Asymmetries of Polarized Electron Scattering at Very High Energies

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The calculations of asymmetries of polarized electron-proton scattering at $s=10^2 \sim 10^3 \text{GeV}^2$ are performed within $SU(2) \times SU(1)$, $SU(3) \times SU(1)$, $SU(2)_L \times SU(2)_R \times SU(1)$ and $SU(3) \times SU(2)_L \times SU(3)_R \times SU(1)$ gauge models. Determining the allowed regions of parameters in each model compatible with the present experiments, we predict the behavior of the asymmetries at high energy ($s \ll 10^6 \text{GeV}^2$). The future experiments on high energy $e-p$ collisions, for example at TRISTAN, may give some important information to discriminate the various gauge symmetry models.

At present, it seems that the weak interactions of order $G_F$ are described by the standard Weinberg-Salam type $SU(2) \times SU(1)$ gauge theory. This has been suggested by model independent phenomenological analyses of neutrino-induced neutral current interactions, and confirmed by an analysis within a single-Z-boson hypothesis using the parity violating effects in polarized $e-d$ scattering observed at SLAC.

However, the present information on neutral currents is concerned only in neutrino reactions and in the parity violating terms of electron reactions. We have no information as for the parity conserving terms of them. Thus, it is possible to consider models in which the weak interactions of order $G_F$ are not described by the Weinberg-Salam type description; the effective Lagrangian of neutrino reactions and the parity violating terms of electron reactions is essentially equivalent to that of the standard model, but that of the parity conserving terms is not. Several authors have proposed such a model within a gauge group $SU(2)_L \times SU(2)_R \times SU(1)_L \times SU(1)_R$. To determine the structure of the weak interactions of order $G_F$, we need to investigate the parity conserving terms of electron reactions.

Furthermore, it is worth considering that there exist some higher gauge symmetries than $SU(2) \times SU(1)$, even though we grant the standard model. At high energies, there may be some higher gauge symmetries, for example, $SU(3) \times SU(1)$ and $SU(2)_L \times SU(2)_R \times SU(1)$, which break down spontaneously to $SU(2) \times SU(1)$ with mass scales larger than $M_\alpha$. In these cases, the weak interactions of order $G_F$ at low energies are approximately described by the standard model, but the deviations from it grow large at very high energies; $Q^2 \gg M_\alpha^2$. $SU(3) \times SU(1)$ gauge models are considered as an expression of the vectorlike theory, and $SU(2)_L \times SU(2)_R \times SU(1)$ gauge model has been proposed from the considerations...
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of the left-right gauge symmetry.

Even though there are various phenomenological constraints to the structure of the weak interactions at low energies, the abundant structure of them is expected at $s \gtrsim M_W^2$. A piece of the important information of them will be obtained from the $e-p$ colliding experiments, for example, at TRISTAN ($s \lesssim 10^3 \text{GeV}^2$).

The purpose of this paper is to predict the asymmetries of polarized $e^\mp p$ scattering at $s=10^2 \sim 10^5 \text{GeV}^2$ based on these gauge models. The asymmetries are defined by

$$A^- = \frac{\sigma(e^+_p) - \sigma(e^-_p)}{\sigma(e^+_p) + \sigma(e^-_p)},$$

(1a)

$$A^+ = \frac{\sigma(e^+_p) - \sigma(e^-_p)}{\sigma(e^+_p) + \sigma(e^-_p)},$$

(1b)

where $\sigma(ep)$ is the deep-inelastic electron-proton scattering cross section at given $x, y$ and $s$, where $x$ and $y$ are usual scaling variables and $Q^2 = s x y$. These asymmetries are caused by the parity violating terms of the weak neutral current interactions. At low energies, they are in proportion to the strength of the parity violating terms as shown by several authors. At high energies, however, the contributions of the parity conserving terms to the denominator are not negligible, so that we can obtain some information on both parity conserving and violating terms.

The calculations are made in the framework of the quark parton model using parton distribution functions parametrized by Barger and Phillips. The QCD corrections give significant deviations to $\sigma(ep)$ from the scaling limit predictions. However, we show later that their corrections to the asymmetries are sufficiently small, which are calculated using the $Q^2$-dependent parton distribution functions parametrized by Buras and Gaemers. Thus, the asymmetries are good sources of information on the weak interactions at high energies.

First, we describe the models studied in this paper briefly. To obtain allowed regions of parameters in each model, we use the results of Abbott and Barnett and the asymmetry data of polarized $e-d$ scattering at SLAC.

A. standard model

$$\begin{pmatrix} y \\ e \end{pmatrix}_L, \begin{pmatrix} u \\ d \end{pmatrix}_L,$$

(2)

We write here only an electron sector, and the other sectors $(\mu, \tau, \cdots)$ form the same type of the multiplets. The same ellipsis is made in the following models. Neutral current interactions are described by

$$\mathcal{L} = \frac{g}{\cos \theta} Z_e \left( \frac{1}{2} \bar{\nu} \gamma^\mu L \nu - \frac{1}{2} \bar{\nu} \gamma^\mu L e \right)$$
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\[ + \frac{1}{2} u^\tau Lu - \frac{1}{2} \bar{d}^\tau Ld \]

\[-\sin^2 \theta \cdot J_{\text{em}}^x, \]

(3)

where \( J_{\text{em}}^x \) is the electromagnetic current, and \( L = \frac{1}{2} (1 - \gamma_5) \) and \( R = \frac{1}{2} (1 + \gamma_5) \).

In the standard scheme, Z-boson mass is given by vacuum expectation values of doublet Higgs scalars as

\[ M_Z^2 = \frac{M_W^2}{\cos^2 \theta} = \frac{g^2}{8 \cos^2 \theta} \sqrt{2} G_F. \]

(4)

In this case, Weinberg angle \( \theta \) is determined as \( \sin^2 \theta = 0.22 \sim 0.24 \). If we take \( M_Z \) as a free parameter, \( \sin^2 \theta = 0.18 \sim 0.24 \) are allowed and a little larger values of \( M_Z \) than Eq. (4) are favored.

B. SU(3) \( \times U(1) \) gauge model

\[
\begin{pmatrix}
\nu_e \\
e
\end{pmatrix}
\begin{pmatrix}
\nu_e \\
e
\end{pmatrix}
;
\begin{pmatrix}
u_e \\
E
\end{pmatrix} U_L \begin{pmatrix}
u_e \\
E
\end{pmatrix} U_R.
\]

(5)

In this model, we have two massive neutral gauge fields \( Z \) and \( X \). One of them, \( Z \), is defined to interact in the same way as Eq. (3). The other gauge field, \( X \), interacts as

\[
\mathcal{L} = \frac{g \cos \theta}{\sqrt{3 - 4 \sin^2 \theta}} X_\mu \left\{ \frac{1}{2} \bar{\nu}_e \gamma^\mu L \nu_e + \frac{1}{2} \bar{\epsilon} \gamma^\mu L \epsilon - \bar{\epsilon} \gamma^\mu R \epsilon \\
+ \frac{1}{2} \bar{\nu}_e \gamma^\mu L \nu_e + \frac{1}{2} \bar{d} \gamma^\mu L d - \bar{d} \gamma^\mu R d \\
+ \frac{1}{2} \tan^2 \theta (\bar{\nu}_e \gamma^\mu L \nu_e - \bar{\epsilon} \gamma^\mu L \epsilon + \bar{u} \gamma^\mu L u - \bar{d} \gamma^\mu L d - 2 J_{\text{em}}^x) \right\}. \]

(6)

The gauge fields \( Z \) and \( X \) are not necessarily mass eigenstates. We define mixing angle \( \alpha \) as

\[
\begin{pmatrix}
Z_1 \\
Z_2
\end{pmatrix} = \begin{pmatrix}
\cos \alpha & -\sin \alpha \\
\sin \alpha & \cos \alpha
\end{pmatrix} \begin{pmatrix}
Z \\
X
\end{pmatrix},
\]

(7)

where \( Z_1 \) and \( Z_2 \) are mass eigenstates of neutral weak bosons, and their masses are \( M_1 \) and \( M_2 \), respectively. \( X \)-boson contributions and mixing effects between \( Z \) and \( X \) give deviations from the standard model predictions. Parameters in the most general case of this model are \( \theta, M_1, M_2 \) and \( \alpha \). The allowed regions of
them are constrained from the results of Refs. 2) and 4), and they are complicat-
edly related with each other. It is not our purpose to show it exactly. Roughly,
the parameters are constrained as \( \sin^2 \theta = 0.18 \sim 0.30 \), \( M_2/M_1 \geq 1.4 \), and \( \alpha \) varies
from \(-0.02\pi\) to \(0.22\pi\) with varying \(\sin^2 \theta\) and \(M_2\). The value of \(M_2\) is arround
that of \(M_Z\) of the standard model.

C. \(SU(2)_L \times SU(2)_R \times U(1)\) gauge model

\[
\left( \begin{array}{c}
\nu_e \\
e \end{array} \right)_{L}, \quad \left( \begin{array}{c}
\nu_e \end{array} \right)_{R} ; \quad \left( \begin{array}{c}
u_e \\
e \end{array} \right)_{L}, \quad \left( \begin{array}{c}
\nu_e \end{array} \right)_{R},
\]
where we denote \(SU(2)_L\)-doublets in the vertical direction and \(SU(2)_R\)-doublets in
the horizontal one. The parameters in this model are defined in the same way
as those of model B. The additional neutral gauge boson, \(X\), interacts as

\[
\mathcal{L} = \frac{g \cos \theta}{\sqrt{1 - 2 \sin^2 \theta}} \mathbf{X} \left\{ \frac{1}{2} \bar{\nu}_e \gamma^5 R \nu_e - \frac{1}{2} \bar{\nu}_e \gamma^5 R e
\right. \\
+ \frac{1}{2} \bar{\nu}_e \gamma^5 R u - \frac{1}{2} \bar{\nu}_e \gamma^5 R d \\
+ \left. \frac{1}{2} \tan^2 \theta (\bar{\nu}_e \gamma^5 L \nu_e - \bar{\nu}_e \gamma^5 L e + \bar{\nu}_e \gamma^5 L u - \bar{\nu}_e \gamma^5 L d - 2 J_{\pi}^m \right\}.
\]

The allowed regions of the parameters are represented roughly as \(\sin^2 \theta = 0.16 \sim 0.34\), 
\(M_2/M_1 \geq 3\) and \(\alpha = -0.2\pi \sim 0\).

D. \(SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R\) gauge model

\[
\left( \begin{array}{c}
\nu_e \\
e \end{array} \right)_{L}, \quad \left( \begin{array}{c}
\nu_e \end{array} \right)_{R} ; \quad \left( \begin{array}{c}
u_e \\
e \end{array} \right)_{L}, \quad \left( \begin{array}{c}
\nu_e \end{array} \right)_{R},
\]
where we denote \(SU(2)_L\)-doublets in the vertical direction and \(SU(2)_R\)-doublets in
the horizontal one; \(Y_R = 0\), \(Y_L = 0\).

The general treatment of this model is complicated, because there are three massive
neutral gauge fields. Thus, we study the special case where the present neutral
current phenomenology at low energies is exactly the same as that of the standard
model. First, we assume that \(SU(2)_R \times U(1)_R\) breaks down to \(U(1)_L\) at sufficient-
ly large mass scale. We choose remaining two massive neutral gauge fields as
interacting by

\[
\mathcal{L} = g \sin \theta \cdot Z^\nu \left\{ \bar{\nu}_e \gamma^\nu \gamma^\nu e - \frac{2}{3} \bar{\nu}_e \gamma^\nu \gamma^\nu u + \frac{1}{3} \bar{\nu}_e \gamma^\nu \gamma^\nu d
\right. \\
+ \frac{g}{\sqrt{1 - 2 \sin^2 \theta}} Z^\nu \left\{ \frac{1}{2} \bar{\nu}_e \gamma^\nu L \nu_e - \frac{1}{2} \bar{\nu}_e \gamma^\nu L e \\
+ \frac{1}{2} \bar{\nu}_e \gamma^\nu L u - \frac{1}{2} \bar{\nu}_e \gamma^\nu L d - 2 \sin^2 \theta (\bar{\nu}_e \gamma^\nu L e \\
+ \frac{2}{3} \bar{\nu}_e \gamma^\nu L u - \frac{1}{3} \bar{\nu}_e \gamma^\nu L d \right\}.
\]
Furthermore, we assume that the mass eigenvalues of neutral weak bosons, $M_i$ and $M_\nu$, are given by

$$\frac{M_\nu^2}{M_i^2} = (1-2 \sin^2 \theta) \left(1 + \sqrt{\frac{\sin^2 \theta}{1-2 \sin^2 \theta}} \cos \frac{\alpha}{\sin \frac{\alpha}{2}}\right), \tag{12a}$$

$$\frac{M_\nu^2}{M_i^2} = (1-2 \sin^2 \theta) \left(1 - \sqrt{\frac{\sin^2 \theta}{1-2 \sin^2 \theta}} \sin \frac{\alpha}{\cos \frac{\alpha}{2}}\right), \tag{12b}$$

where $\alpha$ is the mixing angle of $Z^\prime$ and $Z''$ defined as Eq. (7), and $0 < \tan \alpha < \sqrt{\sin^2 \theta/(1-2 \sin^2 \theta)}$. $M_i$ goes to zero as $\alpha \to 0$, and in this case $M_\nu = (M_{Z^0} \text{ of the standard model})$. By these two assumptions, the neutrino-induced neutral current and the parity violating terms of the electron-induced one in this model have exactly the same structure as the standard model at low energy limit in contrast to the high energy behavior and the parity conserving terms. In this model, $\sin^2 \theta = 0.22 \sim 0.24$ and $\alpha = 0 \sim 0.22 \pi$ from the low energy phenomenology.

Model B is a vectorlike model. However, the parity is still violated at infinite energy as for the ordinary fermions, because the fermion mass eigenstates are in the different representations between left- and right-handed multiplets. Model C and D are left-right gauge symmetric ones. In these models, the parity violation vanishes at infinite energy, where gauge boson masses can be neglected, and the asymmetry predictions are expected to be small compared with those of the standard model.

Now, we describe the results of our calculations. We calculate the asymmetries versus $s$ ($s \leq 10^4 \text{ GeV}^2$) at $x=0.5$ and $y=0.5$. The kinematical behavior of them depends mainly on $Q^2 = s x y$, so that we can see easily their behavior such as propagator effects in $s(Q^2)$-dependence at fixed $x$ and $y$. At smaller $x$, the QCD corrections may give the significant deviations from the exact scaling predictions, and the behavior of them depends on the structure of the quark distribution functions. In models A, B and C, we calculate them in whole allowed region of parameters as mentioned above to clarify the discrimination of gauge models.

The predictions of the standard model are given in Fig. 1, where the QCD corrections are also displayed. As shown in Fig. 1, the QCD corrections do not give significant deviations to the results. The differences between the results using the other parton distribution functions are also small. These are also true in the other gauge models, where we do not show in this paper, but have calculated with some typical cases.

The results of the $SU(3) \times U(1)$ and $SU(2)_L \times SU(2)_R \times U(1)$ gauge models are shown in Figs. 2 and 3, respectively. The predicted regions of these models include those of the standard model, because the structure of neutral currents in these models is consistent with that in the standard model in a limit $M_e \to \infty$ and $\alpha \to 0$.

It should be mentioned that we have used only the results of Abbott and
Bernett's analysis and the asymmetry data at SLAC to obtain allowed regions of the parameters, and that the precise information of the other kinds might give further constraints to them. By a rough estimation, the regions $M_l/M_1 < 3$ in

**Fig. 1.** Predictions of the standard model at $x=0.5$ and $y=0.5$. Parameters are (a) $\sin^2\theta=0.18, M_2=100$ GeV and (b) $\sin^2\theta=0.23, M_2=88.6$ GeV which satisfies Eq. (4). It is noted that the predictions for $A^+$ with these parameters are crossed, but almost of those with the other parameters are between these two curves. Dashed curves are calculated with QCD corrections for case (b).

**Fig. 2.** Predictions of the $SU(3) \times U(1)$ gauge model B. Shaded areas are the predicted regions. Dashed curves are the lower bounds with constraint $M_l/M_1 > 3$. 

Fig. 2 are not favorable to the elastic $\mu^+e^-$ scattering data at Aachen-Padova.\cite{121}

The positron asymmetry $A^+$ in model B and the electron asymmetry $A^-$ in model C change their sign at $Q^2 \gtrsim 10^4 \text{GeV}^2$. Typical behavior of the predictions in these models are shown in Fig. 4.

In the $SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$ gauge model, we calculate in the special case, Eqs. (11) and (12), with parameters $\sin^2 \theta = 0.23$ and some values of $\alpha$. The results are shown in Fig. 5. This model gives the predictions prominently deviated from the standard model at small $\alpha$. This is due to the different structure of the parity conserving neutral current and small value of $M_\xi$.

As a summary, the discrimination of the various gauge symmetry models will be clarified at $s \gtrsim 10^4 \sim 10^5 \text{GeV}^2$ by the asymmetries of $e^+\mu^-$ scattering. Especially, the $SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$ gauge model predicts them from the standard model at relatively low energies ($Q^2 \approx 10^4 \text{GeV}^2$). At the TRISTAN energy ($s = 24000 \text{GeV}^2$), the $SU(3)_C \times U(1)_Y$ and $SU(2)_L \times SU(2)_R$ gauge models are slightly distinguishable. In these models, a neutral gauge boson may have a smaller mass than $M_\xi$, though the parameters of each model are constrained from the present phenomenology. If that is the case, some information of these gauge models may be obtained at TRISTAN by a careful observation of propagator effects. As the QCD corrections of the asymmetries are small at $x = 0.5$, the asymmetries are good observables to find propagator effects of the neutral gauge bosons.

Finally, further constraints on neutral current parameters in models B and C can be obtained from more precise experiments at low energies including the atomic parity violations. However, it is difficult to determine the parameters $\alpha$ and $r = M_\eta/M_\xi$ ($\alpha \lesssim 0.6, r \gtrsim 3$) more precisely. If $\alpha = 0$ and $r = \infty$, gauge symmetry is
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Fig. 4. Predictions of (a) the model B with $\sin^2\theta=0.2$, $M_1=96\,\text{GeV}$, $M_4/M_1=3$ and $\alpha=0$ and (b) the model C with $\sin^2\theta=0.2$, $M_1=96\,\text{GeV}$, $M_4/M_1=3$ and $\alpha=-0.02\pi$. 

Fig. 5. Predictions of the $SU(2)_L\times SU(2)_R \times U(1)_L \times U(1)_R$ gauge model with $\sin^2\theta=0.23$ and (a) $\alpha=0.02\pi$ ($M_1=31.4\,\text{GeV}$, $M_4=108.0\,\text{GeV}$), (b) $\alpha=0.10\pi$ ($M_1=61.0\,\text{GeV}$, $M_4=119.2\,\text{GeV}$) and (c) $\alpha=0.22\pi$ ($M_1=79.1\,\text{GeV}$, $M_4=156.0\,\text{GeV}$). Dashed curves denote sign-exchanged values.

not $SU(3)\times U(1)$ nor $SU(2)_L \times SU(2)_R \times U(1)$, but $SU(2) \times U(1)$. It will be clarified at very high energies ($s\gtrsim 10^4\sim 10^5\,\text{GeV}^2$) whether $\alpha$ and $r^{-1}$ are truly consistent with zero or not.
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