + Eq. (23)) with the data, QFC or TRC alone cannot explain the data and the sum of TRC and QFC can nicely explain the data.

For $p\rightarrow K^+$, we must determine the so-called $\lambda$ suppression factor ($\lambda=g_K^+ /g_{\pi^+}$). The two-component model predicts that QFC explains well $p\rightarrow K^+$ at relatively large $P_T$. Therefore at such $P_T$ we have:

$$E \frac{d\sigma}{dP_T} (p \rightarrow K^+) = 15.3 \lambda e^{-2.4 t} (1-x_F)^{3.5} \text{mb/GeV}^2.$$ (24)

Equation (24) fits well the data at $P_T=0.95\text{GeV/c}$ in the case that $\lambda=0.22$ as shown in Fig. 7. Due to the smallness of QFC, it is expected that an effect of TRC is large in $p\rightarrow K^+$ compared with $p\rightarrow \pi^+$ and $Ed\sigma/dP_T$ differs significantly from $(1-x_F)^{3.5}$ even at not so small $P_T$. This is well supported by the data. The data show that $Ed\sigma/dP_T \sim (1-x_F)^{3.5}$ at $P_T=0.65\text{GeV/c}$. If we assume $\Sigma_d$ trajectory dominance with $\alpha_d=0.85-t-0.12$, the data are well explained by our two-component model with the following TRC as seen in Fig. 7:

$$\text{TRC} (p \rightarrow K^+) = 0.22 e^H (1-x_F)^{1-z_2} \text{mb/GeV}^2.$$ (25)

Contrary to $p \rightarrow \pi^+$, TRC is expected to be dominant at $t=0$ due to the $\lambda$ factor.
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Fig. 7. Comparison of our prediction for $pp \rightarrow K^+X$ (solid line) with the data at $P_T = 0.65$ and 0.95 GeV/c ($\sqrt{s} = 45$ GeV).

Fig. 8. Comparison of our prediction for ratio $K^+/\pi^+$ in $pp$ collision (solid line) with the data.

Fig. 9. $t$ dependence of $E_\theta/dP^*_{+}$ in $pp \rightarrow K^+, \pi^+X$. The solid lines are our prediction. QFC is also plotted (dash-dotted line).
This seems to be also supported by the data\textsuperscript{30} as shown in Fig. 6.

The ratio \( p \rightarrow K^+ / p \rightarrow \pi^+ \) is plotted in Fig. 8. The data\textsuperscript{130} show that it varies with \( x_F \) at \( P_T = 0.65 \) GeV/c and is nearly a constant at \( P_T = 0.85 \) GeV/c. Our model can reproduce nicely these features.

Finally we consider the \( t \) dependence of cross section at fixed \( x_F \). The results of our model are plotted in Fig. 9. The agreement with the experiments\textsuperscript{21} is good. Our model predicts for \( p \rightarrow K^+ \) that \( E d\sigma / dP^2 \) behaves as \( e^{0.17 (1-x_F)} \) at \( t \rightarrow 0 \) and as \( e^{x_F t} \) at large \( -t \). This feature, the break of \( t \)-dependence, is a typical result of the two-component model. Although this feature is not so visible in the data by Singh et al., the combination of the data with \( t \rightarrow 0 \) by Anderson et al.\textsuperscript{131} with them strongly suggests this feature. Of course more experiments, especially at \( t \sim -0.1 \) GeV\textsuperscript{2}, are needed in order to confirm this point.

The ratio \( p \rightarrow \pi^+ / p \rightarrow \pi^- \) is predicted to be 2 in the region where QFC is dominant as in any naive model. According to the data, the ratio is significantly larger than 2 even at \( P_T = 0.95 \) GeV/c. Equation (4) is perhaps too naive. An idea as given by Farrar and Jackson\textsuperscript{10} may be needed. We leave this point for a further study. In this paper we use the simplest distribution, Eq. (4).  

§ 5. \( K^- \rightarrow K^0 \) and \( p \rightarrow n \)

In this section we study \( K^- \rightarrow K^0 \) and \( p \rightarrow n \). For these processes we can predict cross sections without any adjustable parameters.

It is natural to take \( g_s = g_s' \) since the lacking quark is the same in both cases. Therefore we have for QFC of \( K^- \rightarrow K^0 \),

\begin{equation}
\text{QFC} = 2.4e^{-1.3x_F} (1-x_F)^2 \text{mb/GeV}^2,
\end{equation}

where we take \( \sigma_T(K^-p) = 20 \) mb. The relevant trajectories are \( \rho \) and \( A_2 \) in this process. Using the \( SU(3) \) relation \( G_{\rho-x_F} = 2G_{\rho-K^+} \), we obtain for \( \rho \) exchange TRC,

\begin{equation}
\text{TRC}(\rho) = \frac{1}{2} \cdot \frac{70}{\pi} e^{x_F} (1-x_F)^{1-2x_F(t)} \text{mb/GeV}^2.
\end{equation}

For \( A_2 \) exchange term, invoking \( \text{TRC}(A_2, t=0) = \text{TRC}(\rho, t=0) \) due to the \( \rho - A_2 \) exchange degeneracy, we take

\begin{equation}
\text{TRC}(A_2) = \frac{1}{2} \cdot \frac{70}{\pi} 0.5e^{x_F} (1-x_F)^{1-2x_F(t)} \text{mb/GeV}^2.
\end{equation}

Therefore we obtain

\begin{equation}
E \frac{d\sigma}{dP^2} (K^- \rightarrow K^0) = 2.4e^{-1.3x_F} (1-x_F)^2
\end{equation}

\begin{equation}
+ 11e^{x_F} (\alpha_s^2 + 0.25) (1-x_F)^{1-2x_F(t)} \text{mb/GeV}^2.
\end{equation}

Equation (29) is plotted in Fig. 10. Unfortunately there is no high energy data available at present. Future experiments will check our prediction. There exists the data on \( K^{-}\pi^{+}\rightarrow K^{0} \) at \( P_{T}=14.3 \text{ GeV/c} \). In Fig. 11, we compare our prediction for \( d\sigma/dx_{F} \) with the data.\(^{19}\) The agreement is good for \( M_{\pi}^{2}<5 \text{ GeV}^{2} \). Perhaps the triple Regge model cannot be applied to the region \( M_{\pi}^{2}<5 \text{ GeV}^{2} \). It is to be noted that TRC alone cannot explain the steep rise of \( d\sigma/dx_{F} \) with \( (1-x_{F}) \) and we can reproduce the data by adding our QFC. The flatness of TRC is due to the fact that \( 1-2\alpha_{K}(0)=0 \). Of course the value of \( P_{T} \) in the data is small. However, the rule for an early scaling found by Whitmore et al., i.e., \( a\bar{c} \) exotic,\(^{18}\) suggests that we can extract a definite conclusion even from a relatively low energy data in this process.

For \( \pi^{+}\rightarrow K^{+} \) and \( K^{+}\rightarrow\pi^{+} \), our model predicts by using \( \lambda=0.22 \),

\[
\text{QFC}(\pi^{+}\rightarrow K^{+}) = 0.66e^{-1.38x_{F}} \cdot (1-x_{F})^{2} \text{ mb/GeV}^{3}
\]  
(30)

and

\[
\text{QFC}(K^{+}\rightarrow\pi^{+}) = 2.4e^{-1.38x_{F}} \cdot (1-x_{F})^{2} \text{ mb/GeV}^{4}.
\]  
(31)

Since QFC(\( \pi^{+}\rightarrow K^{+} \)) is small due to the \( \lambda \) factor as in \( p\rightarrow K^{+} \), TRC is dominant at low \(-t\). In fact, the data by Anderson et al.\(^{18}\) are well fitted by taking TRC \( =6.7e^{0}(\alpha_{K}^{2}+0.36)(1-x_{F})^{1.39x_{F}} \) with \( \alpha_{K}=0.8t+0.36 \), \( K^{+}\rightarrow\pi^{+} \) is interesting, since \( (1-x_{F})^{2} \) behavior is more visible due to the largeness of QFC.

Next we consider \( p\rightarrow n \). In this process diquark intermediate state \( (ud) \) is dominant. As for \( g_{ud}^{s} \), we assume \( g_{ud}^{s}=g_{ud}^{n} \) for \( x>0.6 \) since a lacking quark is a nonstrange one in both processes. This assumption can be checked by the specia-
tor Jet associated with a large $P_T$ trigger. If we assume $g_{\pi^+\pi^+}^a = g_{\pi^0\pi^0}^a = g_{\pi^0\pi^0}^a$, we obtain from the same procedure as in the paper by Fontannaz et al.,\textsuperscript{17} for forward protons associated with a trigger particle at $90^\circ$ with $x_F$,

$$\frac{x_F}{N} \frac{dN}{dx_F} = \frac{x_F}{1-x_F} \left[ \frac{2}{3} g_{\pi^+\pi^+}^a \left( \frac{x_F}{1-x_T} \right) + \frac{1}{3} g_{\pi^0\pi^0}^a \left( \frac{x_F}{1-x_T} \right) \right] = \frac{1}{4} \sqrt{1-x_F} \left[ 1 - \frac{x_F}{1-x_T} \right]. \quad (32)$$

Equation (32) is compared with the data\textsuperscript{18} for forward positive particles (protons are expected to dominate for large $x_F$) in Fig. 12, where the experimental cutoff on $P_T$ for forward particles are taken into account in the same way as in Ref. 17. The agreement is striking for $x_F > 0.6$. Of course, we cannot expect the agreement for small $x_F$ since pions are expected to be abundant in such region.

From Eqs. (8) and (10), we obtain,

$$F(x_F) (p \to n) = \frac{125}{256} (1-x_F)^2 f(x_F), \quad (33)$$

where

$$g(x_F) = \int_{-1}^{1} \left( \frac{1+x_F}{2} + \frac{1-x_F}{2} t \right)^{1/2} \sqrt{1-t} \sqrt{1+t} dt.$$  

$g(x_F)$ is $\sim 1.2$ for $x_F = 0.7$ and $\pi/2$ for $x_F = 1$. Therefore we have,

$$QFC = 12.5 e^{-x_F} \sqrt{x_F} (1-x_F)^2 g(x_F) \text{ mb/GeV}^2. \quad (34)$$

In this process, the leading trajectories are $\rho$ and $A_2$. At small $-t$ (or $P_T$), however, $\pi$ exchange contributions are known to be large. Certainly, very steep slope $e^{\omega t}$ is observed for $t' \sim 0$ in $p \to J^{++}$ which is characteristic of reactions mediated by pion exchange.\textsuperscript{19} However, for $t' \gg m_\pi^2 (P_T^2 \gg m_\pi^2)$, $\pi$ contributions are expected to become small. In fact, the steep slope vanishes away for $t' > 0.3$ GeV\textsuperscript{2} in $p \to J^{++}$. Therefore we study the data with $P_T^2 > 10 m_\pi^2$ on the assumption of the leading trajectory dominance for TRC.

If we assume the $\rho$ meson universality relation $g_{\rho^0\pi^0} = 1/2 g_{\rho^+\pi^0}^a$ and $\rho - A_2$ exchange degeneracy, TRC of $pp \to nX$ is the same as of $K^-p \to K^0X$. Therefore we obtain for $p \to n$, \[ ... \]
Equation (35) is compared with the available data at $P_T = 0.4, 0.6$ and 0.8 GeV/c in Fig. 13. The agreement is striking both in shape and magnitude. The experiments show that as $P_T$ is larger, $E d\sigma/dP^\parallel$ falls more steeply with $x_F$. This is nicely reproduced. At $P_T = 0.8$ GeV/c, TRC is negligibly small. The fact that we can reproduce the data at $P_T = 0.8$ GeV/c implies the validity of QFC derived from our model. Our model predicts that the shape versus $x_F$ does not change at $P_T \geq 0.8$ GeV/c, i.e., $\sim (1-x_F)^2$ at large $x_F$. Future experiments will be able to check this prediction. In p-n, the data with $P_T = 1.0 \sim 1.5$ GeV exist. The form $(1-x_F)^2$ can describe the data quite well. This strongly supports our model. A detailed analysis will be published elsewhere. In Fig. 13, TRC and QFC are shown separately. TRC becomes smaller as $x_F$ does. This is due to the fact that, at fixed $P_T$, small $x_F$ corresponds to large $-t$. It is clear that TRC alone cannot describe the data even at $P_T = 0.4$ GeV/c.

Recently Ranft has calculated $pp\to nX$ on the basis of the recombination model. The shape versus $x_F$ derived by him is shown in Fig. 13. At $P_T = 0.4 \sim 0.6$ GeV/c it seems hard to distinguish the recombination model and ours. At $P_T = 0.8$ GeV/c, however, the difference between them becomes obvious. The experiments clearly support our model.

Engler et al. have concluded from their experiments that a $\pi$ exchange contribution is dominant in $pp\to nX$. This is due to the fact that the experiments show $\alpha_{\text{eff}}(t) \approx t$. Certainly $\alpha_{\text{eff}}(t = -2) \approx -2$ and $\alpha_{\text{eff}}(t = -0.2) \approx 0$ according to the data. However, we should pay attention to the fact that the fit region of

\[
E \frac{d\sigma}{dP^\parallel} (\text{mb/GeV}^2) = 12.5 e^{-4.4x_F^2} (1-x_F)^2 y(x_F) \\
+ 11 e^{28} (\alpha^2_{\pi} + 0.25) (1-x_F)^{1-2x_F} \text{mb/GeV}^2.
\]
$x_F$ for $\alpha_{\text{eff}}$ is $0.8-0.9$ at $t \sim 0$ and $0.5-0.6$ at $t = -2 \text{GeV}^2$. In our model QFC is dominant at $t \approx -2$. If we define $\alpha_{\text{eff}}$ by $1 - 2\alpha_{\text{eff}} = \partial \ln \text{QFC}/\partial \ln (1-x_F)_{\text{fixed}}$, our model predicts $\alpha_{\text{eff}}(t = -2) \approx -2$ for $x_F \sim 0.6$. $\alpha_{\text{eff}}$ in our model can mimic $\alpha_s$. Engler et al. have not examined the behaviors of residues. If we try to describe the data by TRC only, we are forced to use a too queer residue to accept. This is due to the fact that $E_d\sigma/dP_t$ does not decrease rapidly with $-t$. It seems impossible to describe the data without a term like QFC (factorization of $P_T$ and $x_F$ distributions).

§ 6. $p \rightarrow K^-, \bar{p}, \bar{A}$, and $\pi^+, K^- \rightarrow \pi^-$

At first we study $p \rightarrow K^-, \bar{p}, \bar{A}$ for which only QFC contributes.

For $p \rightarrow K^-$, we must determine $g_u^p$ and $g_d^p$. Considering the threshold behaviors of them are $(1-z)^{3.5}$ in our model, we assume the following form for large $z$:

$$g_u^p = g_d^p = \frac{a}{z} \left(1 - z\right)^{3.5},$$

where $a$ is a free parameter. In this case, we obtain for $p \rightarrow K^-$,

$$E_d\sigma/dP_t = 1.66a e^{-1.4x_F t}(1-x_F)^{3.5}h(x_F)\text{mb/GeV}^2,$$

where

$$h(x_F) = \int_{-1}^{1} \left( \frac{1+x_F}{2} + \frac{1-x_F}{2} t \right)^{-3.5} (1-t)^2(1+t)^3 dt.$$

Equation (37) is compared with the data$^9$ in Fig. 14. The agreement is good in the case $a = 1$. The consistency of Eq. (36) with large $P_T$ hadron production will be discussed in § 7. The two-component model predicts that the behavior of $p \rightarrow K^-$ versus $x_F$ does not change with $P_T$. This is supported by the data.

For $p \rightarrow \bar{p}$, we must know $g_u^{\bar{p}}$. Since $g_u^{\bar{p}} \sim (1-z)^{3.5}$ near $z = 1$ in our model, we assume

$$g_u^{\bar{p}} = g_d^{\bar{p}} = \frac{b}{z} \left(1 - z\right)^{3.5},$$

where $b$ is a free parameter. For $p \rightarrow \bar{p}$, we obtain

$$E_d\sigma/dP_t = 0.59b e^{-1.0x_F t}(1-x_F)^{3.5}\rho(x_F)\text{mb/GeV}^2,$$

where

$$\rho(x_F) = \int_{-1}^{1} \left( \frac{1+x_F}{2} + \frac{1-x_F}{2} t \right)^{-4.5} (1-t)^2(1+t)^3 dt.$$
In the case \( b = 1 \), Eq. (39) can reproduce the data\(^{31} \) as shown in Fig. 15.

For \( p \rightarrow \Lambda \), we expect that cross sections of \( p \rightarrow \Lambda \) are smaller by the \( \lambda \) suppression factor than of \( p \rightarrow \bar{p} \). Since \( \lambda = 0.22 \), we get for \( p \rightarrow \Lambda \),

\[
E \frac{d\sigma}{dP^2} = 0.13 e^{-1.4x_p^2} (1-x_p)^3 p(x_p) \text{mb/GeV}^2.
\]  

Equation (40) is in reasonable agreement with the data\(^{31} \) as seen in Fig. 15, although measured values of \( x_p \) are not large. The small discrepancy in Fig. 15 is improved if we take \( \lambda = 0.26 \).

For \( \pi^+, K^+ \rightarrow \pi^- \), we need \( g_u^- \), \( g_s^- \), and \( g_t^- \). Considering Eq. (36) and \( \lambda = 0.22 \), we take

\[
g_u^- = g_s^- = g_t^- = \lambda^{-1} g_s K^+ = 0.22^{-1/4} (1 - z)^{3.2}/z. \tag{41}
\]

Therefore we have for QFC of \( \pi^+ \rightarrow \pi^- \)

\[
\text{QFC} = 2.16 (1-x_p)^3 e^{-1.5x_p^2} p(x_p) \text{mb/GeV}^2,
\]  

where
Equation (42) is very small at large $x_F$ (say $x_F \approx 0.8$) compared with the experimental data at $P_L = 22$ GeV/c. However, in this process, QFC is very small compared with that in $\pi^+ \rightarrow \pi^0$ and TRC does not exist. Therefore at large $x_F$ a resonance decay contribution cannot be neglected. Since $f$ meson production cross sections are small compared with $p$ meson according to the experiments, we consider only $p$ meson contributions. The experiments show that $x_F \frac{d\sigma}{dx_F} (\pi^+ \rightarrow \rho^0) \approx 2.8$ mb for $x_F = 0.5 - 1$ at $P_L = 16$ GeV/c. Here we take an approximation that $\rho$ is produced with $P_\rho = 0$ only and decay isotropically at the rest system of $p_0$. In this case we can easily estimate the contribution of the cascade decay $\pi^+ \rightarrow \rho^0 \rightarrow \rho^- \rightarrow \pi^- \pi^0$ to $\pi^-$ spectrum. The sum of QFC and $\rho^0$ decay contribution agrees reasonably with the data as seen in Fig. 16. In the integrated cross section $\frac{d\sigma}{dx_F}$, the contribution of QFC is not so visible. The data at $P_L \approx 0.8$ GeV/c are desired.

In $K^+ \rightarrow \pi^-$, QFC is that of $\pi^+ \rightarrow \pi^-$ multiplied by $\frac{\sigma_T (K^+ p) / \sigma_T (p p)}{0.7}$. Since we have no information for $K^+ \rightarrow \rho^0$, we assume that $x_F \frac{d\sigma}{dx_F} (K^+ \rightarrow \rho^0)$ is a constant for $x_F > 0.4$. We obtain a good fit for the data in the case that the constant $= 0.55$ mb as shown in Fig. 17. Due to the small cross section of $K^+ \rightarrow \rho^0$, QFC is more visible in this process. If $\sigma (K^+ \rightarrow \rho^0)$ is measured, it will test our QFC for $K^+ \rightarrow \pi^-$, though indirectly. For $p \rightarrow K^-, p \rightarrow f, A \rightarrow K^-$ can contribute. This is, however, negligibly small due to the large mass of $K$ meson and the

---

\[ r(x_F) = \int_1^{1/t} \left( \frac{1 + x_F}{2} + \frac{1 - x_F}{2} \right) \sqrt{1 - t(1 + t)^{-1}} dt. \]

---

*) A similar study has been performed for $\pi^- \rightarrow \pi^-$ using different QFC by Kinoshita et al. 36
smallness of the branching ratio of $f, A_s \rightarrow K \bar{K}$, unless $\sigma(p \rightarrow f, A_s) \gg \sigma(p \rightarrow \pi)$.

§ 7. Consistency with large $P_T$ hadron production

The quark decay functions we have used are summarized and compared with the Field-Feynman (FF) parametrizations in Fig. 18. $g_\pi^s$ has a magnitude similar to the FF parametrization. However, $\omega(z) = g_{s^+}/g_{s^0} = g_{s^+}/g_{s^0}$ is very small at large $z (z \sim 0.8)$ compared with the FF parametrization. Our model predicts a considerably larger value for $K^+/K^-$ ratio in large $P_T$ hadron productions than the FF model. Is this a trouble? According to the large $P_T$ experiments, the ratio $K^+/K^-$ is significantly larger than the prediction of the FF model, e.g., it is $\sim 15$ at $x_T=0.5$ in the data and $\sim 7$ in the FF model, although the data are not accurate. Our model predicts for $K^+/K^-$ ratio at 90° in large $P_T$ hadron productions in $p\bar{p}$ collisions,

$$K^+/K^- \sim (1-x_T)^{4-5},$$

which is just an observed behavior. Therefore, the smallness of $\omega(z)$ is not a trouble but rather an advantage.

We have used $g_\pi^s = 1/4 (1-z)^{1/2}$. This predicts for the particle ratio in large $P_T$ hadron productions in $p\bar{p}$ collisions,

$$\frac{\bar{p}}{\pi^-} \sim (1-x_T)^{1.5},$$

which is in good agreement with the data $((1-x_T)^{4-2})$.

While the value of $\lambda (=0.22)$ is consistent with the one in deep inelastic electro-production $\lambda = 0.2 \sim 0.3$, it is smaller than the value in large $P_T$ experiments $\lambda = 0.5$. However, this may not be a trouble since $\lambda$ in large $P_T$ experiments is the one near $z = 1$ due to the trigger bias effect.

As a whole, our quark decay functions are consistent with large $P_T$ experiments.

§ 8. Summary and remarks

In the previous sections we have studied small $P_T$ inclusive spectra based on
the two-component model with QFC predicted by our model. We have explained many processes successfully. Our model enjoys this success keeping the consistency with large $P_T$ hadron productions, not only in quark decay functions but also in $P_T$ distributions.

We have explained interesting $P_T$ (or $t$)-dependences by the two-component hypothesis. If the predicted dip-bump structure of $\alpha_{\text{eff}}$ in $\pi^+ \rightarrow \pi^0$ is confirmed experimentally, it will be a powerful evidence of this hypothesis.

The $a\bar{c}$ exotic rule for an early scaling found by Whitmore et al.,\textsuperscript{16} can be understood in QFM by the following argument. In the case that $a\bar{c}$ is not exotic, a valence quark in hadron $a$ can flow into detected hadron $c$ as shown in Fig. 19, where we take the center of mass system of $ab$. The contribution of this quark exchange diagram should vanish at high energy in a parton view since quark $f$ changes its direction by $180^\circ$ and receives a momentum transfer squared of $O(s)$. However, this diagram can give some contributions at relatively low energy.

In the two-component model, a rule for an early scaling is divided into 3 patterns. In the region where QFC is dominant, $a\bar{c}$ exotic rule works. If TRC is dominant, $ab\bar{c}$ exotic rule is expected to work. If both contribute significantly, both $a\bar{c}$ and $ab\bar{c}$ exotic must be satisfied. Whitmore et al. have studied $p \rightarrow \pi$ for $x_T<0.85$. In this process QFC is dominant as shown in this paper. Therefore $a\bar{c}$ rule holds. There is, however, the case that $a\bar{c}$ is exotic but quark $f$ exists, e.g., $p^+ \rightarrow \Sigma^-$. Therefore our rule does not coincide with $a\bar{c}$ rule in special cases. This point should be further studied.

Many important processes remain to be studied, e.g., $p \rightarrow$ hyperon and $M \rightarrow B$. Especially, $M \rightarrow B$ is important for the discrimination of models. While in our model $\text{QFC}(\pi^+ \rightarrow \rho) \sim (1-x_T)^{b_1}$ (see Table 1) as in $p \rightarrow \pi$, it is expected that the behavior of $\pi \rightarrow p$ is very different from that of $p \rightarrow \pi$ in the recombination model. These will be discussed elsewhere.

A comparison of $b \rightarrow c$ with its line reversed process $\bar{c} \rightarrow \bar{b}$ is very interesting. In the Regge pole model, cross sections of these processes are expected to be equal. In QFM, however, they are not equal in many cases. For example, $\text{QFC}(\pi^+ \rightarrow K^+)$ is significantly smaller due to the $\lambda$ factor than $\text{QFC}(K^- \rightarrow \pi^-)$ (see Eqs. (30) and (31)). Future experiments will be able to easily test this point.

References

A Model for Small $P_T$ Inclusive Hadron Productions

   See also, Prog. Theor. Phys. 46 (1971), 550.
18) M. G. Albrow et al., CERN Preprint (1977),