Calculating palaeomagnetic poles for oceanic plates

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Summary. The information available for determining palaeopoles for oceanic plates includes data of four types: (1) palaeomagnetic measurements of inclinations but not declinations from core samples; (2) palaeopoles determined from magnetic anomalies over seamounts; (3) phase shifts of linear magnetic anomalies; and (4) palaeoequators determined from geological analysis of sediment cores. A method is presented which combines these different kinds of data and their respective confidence limits to yield a best fit pole, a confidence ellipse, estimates of the data importances and goodness of fit parameters. Using this method we calculate the Campanian pole position for the Pacific plate which is shown by a chi-squared test to be a reasonable fit to the data. We conclude that the method should be a useful tool for calculating pole paths for oceanic plates such as the Pacific and Indian plates.

Introduction

Polar wander paths for oceanic plates are presently known less accurately than are the paths for continental plates for several reasons. A continental palaeopole may be determined from the mean inclination and declination of a set of completely oriented samples of appropriate age at one tectonically stable site. Moreover, well established analytical techniques are available for combining all of the palaeomagnetic data from a given plate to yield a continental polar wander curve with well determined confidence limits. On oceanic plates, on the other hand, the geological or palaeomagnetic data from one site rarely determines a palaeopole completely, the data available at different sites are generally qualitatively different, and as yet analytical techniques are not available for combining the data to determine palaeopoles.

The following are the four main types of palaeopole data available from oceanic plates. (1) Cores from deep sea drilling are generally azimuthally unoriented and therefore yield only a mean palaeomagnetic inclination. This constrains the palaeopole to lie on a small *Present address: Department of Geological Sciences, Northwestern University, Evanston, Illinois 60201, USA.
circle centred on the sample site. (2) The shape of lineated magnetic anomalies may be parameterized as a phase shift which constrains the palaeopole to lie on a specific half great circle (Schouten & Cande 1976). (3) Palaeoequators inferred from marine sediment cores (van Andel 1974) constrain the palaeopole to be on a great circle centred on the sampling site. (4) The palaeomagnetic inclination and declination obtained by the inversion of magnetic anomalies over seamounts yield palaeopoles comparable with those available from continental palaeomagnetic studies, but with less well determined confidence limits.

In this paper we will present a method for combining such dissimilar types of data from many sites into a best fitting pole position. The technique is based on the principle of maximum likelihood and utilizes non-linear least squares and standard statistical techniques. Our approach is similar to that of Chase (1972) and Minster et al. (1974) for determining instantaneous relative plate motions. Palaeomagnetic data, palaeoequator data, and their error estimates are used as input, and the best fitting pole position, confidence limits, data importances, and goodness of fit parameters are output.

**Formulation of the problem**

Our method depends on the fact that given the coordinates of a 'model' palaeomagnetic pole, then all four types of palaeopole observations can be predicted by functions of that modelled pole position. We represent this palaeomagnetic pole model by a unit vector, \( \mathbf{m} \), which has coordinates, \( \lambda \) (latitude) and \( \phi \) (longitude). Observations at a given site with coordinates \( \lambda_i \) and \( \phi_i \) can be represented by either one or two numbers, depending on the type of data available from that site. For example, the mean inclination from a drilled core can be represented by one number, the colatitude, which is the distance from the site to the pole and is uniquely determined by the mean inclination. Palaeomagnetic data from a seamount can be represented by two numbers, the latitude and longitude of the palaeopole determined from anomaly measurements made over the seamount. In both cases, the 'observed data' in the present sense represent raw observational data that have been transformed by simple calculations into a form that can be compared with values predicted by the model pole.

In this paper we will use the same symbols to represent a particular data type whether it is observed or predicted. We will use the superscript 0 to distinguish between these two cases. For example we will use the symbol \( c^0_i \) to represent the \( i \)th datum, which is a palaeomagnetic colatitude (the symbol \( c \)). In contrast, \( c(m)_i \) is the predicted value of the \( i \)th datum which is a colatitude which depends on the model \( \mathbf{m} \). When we are talking about data and 'predicted data' in the general case, we will use the symbol \( s^0_i \) to represent the \( i \)th datum and \( s(m)_i \) the prediction of the \( i \)th datum given \( \mathbf{m} \).

We can describe the complete set of observations from \( N \) sites represented by one number and \( M \) sites represented by two numbers as an \((N + 2M)\) dimensional vector \( \mathbf{s}^0 \). The predicted value, \( s(m)_i \), of the \( i \)th datum is determined from a function which is different for each different type of observation. The value of a 'predicted' observation depends on \( \mathbf{m} \), on the type of observational data, on the coordinates of the site, and on other site-specific parameters such as the azimuth of lineated magnetic anomalies. For example, at a seamount the usual 'observed data' are the coordinates of a palaeopole, found by least squares inversion of the magnetic anomaly over the seamount, so the 'prediction' is given by

\[
\mathbf{s}_i = \mathbf{m} = (\lambda_p, \phi_p).
\]  

For data of this type, the appropriate function linking the data to the model is a unit multiplier. For other types of data, the appropriate functions are more complicated.
If the model fits the observed data perfectly, we have
\[ s^0 = s(m). \] (2)
Generally no value of \( m \) will exactly satisfy the above equation, so we will seek the value of \( m \) that minimizes the squared 'distance' between the vectors \( s^0 \) and \( s(m) \) taking into account the estimated error, \( \sigma \), of the data at each site. For observations described by a single number, the squared error, \( (\delta s_i)^2 \), is given simply by
\[ (\delta s_i)^2 = [s^0_i - s_i(m)]^2. \] (3)
For observations described by two numbers (generally the latitude and longitude of a pole position) such a simple relationship does not, in general, exist if ordinary geographical coordinates are used. However, by describing the observed datum and 'predicted' datum in coordinate systems described later in this paper, the squared error, which is the squared distance between observed and predicted data, can be written as
\[ (\delta s_i)^2 = (\delta s'_i)^2 + (\delta s''_i)^2 = [(s^0_i)' - (s_i(m))']^2 + [(s^0_i)'' - (s_i(m))'']^2, \] (4)
where \((s^0_i)'\) and \((s^0_i)''\) are the two components of the \( i \)th observation. In our inversion the quantity to be minimized is
\[ \chi^2 = \sum_{i=1}^{N} \left( \frac{\delta s'_i}{\sigma'_i} \right)^2 + \sum_{i=N+1}^{N+M} \left( \frac{\delta s''_i}{\sigma''_i} \right)^2, \] (5)
where \( \sigma'_i \) and \( \sigma''_i \) are the standard errors in the two components of the \( i \)th observation.

The minimum occurs at a stationary value of \( \chi^2 \), i.e. when
\[ \frac{\partial \chi^2}{\partial \lambda_p} = \frac{\partial \chi^2}{\partial \phi_p} = 0. \] (6)
(Maxima, saddle surfaces, etc., also satisfy equation (6) but are easily distinguished from minima by evaluation of second derivatives and various numerical tests and present no problem in practice.)

The above minimization problem has several interesting and useful statistical interpretations, the discussion of which is simplified if we assume that uncertainties in our observations are normally distributed, an excellent approximation if the angular dispersions are small. We further assume that the random errors from different sites are mutually independent and that all palaeomagnetic and palaeoequator data of approximately the same age were produced by the same palaeomagnetic pole. Since the time interval that a particular data set encompasses will often be long enough for significant apparent polar wander to have taken place, this last assumption will, in general, be only approximately true.

Under the assumption of normality, given a 'true' pole \( m' \) the probability density, \( P_i \), of making a particular observation, \( s^0_i \), is given by
\[ P_i(m') = \frac{1}{o_i \sqrt{2\pi}} \exp \left( -\frac{1}{2} \left( \frac{\delta s'_i}{\sigma'_i} \right)^2 \right) \] if \( s^0_i \) is a single component datum and
\[ P_i(m') = \frac{1}{o'_i o''_i \sqrt{2\pi}} \exp \left( -\frac{1}{2} \left[ \left( \frac{\delta s'_i}{\sigma'_i} \right)^2 + \left( \frac{\delta s''_i}{\sigma''_i} \right)^2 \right] \right) \] if \( s^0_i \) is a two component datum. The likelihood function for making a given set of measurements is given by the product of the individual probability densities
\[ P(m') = \prod P_i(m'), \] (9)
where the product is taken for \( i \) ranging from 1 to \( N + M \). Although the coordinates of the
true pole $\mathbf{m}$ are not known, for any assumed model pole position we can always calculate by equation (9) the likelihood of making the observations that comprise our data set. In the method of maximum likelihood, the best estimate of the model vector, $\mathbf{m}$, is that which yields the maximum probability density or likelihood of making the observed set of measurements, $s^0$. Substituting equations (7) and (8) into equation (9) and rewriting, we obtain

$$P(\mathbf{m}) = \left( \frac{1}{\sqrt{2\pi}} \right)^{N+2M} \exp \left\{ -\frac{1}{2} \left[ \sum_{i=1}^{N} \frac{\delta s_i^p}{\sigma_i} \right]^2 + \sum_{i=N+1}^{N+M} \left[ \frac{\delta s_i^p}{\sigma_i} + \frac{\delta s_i^n}{\sigma_i} \right]^2 \right\}. \quad (10)$$

$P(\mathbf{m})$ is maximized when the sum

$$\chi^2 = \sum_{i=1}^{N} \left[ \frac{\delta s_i}{\sigma_i} \right]^2 + \sum_{i=N+1}^{N+M} \left( \frac{\delta s_i^p}{\sigma_i} + \frac{\delta s_i^n}{\sigma_i} \right)^2 \quad (11)$$

is minimized. Thus the least squares minimization problem is equivalent to maximizing the likelihood of the parent probability distribution.

In a strictly linear theory the variable $\chi^2$ would be chi-squared distributed with $N+2M-2$ degrees of freedom because we estimate two parameters, $\lambda_p$ and $\phi_p$, in minimizing the sum of squares of $N+2M$ standard normal variables. We will assume for our problem that any non-linearity is weak. Thus we may assume that $\chi^2$ is approximately chi-squared distributed with $N+2M-2$ degrees of freedom by analogy with linear statistical theory. Chi-squared distributions as tabulated in standard references can be used to test whether or not the modelled pole position and the assumed parent probability function fit the data in a reasonable manner.

The chi-squared statistic permits us to test if the fit of the model to the data is not good enough, sufficiently good, or 'too good'. If the fit is not good enough, one of our underlying assumptions may be in error. For example, we might suspect that our assigned uncertainties are too small, that the data are from rocks of different ages, or even that the Earth's average palaeomagnetic field is not dipolar. A value of $\chi^2$ which is too small indicates that our fit is 'too good' and would lead us to suspect that we have overestimated the uncertainties in the data.

For the rest of the summations in this paper it will prove simpler to adopt the following convention for subscripts of two-component data: we will drop the primes so that equation (11) becomes

$$\chi^2 = \sum_{i=1}^{N+2M} \left[ \frac{s_i^0 - s_i(\mathbf{m})}{\sigma_i} \right]^2 = \sum_{i=1}^{N+2M} \left[ \frac{\delta s_i}{\sigma_i} \right]^2, \quad (12)$$

where $\sigma_{N+1}$ and $\sigma_{N+2}$ are the uncertainties of the first and second components, respectively, of the first two-component datum. Similarly $\sigma_{N+3}$ and $\sigma_{N+4}$ are the uncertainties of the first and second components, respectively, of the second two-component datum. We adopt the same convention for the subscripts of $s_i^0$, $s_i(\mathbf{m})$, $\delta s_i$, etc.

**Numerical solution**

We now return to the problem of minimizing the sums of squares of the difference between observed and 'predicted' palaeopole data. We must simultaneously solve the equations

$$\frac{\partial \chi^2}{\partial m_k} = -2 \sum_i \left[ \frac{s_i^0 - s_i(\mathbf{m})}{\sigma_i^2} \right] \frac{\partial s_i(\mathbf{m})}{\partial m_k} = 0, \quad k = 1, 2, \quad (13)$$
where \( m_1 = \lambda_p \) and \( m_2 = \phi_p \). By \( \partial s_i(m) / \partial \lambda_p \), we mean the derivative of the appropriate function which predicts the \( i \)th observation evaluated at \( m \) and the appropriate site dependent parameters. To keep from proliferating more symbols, we will use \( s_i(m) \) to mean both the value of the predicted observation of the \( i \)th datum and the appropriate function which yields the prediction, given \( m \) and the site-dependent parameters. The sense should be clear from the context. In general the functions, \( s_i(m) \), are not linear and the above equations do not have a closed form solution.

To find the minimum we linearize the problem by starting with a good trial solution, \( m^* \), solving for the best incremental improvement, \( \delta m \), and iterating. (If \( m^* \) is near one of the geographic poles of the spherical coordinate system, numerical problems can arise because of strong non-linearities. Difficulties are avoided in such cases by rotating to a spherical coordinate system for which \( m^* \) is near the equator.) Let

\[
m = m^* + \delta m.
\]

It can be shown (Bevington 1969) that the solution is given by

\[
\delta m = (A^T V^{-1} A)^{-1} A^T V^{-1} \delta s
\]

where

\[
A_{ik} = \frac{\partial s_i(m^*)}{\partial m_k},
\]

\[
V_{ij} = \begin{cases} \sigma_i^2, & \text{if } i = j \\ 0, & \text{if } i \neq j \end{cases}
\]

and \( \delta s \) is evaluated at \( m = m^* \). To find the solution to the original non-linear problem, we form a new trial estimate

\[
m_{\text{new}} = m_{\text{old}} + \delta m
\]

and iterate the process until \( \delta m \) is smaller than some \textit{a priori} criterion. To guard against the possibility of converging in a local minimum, we have routinely started the program off in several different trial pole positions and have always found convergence to the same minimum.

**Estimation of confidence limits**

Confidence limits were calculated by estimating the uncertainties in the final model induced by the uncertainties in the input data. To do this we have used standard linear propagation of errors, namely

\[
\sigma_\lambda^2 = \sum_i \sigma_i^2 \left( \frac{\partial \lambda}{\partial s_i} \right)^2,
\]

\[
\sigma_\phi^2 = \sum_i \sigma_i^2 \left( \frac{\partial \phi}{\partial s_i} \right)^2,
\]

\[
\sigma_{\lambda \phi} = \sum_i \sigma_i^2 \left( \frac{\partial \lambda}{\partial s_i} \frac{\partial \phi}{\partial s_i} \right).
\]

where \( \sigma_i^2 \) refers to the assigned variance of the \( i \)th observation, \( \sigma_\lambda^2 \) is the estimated variance in the model latitude, \( \sigma_\phi^2 \) is the estimated variance in the model longitude, and \( \sigma_{\lambda \phi} \) is the latitude–longitude covariance.
These values are most conveniently calculated from the matrix \( W \), where

\[
W = (A^T V^{-1} A)^{-1}
\]

which is the variance–covariance matrix of \( \lambda \) and \( \phi \) (Matthews & Walker 1965), i.e.

\[
W = \begin{pmatrix}
\sigma_\lambda^2 & \sigma_{\lambda\phi} \\
\sigma_{\lambda\phi} & \sigma_\phi^2
\end{pmatrix}.
\]  

Non-zero values of \( \sigma_{\lambda\phi} \) indicate that the semi-axes of the confidence ellipse are not north–south and east–west. The orientation of the major axes may be found in \((\lambda, \phi)\) space by diagonalizing \( W \). Details of calculation of the magnitude and orientation of the principal axes of the error ellipses are given in Appendix B.

**Estimation of data importances**

The data importances (Minster et al. 1974) are defined as the diagonal elements of a matrix, \( Q \), where

\[
Q = V^{-1/2} A W A^T V^{-1/2}.
\]

\( Q \) is a square matrix of dimension \( N + 2M \) so that data described by one or two numbers have one or two importances, respectively. Data importances are a measure of the information content of each datum. The two importances of palaeomagnetic poles used as input can be summed to determine the total importance for that pole. The importances of all data sum to two in the present problem in which \( m \) has two degrees of freedom. The importance of a datum depends on the assigned uncertainty of that datum and the effect which that uncertainty induces into the uncertainty of the best fit model. In addition, for models with more than one parameter, such as our estimation of a pole position, the importances depend on how linearly independent each datum is from the others when mapped into model space. For example if 10 data constrain the latitude of a pole position and only one datum constrains the longitude, the latter datum will be as 'important' as the sum of the former 10 data, even if they have much smaller uncertainties, as the one datum completely determines one component of the model. The properties of data importances are discussed in more detail by Minster et al. (1974).

**Discussion of data types**

In the following sections, we discuss all of the types of data available as input in the calculation of palaeopoles for an oceanic plate. For each data type we will discuss how data are collected and reduced and how uncertainties are estimated. We then present the functions, \( s_j(m) \), which 'predict' the data given a pole position, \( m \). The partial derivatives, \( A_{ik} \), of these functions with respect to the model parameters, \( \lambda_p \) and \( \phi_p \), are given in Appendix A.

**PALAEOMAGNETIC INCLINATIONS FROM CORES**

The most abundant type of palaeomagnetic data from oceanic plates is the inclination-only data obtained from measurements of azimuthally unoriented samples obtained from deep sea drilling. From the inclination, \( I \), measured for each specimen or stratum, a single palaeocolatitude, \( c \), is calculated using the equation

\[
c = \cot^{-1}(0.5 \tan I).
\]
The mean colatitude for the site must be corrected for bias introduced by geometrical effects due to the angular dispersion of palaeomagnetic vectors. A method for determining this correction is given by Cox (1980). The observed datum for one drill site consists of the corrected palaeocolatitude \( c_i^0 \).

As a preliminary to defining the function \( c_i(m) \), we first define the unit site location vector

\[
u_i = u_i(\lambda_i, \phi_i)
\]

(26)
determined by the coordinates of the sampling site. We next define the cosine of the angle between \( u_i \) and \( m \):

\[
g_i(u_i, m) = (u_i \cdot m) = \cos \lambda_p \cos \lambda_i \cos(\phi_p - \phi_i) + \sin \lambda_p \sin \lambda_i.
\]

(27)

Then the function that predicts a palaeocolatitude from drill core data is simply

\[
c_i = \cos^{-1} g_i.
\]

(28)

In the absence of noise, the predicted value, \( c_i(m) \), will equal the observed datum \( c_i^0 \). This condition is satisfied provided the model pole, \( m \), lies on a small circle of radius \( c_i^0 \) centred on the site, \( u_i \) (Fig. 1). An observed datum of this type constrains one degree of freedom of the model pole, \( m \). The quantity to be minimized in the least squares solution is

\[
\frac{\delta s_i^2}{\sigma_i} = \left( \frac{c_i^0 - c_i(m)}{\sigma_i} \right)^2,
\]

(29)

where \( \sigma_i \) is the standard error of \( c_i^0 \).

Figure 1. Solid circle at centre of plot is DSDP site 192 on Meiji seamount at the northern end of Emperor seamount chain. Given a mean colatitude of \( 72^\circ \pm 12^\circ \) (95 per cent confidence limits) calculated from the palaeomagnetism of basalts obtained by drilling at site 192, any pole position lying on the bold circle centred on site 192 will agree with the mean colatitude exactly. Any pole position lying within the circular annulus shaded by diagonal lines will yield colatitudes at site 192 which are within the 95 per cent confidence limits of the observed colatitude.
A method for estimating confidence limits on colatitudes is given by Cox (1980). The main point of his procedure is that, in general, basalt flows obtained by deep sea drilling rarely span enough time to sample the full range of secular variation. Hence, the between-flow scatter of directions tends to underestimate the actual range of variation of the palaeomagnetic field. Cox (1980) suggests that if the between-flow scatter in the data is less than that of present secular variation, then the larger (present) estimate of scatter should be used in calculating confidence limits. He also points out that if the between-flow directions show serial correlation then the number of independent samples in time of the palaeomagnetic field is less than the number of cooling units. These points will be addressed further in the context of the Campanian pole for the Pacific plate.

**Palaeoequators**

The coordinates of palaeopoles are also constrained by palaeoequators as determined from the sedimentary record. The equatorial zone of upwelling is a region of high biological productivity. Hence sedimentation along this belt is characterized by high sedimentation rates and higher biogenic content than at other latitudes. For the Pacific plate it has been possible to identify in some DSDP cores the age at which the drill site crossed the equator and this, in turn, determines a great circle of possible palaeopole positions. A datum from a single site can thus be considered to be a palaeocolatitude of 90°. Where a large number of sites have been jointly analysed to give both the position and azimuth of palaeoequators (e.g. van Andel 1974), the datum can be considered to be both a palaeocolatitude of 90° and a ‘declination’ equal to the observed azimuth plus or minus 90°.

**Palaeopoles from Seamount Anomalies**

The palaeomagnetic inclination, \( I \), and declination, \( D \), of a seamount can be obtained by inversion of the magnetic anomaly measured over the seamount. From \( I \), \( D \) and the coordinates of the seamount the coordinates of the corresponding palaeomagnetic pole, \( p^0 \), can be found using a standard transformation. This pole, \( p^0_m \), comprises the observed data. Since in the absence of error \( p^0_m \) will correspond to the model pole, \( m \), the appropriate function for ‘predicting’ the data from the model is a unit multiplier as discussed previously. If the confidence interval about the pole is a circle, the radius of this circle provides a measure of the standard error to be used as a weighting factor in combining data in the analysis.

In the case where circular confidence limits are applicable, the squared distance between the modelled and the observed pole is given by

\[
(\delta s_I)^2 = \cos^{-1}(p^0_m \cdot m)^2. \tag{30}
\]

To write this in the form of a difference between a modelled and observed datum as in equation (4), we need to transform to a spherical coordinate system in which \( p^0 \) is at the north pole. In this case the ‘observed’ colatitude is 0° and the predicted colatitude is the colatitude of \( m \) in the new coordinate system. The problem thus reduces to the form of equation (29). The main difference is that the pole position is counted as two degrees of freedom and the ordinary colatitude is counted as one degree of freedom.

For Cretaceous seamounts in the Pacific, Harrison et al. (1975) have pointed out that groups of seamount poles may not be distributed with azimuthal symmetry about their mean, elliptical confidence intervals being more appropriate than circular ones. To analyse poles with elliptical confidence intervals, we begin by noting that for an observed palaeo-
magnetic pole, \( p_0 \), two model poles \( m_1 \) and \( m_2 \) equidistant from \( p \) will not in general be equally likely. In Fig. 2, pole \( m_2 \) aligned with the major axis is the more likely. To quantify this in a manner which yields uncorrelated errors in the observed data, we need to measure the distance of the model pole, \( m \), for the observed pole \( p_0 \) in coordinates parallel to the major and minor axes. Since \( m \) and \( p_0 \) are constrained to lie on a sphere, the coordinates of \( m \) will not be exactly parallel to the major and minor axes of the error ellipse, which lie in the plane tangent to the sphere at \( p_0 \). Therefore, one must choose between posing the problem in spherical coordinates or mapping a portion of the sphere on to a plane, where orthogonal coordinates can be used.

For consistency with the treatment given other types of data, we have chosen analysis on a sphere. To define the metric for the distance of a model pole \( m \) from \( p_0 \), we first designate two points \( t' \) and \( t'' \) which are 90° from \( p_0 \). \( t' \) is located on the great circle aligned with the major axis of the confidence ellipse and \( t'' \) on the great circle aligned with the minor axis. The coordinate system for finding the distance of \( m \) from \( p_0 \) consists of the two sets of

Figure 2. The pole position, \( p \), with elliptical confidence limits inferred from data from one site are shown equidistant from two possible modelled pole positions \( m_1 \) and \( m_2 \). If the confidence limits were circular, model poles \( m_1 \) and \( m_2 \) would fit the data equally well. However, with elliptical confidence limits on \( p \), pole \( m_1 \) is clearly a worse fit to the observations.

Figure 3. Observed pole position, \( p \), and its elliptical confidence limits is shown on the coordinate system used to measure distance on a sphere parallel to the axes of the ellipse. One set of small circles is concentric about axis \( t' \), which lies on the great circle parallel to the major axis of the ellipse. Similarly, the other set of small circles is concentric about axis \( t'' \), which lies on the great circle parallel to the minor axis of the ellipse. \( h' \) is the distance that the illustrated model pole misfits parallel to the major axis. \( h'' \) is the distance of misfit parallel to the minor axis.
small circles concentric about \( t' \) and \( t'' \) (Fig. 3). The displacement of \( m \) from \( p_i^0 \) in the direction of the major axis is
\[
h' = 90^\circ - \cos^{-1}(t' \cdot m)
\] (31)
and the displacement of \( m \) from \( p_i^0 \) in the direction of the minor axis is
\[
h'' = 90^\circ - \cos^{-1}(t'' \cdot m),
\] (32)
h' and \( h'' \) being the measures of the misfit of \( m \) and \( p_i^0 \). The quantity to be minimized in the least squares analysis is
\[
\frac{[\delta s'_i]^2}{\sigma'_i} = \frac{[\delta s''_i]^2}{\sigma''_i} = \frac{[h']^2}{\sigma'_i} + \frac{[h'']^2}{\sigma''_i},
\] (33)
where \( \sigma'_i \) and \( \sigma''_i \) are the standard errors along the major and minor axes of the confidence ellipse.

Seamount poles, unlike the palaeocolatitude and skewness data, constrain two degrees of freedom in the position of \( m \). In the present analysis, the seamount data with elliptical confidence limits from one site are equivalent to the palaeoequator data from two sites, \( t' \) and \( t'' \), the errors being \( \sigma' \) and \( \sigma'' \). Computationally, therefore, the analysis of poles from seamount sites, the analysis of palaeomagnetic data from cores, and the analysis of palaeo-equators are all reduced to the analysis of palaeocolatitudes.

Sometimes the results of the inversion of a seamount anomaly is presented in terms of palaeomagnetic declinations, inclinations, and associated confidence limits, rather than in terms of a pole position (i.e. Kodama, Uyeda & Isezaki 1978). It then may be more convenient to analyze the declination and inclination directly. The declination is given by
\[
D = -\tan^{-1}\left\{\frac{\cos\lambda_p \sin(\phi_i - \phi_p)}{-\sin\lambda_i \cos\lambda_p \cos(\phi_i - \phi_p) + \cos\lambda_i \sin\lambda_p}\right\},\quad -180^\circ < D < 180^\circ.
\] (34)
(The numerator and denominator in equation (34) yield the correct quadrant in two-variable arctangent functions on computers.) The inclination is given by
\[
I = \tan^{-1}\left\{2 \cot(c)\right\} = \tan^{-1}\left\{2 \frac{g}{(1 - g^2)^{1/2}}\right\},\quad -90^\circ < I < 90^\circ.
\] (35)

ANOMALY SKEWNESS

The shape of linear seafloor anomalies can be described by a 'skewness parameter', \( \theta_i \) (Schouten 1971; Schouten & Cande 1976), which is determined experimentally by finding the phase shift of the linear filter required to transform the anomaly into a symmetrical shape. The shape or skewness of an anomaly is determined by the following two quantities: (1) the 'effective inclination', \( e' \), of the present field, which is the inclination below the horizontal of the projection of the present geomagnetic field on to a vertical plane perpendicular to the trend of the lineations (Fig. 4a); and (2) the effective inclination, \( e^0 \), of the remanent magnetization of the seafloor, which in the case of ancient seafloor is substantially different from \( e' \). The effective inclination, \( e^0 \), of the palaeomagnetism of the seafloor is the observed datum \( (q^0) \) to be modelled. We calculate \( e^0 \) from the measured value \( \theta_i \) of the skewness parameter and the effective inclination \( e' \) of the present magnetic field at the survey site by the equation (Schouten & Cande 1976)
\[
e^0 = \theta_i - e' + 180^\circ.
\] (36)
Figure 4. (a) Block of magnetized seafloor with palaeomagnetic (or present field) vector, $F$. The $Z$ axis is down and the $X$ axis normal to the strike of the magnetized body, the $+X$ axis being in the direction in which the anomalies decrease in age. The vector, $H$, is the projection of $F$ on to the horizontal plane and $V$ is the projection of $F$ on to the (vertical) $X-Z$ plane. The inclination, $I$, is the angle between $H$ and $F$. The effective inclination, $e$, is the angle between $V$ and $X$ (after Schouten & Cande 1976). (b) Although the range of $I$ is $-90^\circ < I < 90^\circ$, the range of $e$ is $-180^\circ < e < 180^\circ$. Solid arrows show some different directions of $V$ and adjacent numbers are the corresponding values of $e$.

To be perfectly consistent with the observed datum $e_0^0$, model poles $m$ must lie on a half great circle (Fig. 5) bracketed by confidence limits which are also half great circles. The effective inclination corresponding to a modelled pole position, $m$, is given by

$$e = \tan^{-1}\left(\frac{\tan I}{\sin \alpha}\right), \quad -180^\circ < e < 180^\circ,$$

$$\alpha = a - D(m),$$

where $a$ is the azimuth of the lineations which is $90^\circ$ clockwise from the direction in which the anomalies become younger. The numerator and denominator in equation (37) yield the correct quadrant in two variable arctangent functions. Expressions for $I(m)$ and $D(m)$ were

Figure 5. Semi-great circle of possible pole positions, given the effective inclination for magnetic lineations at $L$. Lighter small circles form a lune-shaped confidence interval about the semi-great circle of possible pole positions (after Schouten & Cande 1976).
The quantity to be minimized in the least squares solution is

\[
\frac{\delta t^2}{\sigma_t} = \left( \frac{e_i^0 - e_i(m)}{\sigma_t} \right)^2,
\]

where \(\sigma_t\) is the error in the observed datum \(e_i^0\).

In practice, confidence limits on skewness (and therefore on effective inclination) have been determined in two ways. Larson & Chase (1972) assigned confidence limits for skewness by comparing an observed profile having many anomalies with a sequence of models in which skewness was varied by 5° increments. From these models they subjectively determined a range of skewness which was an acceptable fit to the observed anomaly profiles. Cande (1976) followed a more statistical approach. He determined (with 5° precision) the skewness correction needed to bring individual anomalies contained in the profile into symmetry. Once he had calculated the skewness for a number of anomalies, he calculated the mean and standard error of the skewness estimate. In general, the resulting confidence limits are smaller than those assigned by Larson & Chase (1972).

In order to determine effective palaeomagnetic inclination from anomaly skewness, it is necessary to make several geological assumptions about the oceanic crust: the boundaries between normally and reversely magnetized strips of crust are vertical; within a strip of crust of constant polarity, the intensity of magnetization does not vary laterally (although it may vary with depth); and the crust has not been tectonically rotated. If these are not valid, then part of the observed skewness is anomalous. On the basis of internal consistency checks, Cande (1976) has shown that 12°–28° of anomalous skewness is present in anomalies 27–32 in the South Pacific and Cande & Kristofferson (1977) have shown that about 80° of anomalous skewness is present in anomalies 33 and 34 in the North Atlantic. Therefore additional uncertainty must be added to skewness of anomalies of these ages. If \(\sigma_{\theta A}\) is the standard error in skewness due to ‘anomalous skewness’ and \(\sigma_{\theta P}\) is the uncertainty (standard error) in mean skewness due to the limits of accuracy to which we can measure skewness, the total uncertainty, \(\sigma_{\theta T}\), is given by

\[
\sigma_{\theta T} = \left( (\sigma_{\theta A})^2 + (\sigma_{\theta P})^2 \right)^{1/2}
\]

which, for our purposes, is also the uncertainty in effective inclination.

**Application: the Campanian pole for the Pacific plate**

We have used the following data to calculate a Campanian pole for the Pacific plate:

1. Palaeomagnetic colatitude determined from the basalt from DSDP site 163 (Marshall 1978).
2. Palaeomagnetic colatitude determination from the basalt from DSDP site 192a (Marshall 1978).
3. Seamount poles from the reversely magnetized seamounts of the Hawaiian seamount group and seamount A of the north Japan group (Harrison et al. 1975).

The age of the basalt of site 163, situated between the Clipperton and Clarion fracture zones and west of youngest Cretaceous magnetic anomalies, is not well determined by either K–Ar age dates or magnetic anomalies. However, a minimum age of Campanian is indicated by the age of the overlying sediments (\(G. elevata\) foraminiferal zone tentatively identified in the site report) and the observation that the basalt is clearly extrusive (van Andel, Heath et al. 1973). Westward extrapolation of basement ages determined from DSDP legs 9 and
Oceanic palaeomagnetic poles

Figure 6. 'Middle' and late Cretaceous magnetic reversal time-scale and geological time-scale (after La Brecque et al. 1977; Alvarez et al. 1977; Larson & Hilde 1975). Stage boundaries shown by solid lines are from La Brecque et al. (1977), dashed stage boundaries are from van Hinte (1976) with both scales corrected for new decay constants (as in Mankinen & Dalrymple 1979).

16 and inferred spreading rates suggest that at site 163 the crust is about 67–72 Myr and hence late Cretaceous in age (Dymond 1973).

Turning to the question of the magnetic polarity at site 163, the Campanian minimum age and the lack of mapped magnetic lineations in this region suggests that the crust may have formed during either Gubbio long normal, A−, or B+ polarity epochs (Alvarez et al. 1977) which are all characterized by rare or no anomalies in the seafloor (Fig. 6). Gordon, Cockerham & Cox (1979) argue that the site 163 basalts must be reversely magnetized in order to be consistent with the amount of northward motion implied by the palaeomagnetism of site 315 basalts (Cockerham 1979), for which polarity is constrained by age, and with Tertiary palaeoequators in the central equatorial Pacific (van Andel 1974). With reversed polarity the site 163 datum is also more consistent with the palaeomagnetic poles determined from the anomalies over seamounts of probable Campanian age, which are discussed below. The generally accepted Cretaceous magnetic polarity time scales (Helsley & Steiner 1968; Couillard & Irving 1975; Larson & Hilde 1975; La Brecque, Kent & Cande 1977) indicate a continuous normal polarity zone (Fig. 6) throughout the late Cretaceous prior to the young edge of anomaly 34, about 82 Myr, near the Campanian/Santonian boundary (using the LaBrecque et al. (1977) time scale as corrected for new decay constants by Mankinen & Dalrymple (1979)). The youngest reversal which is older than anomaly 34
is anomaly M-0, which is overlain by Aptian sediments and assigned an age of 111 Myr by Larson & Hilde (1975). Since an age as old as 111 Myr for site 163 basalt seems geologically improbable and since the age of the overlying sediments is Campanian, we conclude that the reversely magnetized basalt at site 163 probably correlates with the Gubbio A− polarity zone of Alvarez et al. (1977). The polarity of the basalt at site 192a (Meiji seamount) is clearly normal. If it were reversed it would imply a palaeolatitude of the Hawaiian hotspot drastically different from all the other measured values. Its age is constrained by overlying sediments of early Maastrichtian age. Extrapolation along the radiometric ages of the Hawaiian—Emperor chain (Dalrymple, Lanphere & Clague 1979) suggests an age of about 70−75 Myr. Our interpretation, which is consistent with all these constraints, is that the basalts were extruded during the Upper Campanian Gubbio B+ polarity epoch. The small circles describing possible pole positions consistent with the paleomagnetic data from the basalts of sites 163 and 192a are shown in Fig. 7.

The next datum is the pole determined from the magnetic anomaly over seamount A (see Table 3 of Harrison et al. 1975) near Japan. It has a reliable K−Ar age of 80−82 Myr which places it in or slightly older than the Campanian in all commonly used Cretaceous time-scales.

The last group of data come from three reversely magnetized seamounts (Francheteau, Sclater & Craig 1969; Francheteau et al. 1970) near Hawaii (W, BB and DD from Table 3 of Harrison et al. 1975), the ages of which are determined on the following basis. Seamounts that form during times of rapidly changing polarity do not generally yield consistent palaeo-pole data. The anomaly inversions of these seamounts gave highly consistent results, indicating that they formed during a reasonably long interval of reversed polarity such as M0, Gubbio A−, Gubbio E−, or possibly Gubbio C− (Fig. 6). Since these seamounts rest on crust which is late Cretaceous in age (and outside the area inferred by Larson & Chase 1972, to be bounded by the M anomalies of the Pacific plate), they are younger than anomaly M0 and hence can be no older than Gubbio A−, the oldest Cretaceous reversed polarity epoch younger than M0. Fossil ages and radiometric ages from other seamounts in the same group as summarized by Harrison et al. (1975) suggest an age of about 82 to 87 Myr, contemporaneous with or somewhat older than Gubbio A−. We conclude that Gubbio A− is the most likely age for these seamounts.

CONFIDENCE LIMITS

The method of calculating the uncertainties in colatitudes from the DSDP sites are detailed in Cox (1980) and briefly discussed above. In addition to the standard error estimated by this method we have added a 3 deg uncertainty (s.d.) in observed inclination, $I$, to allow for inevitable small departures from vertical of the DSDP coring. For example, DSDP hole 433 deviated 2.5 deg from vertical when logged for inclination (W. J. Morgan 1979, private communication). The resulting total standard deviation of the mean colatitude, $\sigma_{TC}$, is given by

$$
\sigma_{TC} = \left[ \sigma_C^2 + \left( \frac{\partial c}{\partial I} \right)^2 (3°)^2 \right]^{1/2}
$$

(41)

where

$$
\frac{\partial c}{\partial I} = \frac{-2}{1 + 3 \cos^2 I}
$$

(42)

and where $\sigma_C$ is the standard error of the colatitude before the tilt correction.
Figure 7. Small circles of possible pole positions consistent with the palaeomagnetism of the basalt of sites 163 and 192. 163R and 163N refer respectively to either reversed or normal magnetization assumed for the basalts of DSDP site 163. Solid circles are the four seamount poles used in the calculation of the Campanian pole position.

The estimation of confidence limits for the seamount poles is very difficult, especially since we consider only a few pole positions. We did not use the goodness of fit ratios to estimate confidence limits since a seamount with a high goodness of fit ratio can have an unreliable pole position (e.g. Blakely & Christiansen 1978). Our approach was to estimate the shape and orientation of the elliptical confidence region of the seamounts from the distribution of pole positions of all of the reliable seamount poles from the group which they were placed in by Harrison et al. (1975). For instance we estimated the uncertainty in the three reversed Hawaiian poles from the scatter in the pole positions of all nine reliable results for the Hawaiian group. We then assigned this elliptical uncertainty to each of the seamount poles. We expect this estimate to be conservative because the seamount poles which we used to estimate the variance likely span a time interval long enough that polar wander has occurred, increasing the scatter in the pole positions. The input data and uncertainties are summarized in Table 1 and illustrated in Fig. 7.

Results and discussion

Combining the four seamount poles and two colatitudes yields the maximum likelihood pole position and statistics presented in Table 2. The value of $\chi^2$ for the combined data set is 6.6 with 8 degrees of freedom. Tables show that $\chi^2$ is expected to be greater than this value more than 50 per cent of the time, hence this is a reasonable fit that is neither too good nor too bad. For 8 degrees of freedom chi-square is expected to be less than 2.73 5 per cent of the time and less than 15.5 95 per cent of the time. Thus if $\chi^2$ had been 16 we would have concluded that the fit was unreasonably bad since $\chi^2$ was too big at the 5 per cent risk level. Similarly if $\chi^2$ had turned out to be 2.5 we would have concluded that the fit was 'too good' at the 5 per cent risk level.
Table 1. Data used as input for Campanian pole.

Results from anomalies over seamounts

<table>
<thead>
<tr>
<th>Seamount symbol</th>
<th>A</th>
<th>W</th>
<th>BB</th>
<th>DD</th>
</tr>
</thead>
<tbody>
<tr>
<td>N latitude of pole</td>
<td>56.0°</td>
<td>60.0°</td>
<td>56.5°</td>
<td>49.8°</td>
</tr>
<tr>
<td>E longitude of pole</td>
<td>359.0°</td>
<td>359.0°</td>
<td>350.1°</td>
<td>1.5°</td>
</tr>
<tr>
<td>Major semi-axis length (standard error)</td>
<td>6.4°</td>
<td>11.7°</td>
<td>11.7°</td>
<td>11.7°</td>
</tr>
<tr>
<td>Minor semi-axis length (standard error)</td>
<td>1.5°</td>
<td>6.6°</td>
<td>6.6°</td>
<td>6.6°</td>
</tr>
<tr>
<td>Major axis orientation (clockwise from north)</td>
<td>99.0°</td>
<td>67.0°</td>
<td>67.0°</td>
<td>67.0°</td>
</tr>
</tbody>
</table>

Results from drill cores

<table>
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<tr>
<th>Hole number</th>
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<th>192</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assumed number of independent samples</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Mean colatitude</td>
<td>93.4°</td>
<td>72.2°</td>
</tr>
<tr>
<td>Corrected colatitude</td>
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<td>71.9°</td>
</tr>
<tr>
<td>Standard error of colatitude</td>
<td>5.6°</td>
<td>5.9°</td>
</tr>
<tr>
<td>Standard error including tilt of hole</td>
<td>5.8°</td>
<td>6.2°</td>
</tr>
<tr>
<td>( \delta c/\delta f )</td>
<td>-0.51</td>
<td>-0.65</td>
</tr>
<tr>
<td>N latitude of site</td>
<td>11.2°</td>
<td>53.0°</td>
</tr>
<tr>
<td>E longitude of site</td>
<td>-150.3°</td>
<td>164.7°</td>
</tr>
</tbody>
</table>

Table 2. Best fit pole combining core and seamount data.

The best fit pole:

56.7° N, -8.3° E

95 per cent confidence ellipse:

Length of semi-axis Orientation (CW from N)
3° -1°
8° 89°

\( \chi^2 = 6.6 \) with 8 degrees of freedom

Colatitudes: errors, residuals and importances

<table>
<thead>
<tr>
<th>Identification</th>
<th>Location</th>
<th>Datum</th>
<th>Standard error</th>
<th>Model</th>
<th>Residual</th>
<th>Importance</th>
</tr>
</thead>
<tbody>
<tr>
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<td>Lat. N</td>
<td>Long. E</td>
<td>93.5</td>
<td>5.8</td>
<td>105.2</td>
<td>-11.7</td>
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<tr>
<td>192 basalt</td>
<td>53.0</td>
<td>164.7</td>
<td>71.9</td>
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<td>70.2</td>
<td>1.7</td>
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Seamount poles: errors, residuals and importances

<table>
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<th>Identification</th>
<th>Type</th>
<th>Datum</th>
<th>Standard error</th>
<th>Model</th>
<th>Residual</th>
<th>Importance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seamount A</td>
<td>major axis</td>
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<td>359.0</td>
<td>6.4</td>
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</tr>
<tr>
<td>Seamount W</td>
<td>minor axis</td>
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<td>359.0</td>
<td>11.7</td>
<td>56.7</td>
<td>-8.3</td>
</tr>
<tr>
<td>Seamount BB</td>
<td>major axis</td>
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<td>56.7</td>
<td>-8.3</td>
</tr>
<tr>
<td>Seamount DD</td>
<td>minor axis</td>
<td>49.8</td>
<td>1.5</td>
<td>11.7</td>
<td>56.7</td>
<td>-8.3</td>
</tr>
</tbody>
</table>
Table 3. Best fit pole omitting site 163.

The best fit pole:

55.9° N    -1.4° E

95 per cent confidence ellipse:

<table>
<thead>
<tr>
<th>Length of semi-axis</th>
<th>Orientation (CW from N)</th>
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</thead>
<tbody>
<tr>
<td>3°</td>
<td>6°</td>
</tr>
<tr>
<td>8°</td>
<td>96°</td>
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</tbody>
</table>

\[ \chi^2 = 1.5 \] with 7 degrees of freedom

Colatitudes: errors, residuals and importances

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<th>Standard error</th>
<th>Model</th>
<th>Residual</th>
<th>Importance</th>
</tr>
</thead>
<tbody>
<tr>
<td>192 basalt</td>
<td>53.0</td>
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<td>6.2</td>
<td>70.5</td>
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Seamount poles: errors, residuals and importances

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<th>Type</th>
<th>Location</th>
<th>Datum</th>
<th>Standard error</th>
<th>Model</th>
<th>Residual</th>
<th>Importance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seamount A</td>
<td>major axis</td>
<td>56.0</td>
<td>359.0</td>
<td>6.4</td>
<td>55.9</td>
<td>-1.4</td>
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<td></td>
<td>minor axis</td>
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<td>1.5</td>
<td></td>
<td></td>
<td>0.1</td>
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<tr>
<td>Seamount W</td>
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<td>359.0</td>
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<td>55.9</td>
<td>-1.4</td>
<td>-1.8</td>
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<td>minor axis</td>
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<td>3.6</td>
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<td>-1.4</td>
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<td>minor axis</td>
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<td>6.6</td>
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<td></td>
<td>-6.3</td>
</tr>
</tbody>
</table>

Even though the general level of dispersion in the data as assessed by the value of \( \chi^2 \) is reasonable, a closer examination of the residuals shows that a suspiciously large amount of the total error is concentrated in one datum, the colatitude from DSDP site 163. To check whether this datum and its error are consistent with the other data, we evaluated its contribution to the total \( \chi^2 \) by the following procedure. First, the data set was reanalysed without site 163, yielding a \( \chi^2 \) value of 1.5 with 7 degrees of freedom (Table 3), which implies that the contribution of site 163 to the total \( \chi^2 \) is 5.1 with 1 degree of freedom. Noting that the difference between chi-squared distributed variables is chi-squared distributed (Bevington 1969) and referring to standard tables, \( \chi^2 \) is expected to be greater than 5.0 only 2.5 per cent of the time, providing a basis for rejecting at the 97.5 per cent confidence level, the mutual consistency of the site 163 datum with the other data. The site 163 results may also be shown to be inconsistent with the seamount data by combining the seamount poles into a mean pole with a circular confidence circle and using a chi-squared test to compare this with the result from site 163. Both tests suggest that a significant source of error was overlooked in the site 163 datum. Repeating the analysis omitting site 163 yields a pole (Table 3) 3.9° away from the previous pole.

The difference between the observed and predicted colatitude at site 163 is 11.7° whereas the standard error at site 163 is only 5.8°. If the standard error had been as large as 6.9°, then using the analysis outlined in the previous paragraph the result at site 163 would not have been inconsistent with the other data at the 95 per cent confidence level. This standard
Table 4. Best fit pole combining core and seamount data.

The best fit pole:

$56.5^\circ \text{N} \quad -6.5^\circ \text{E}$

95 per cent confidence ellipse:

<table>
<thead>
<tr>
<th>Length of semi-axis</th>
<th>Orientation (CW from N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3^\circ$</td>
<td>$1^\circ$</td>
</tr>
<tr>
<td>$8^\circ$</td>
<td>$91^\circ$</td>
</tr>
</tbody>
</table>

$\chi^2 = 5.3$ with 8 degrees of freedom

Colatitudes: errors, residuals and importances

<table>
<thead>
<tr>
<th>Identification</th>
<th>Location</th>
<th>Standard error</th>
<th>Model</th>
<th>Residual</th>
<th>Importance</th>
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<td>Lat. N</td>
<td>Long. E</td>
<td>Datum</td>
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<tr>
<td>163 basalt</td>
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<td>164.7</td>
<td>71.9</td>
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Seamount poles: errors, residuals and importances

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<td>Seamount A</td>
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</table>

Figure 8. Campanian palaeomagnetic pole and 95 per cent confidence limits for the Pacific plate.
error of 6.9° would be consistent with the presence of an additional independent random error of 3.7° in colatitude (7.3° in inclination) besides the known error of 5.8°. There are several possible sources for such an additional error:

(1) Relative motion between the part of the Pacific plate where site 163 is situated and the part of the Pacific plate where the other sites are located.

(2) Inadequate sampling in time. If the time intervals between the ages of the sampled strata are less than the longest periods present in the geomagnetic secular variation, about $10^4$ yr, the palaeomagnetic measurements will not be statistically independent and the calculated confidence limits will be too small.

(3) Local tectonic tilt of the strata at site 163 at some time subsequent to the original magnetization of the strata. The first is unlikely because of the proximity of site 163 to the seamounts used in the analysis — there is scarcely room for an undetected plate boundary between the sites, and there is no evidence for the presence of a plate boundary along which 410 km (3.7°) of north—south motion has taken place. Concerning (2), although we tried to obtain a full estimate of the error due to secular variation, the possibility always remains that all of the basalt strata sampled were formed in a very short period of time when the direction of the field was several tens of degrees from the direction of an axial dipole field. Local tectonic tilts of the order of 10° associated with the migration of ocean floor away from the spreading centre appears to us the most likely source of the additional noise. This analysis points to the possibility that the errors calculated for palaeomagnetic results from the ocean floor may be too small, even after a careful attempt has been made to include the noise due to geomagnetic secular variation.

Using the larger error of 6.9° for site 163 and repeating the analysis yields the model pole given in Table 4 and Fig. 8, which is displaced 1.0° from the original pole found using the smaller error for site 163. This is our best estimate of the Campanian pole. In the original analysis the importance of site 163 was 0.19, almost four times the value of 0.05 for site 192. In the analysis with the larger error, the importance of site 163 was 0.14, still much larger than site 192 although the assigned standard error of site 163 is now larger than that of site 192. Numerical experiments showed that the greater importance was not due to the misfit of site 163 with respect to the other data but is due to the relative geometry of site 163 and the elliptical uncertainties assigned to the seamount poles. This illustrates how data importances might be used in the planning of future palaeomagnetic research for the oceanic plates. Since we do not as yet have enough experience in analysing data from oceanic plates to assess the reliability of error estimates given in the literature, analyses like the one given above are especially useful for identifying anomalous error estimates.

Conclusions

The use of maximum likelihood estimation provides a framework within which to combine and compare heterogeneous palaeopole data from oceanic plates. In applying this method to determine a Campanian palaeopole for the Pacific plate, we have found that the published data from seamount inversions and from palaeomagnetic study of DSDP core 192 are mutually consistent on the basis of a chi-squared test. Our study also highlights the difficulties in estimating the uncertainties of palaeomagnetic results on samples obtained by deep sea drilling. The results for the Campanian appear to be encouraging enough to justify using this method of analysis to obtain pole paths and confidence limits for the major oceanic plates and for using numerical experiments with the data importances for planning further palaeomagnetic study of the oceanic plates.
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References


Oceanic palaeomagnetic poles


Appendix A: partial derivatives

\[
\frac{\partial c}{\partial \lambda_p} = -\frac{1}{(1 - g^2)^{1/2}} \frac{\partial g}{\partial \lambda_p}, \tag{A1}
\]

\[
\frac{\partial c}{\partial \phi_p} = -\frac{1}{(1 - g^2)^{1/2}} \frac{\partial g}{\partial \phi_p}, \tag{A2}
\]

where

\[
\frac{\partial g}{\partial \lambda_p} = - \sin \lambda_p \cos \lambda_i \cos (\phi_p - \phi_i) + \cos \lambda_p \sin \lambda_i \tag{A3}
\]

and

\[
\frac{\partial g}{\partial \phi_p} = - \cos \lambda_p \cos \lambda_i \sin (\phi_p - \phi_i). \tag{A4}
\]

\[
\frac{\partial D}{\partial \lambda_p} = \frac{\cos \lambda_i \sin (\phi_i - \phi_p)}{1 - g^2}, \tag{A5}
\]

\[
\frac{\partial D}{\partial \phi_p} = -\cos \lambda_p \left[ \cos \lambda_p \sin \lambda_i + \sin \lambda_p \cos (\phi_i - \phi_p) \cos \lambda_i \right], \tag{A6}
\]

\[
\frac{\partial I}{\partial \lambda_p} = \frac{2}{(1 + 3g^2)(1 - g^2)^{1/2}} \frac{\partial g}{\partial \lambda_p}, \tag{A7}
\]

\[
\frac{\partial I}{\partial \phi_p} = \frac{2}{(1 + 3g^2)(1 - g^2)^{1/2}} \frac{\partial g}{\partial \phi_p}, \tag{A8}
\]

\[
\frac{\partial e}{\partial \lambda_p} = \frac{2}{\sin^2 \alpha + (4 - \sin^2 \alpha)g^2} \left[ \frac{\sin \alpha (\partial g/\partial \lambda_p)}{(1 - g^2)^{1/2}} + g (1 - g^2)^{1/2} \cos \alpha \frac{\partial D}{\partial \lambda_p} \right], \tag{A9}
\]

\[
\frac{\partial e}{\partial \phi_p} = \frac{2}{\sin^2 \alpha + (4 - \sin^2 \alpha)g^2} \left[ \frac{\sin \alpha (\partial g/\partial \phi_p)}{(1 - g^2)^{1/2}} + g (1 - g^2)^{1/2} \cos \alpha \frac{\partial D}{\partial \phi_p} \right]. \tag{A10}
\]

Appendix B: error ellipse

For the purpose of finding the trace of an ellipse on a sphere it is convenient to transform first to a local spherical coordinate system which places \( m \) at the equator so that both semi-
axes of the ellipse may be described by great circle lengths. A simple transformation which corrects degrees of longitude to great circle degrees yields the matrix, $R$, where

$$
R = \begin{pmatrix}
\sigma_\lambda^2 & \sigma_{\lambda \phi} \cos \lambda \\
\sigma_{\lambda \phi} \cos \lambda & \sigma_\phi^2 \cos^2 \lambda
\end{pmatrix}.
$$  \hspace{1cm} (B1)

On diagonalizing $R$ the orientation of one axis is found to be

$$
\omega_1 = 0.5 \tan^{-1} \left[ \frac{2R_{12}}{(R_{11} - R_{22})} \right] = 0.5 \tan^{-1} \left[ \frac{2W_{12} \cos \lambda}{(W_{11} - W_{22} \cos^2 \lambda)} \right] \hspace{1cm} (B2)
$$

where $\omega_1$ is measured clockwise from north. The semi-axis length of the standard error along this direction is given by

$$
\sigma_1 = \left( 0.5 \left[ R_{11} + R_{22} + \frac{(R_{11} - R_{22})}{\cos 2\omega_1} \right] \right)^{1/2} = \left( 0.5 \left[ W_{11} + W_{22} \cos^2 \lambda + \frac{(W_{11} - W_{22} \cos^2 \lambda)}{\cos 2\omega_1} \right] \right)^{1/2}.
$$  \hspace{1cm} (B3)

Similarly the semi-axis length in the $\omega_2 = \omega_1 \pm \pi/2$ direction is given by

$$
\sigma_2 = \left( 0.5 \left[ W_{11} + W_{22} \cos^2 \lambda - \frac{(W_{11} - W_{22} \cos^2 \lambda)}{\cos 2\omega_1} \right] \right)^{1/2}.
$$  \hspace{1cm} (B4)

The direction of the major axis will be $\omega_1$ if $W_{11} > W_{22} \cos^2 \lambda$ and $\omega_2$ otherwise.