Finite-Range DWBA Analysis of Anomalous Analyzing Powers in \((p, \alpha)\) Reactions

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An anomalous analyzing power in the reaction \(^{60}\text{Ni}\,(p, \alpha)^{57}\text{Co}\,(3/2^+)\) is studied in the framework of distorted-waves Born approximation. The anomaly is well reproduced by calculations based on the form factors and spectroscopic amplitudes which are derived from microscopic model wave functions.

§1. Introduction

Analyzing powers have been recently measured for \((p, \alpha)\) reactions on even-even targets using a 22-MeV polarized proton beam.\(^1\)\(^-\)\(^2\) The measured analyzing powers show a strong \(J\)-dependence and are well reproduced, except for a few data, by the zero-range distorted-waves Born approximation (DWBA), which assumes that a triton cluster bound in a Woods-Saxon well is transferred.\(^1\) The analyzing power for the reaction into the excited state \(3/2^+\) of cobalt isotopes is the exception (see Fig. 1), therefore it may be called "anomalous". Although the reaction \(^{40}\text{Ca}\,(p, \alpha)^{37}\text{K}\,(3/2^+)\) yields a similar analyzing power, it is well reproduced by the zero-range DWBA calculation.\(^7\) The zero-range DWBA seems to be better for the reaction \(\text{Ca}\,(p, \alpha)^{37}\text{K}\,(3/2^+)\) than for \(\text{Ni}\,(p, \alpha)^{57}\text{Co}\,(3/2^+)\); in the latter where one proton is picked up from a deeply bound orbit and two neutrons are picked up from an open shell, internal motions between the three nucleons are more

Fig. 1. The analyzing power of \(^{60}\text{Ni}\,(p, \alpha)^{57}\text{Co}(3/2^+; E_x=3.54\text{MeV})\). The solid line is obtained from the finite-range DWBA calculation with only one component (3200, 2). (See § 4.) The dot-dashed line shows the result of the zero-range DWBA calculation in which a Woods-Saxon bound state is used.
important and the triton cluster is not good assumption. Moreover it should be noticed that a $1/2^+$ state is lying lower than the $3/2^+$ state for cobalt isotopes, on the other hand the $3/2^+$ state is the ground state in $^{34}$K. This suggests that the $3/2^+$ state of cobalt isotopes is not a pure $1d_{3/2}$ hole state, but consists partially of a $2s_{1/2}$ hole state coupled with a distortion of the nuclear surface. The Hartree-Fock calculation supports that $^{60}$Ni is not inert, but rather deformable in oblate shape.

The main purpose of the present paper is to investigate how the anomalous analyzing power of the $3/2^+$ state reflects the nuclear structure of the state through the internal motions between transferred three nucleons. We employ the finite-range DWBA and formulate in the next section the form factor and the spectroscopic amplitude for the reaction $(p, \alpha)$, according to the definition given by Satchler. The microscopic description of three-nucleon transfer reactions has been already formulated by many authors, therefore the details of the formulation will be omitted. In § 3 we shall introduce a nuclear model to estimate the effect of the surface distortion on the spectroscopic amplitude. Numerical results are given in § 4.

§ 2. Formulation of transition matrix

We analyze the $(p, \alpha)$ reaction as a pick-up process. One proton and two neutrons, hereafter referred to as a triton, in the ground state of a target nucleus are picked up through direct interactions with an incoming proton; two nucleons behave as spectators and are transferred by the overlapping.

The nuclear matrix element for the DWBA transition matrix of a reaction $A(p, \alpha)B$ has the form

$$
\langle B, \alpha | V | A, p \rangle = \int d\xi \hat{m} d\xi \bar{\Psi}_a(\xi) \Phi_a^*(\xi, r, \xi) V_T(\xi, \xi) \psi_m(\xi, \xi) \chi_{\gamma_m}(\xi) \cdot (2.1)
$$

Here $\Psi_T(\xi)$ represents the internal state of the outgoing alpha-particle and $\chi_{\gamma_m}(\xi)$ the spin state of the incoming proton. The symbols $\xi$ and $\xi$ stand for the internal coordinates of the residual nucleus $B$ and the triton. The definition of relative coordinates are shown in Fig. 2. The relative coordinates $r_{pl}$ and $r_{ab}$ written in terms of the channel coordinates, $r_{pl}$ and $r_{ab}$, and the mass number $A$ of the target:

$$
r_{pl} = \frac{4A}{3(A+1)} \left( \frac{r_{pl} - A - 3}{A} r_{ab} \right),
$$

$$
r_{ab} = \frac{4A}{3(A+1)} \left( - \frac{1}{4} r_{pl} + r_{ab} \right).$$

The integration over $\xi$ leads to the form
The sum \( \sum \) is over the angular momenta \( j_0, j_1, j_2 \), and \( J_n \). The former three specify the single-particle states occupied by one proton and two neutrons respectively, and the last is the angular-momentum transfer by the two neutrons. The target is chosen to be an even-even nucleus, and so \( J_\lambda = 0 \) for the ground state. The Pauli principle is taken into account by \( \langle B [j_0, j_1, j_2] | A \rangle \). This expansion coefficient depends on the nuclear structure of the target and residual nuclei and the details will be given in the next section.

We use a harmonic oscillator basis to describe single-particle states. It is well known that this basis makes the transformation of the coordinates by the Talmi coefficients easy, though the tail of this wave function is not realistic. As \( \phi_a \) we choose the relative part of the shell model state \( (1s_1 1l_2) \):  

\[
\phi_a(\xi, r_{pq}) = \phi(\mathbf{r}_{12}, \frac{1}{2} \nu_u) \phi(\mathbf{r}_{01}, \frac{2}{3} \nu_u) \phi(\mathbf{r}_{02}, \frac{3}{4} \nu_u) \chi(\rho) \chi(0) \chi(1) \chi(2) \to \chi^{s},
\]

where \( \phi(\mathbf{r}, \nu) \) represents the \( 1s \) state with an oscillator constant \( \nu \), and \( \chi(\rho) \chi(0) \chi(1) \chi(2) \to \chi^{s} \) the spin state coupled to spin zero.

At first let us consider the part of proton-proton interaction, \( V_{pp} \). The zero angular momenta of \( \phi(\mathbf{r}_{12}, \frac{1}{2} \nu_u) \) and \( \chi(1) \chi(2) \to \chi^{s} \) in Eq. (2·4) are then conserved. In order to integrate on \( \xi \), we transform the wave function of the triton in Eq. (2·3) as follows:

\[
\Phi_{j_0, j_1, j_2}^{(d)}(r_{01}, r_{12}, r_{23}) = \sum \langle j_0 | j_2 \rangle \frac{1}{2} \frac{1}{2} (0) J_\lambda \frac{1}{2} \frac{1}{2} (j_1) \frac{1}{2} \frac{1}{2} (j_2) J_\lambda
\]
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\[ x \left\{ \frac{1}{4} J_0 \left( \frac{1}{2} \right) J_0 \frac{1}{2} (j) J_0 0 \right\} J_0 \]

\[ \times \left\{ N_1 J_0 (r) 00 (r_0) J_0 n d_1 (r_0) n d_2 (r_2) J_2 \right\} \]

\[ \times \left\{ N L (r_0) n A (r_0) l | n d_0 (r_0) N_2 J_2 (r) l \right\} \]

\[ \times \phi \left( r_{12}, \frac{1}{2} J_2 \right) [Z (1) \chi (2)] \]

\[ \times \sum_{n, m} \left( l, -m \right) \frac{1}{2} m_p | J_2, -M_2 \right\} \left[ \phi_{NL} (r_{12}, 3 \nu_2) \phi_{NL} (r, \nu) \right] \]

\[ \times m, m' \left( 1, \frac{2}{3} \nu_2 \right) \mathcal{X}_m (0). \]

(2.5)

Here the first sum is over \( n, A, N, L \) and \( l \). We have omitted the terms to vanish as integrated on \( \xi \), and taken the only zero node state \( \phi (r_{12}, \frac{1}{2} \nu_2) \), assuming that the difference between oscillator constants \( \nu_2 \) and \( \nu_4 \) is not essential. The first two factors in the right-hand side of Eq. (2.5) are the coefficients to transform \( j-j \) coupling states to \( L-S \) coupling states, and the next two factors are the Talmi coefficients. The wave functions \( \phi_{NL} \) and \( \phi_{NL} \) represent the bound state of the triton and the relative motion between the proton and the centre of the two neutrons respectively.

We can then write the nuclear matrix element in the standard form:

\[ \langle B, \alpha | V | A, \rho \rangle = \sum l^{-1} \left( l m \frac{1}{2} m_p | J_2, M_2 \right) \sum \mathcal{A}_{NL} \mathcal{A}_{NL} (r_{12}, r_{12}, r_{12}), \]

where the form factor is given by:

\[ \mathcal{A}_{NL} (r_{12}, r_{12}, r_{12}) = i \left( -1 \right)^{l+m} \sum \mathcal{L} \left( L M A, -m - M | l, -m \right) \]

\[ \times \phi_{NL} (r_{12}, 3 \nu_2) \phi_{NL} (r_{12}, 3 \nu_2) \]

\[ \times \int dr \phi_{NL} (r, \frac{2}{3} \nu_2) \mathcal{V} \left( \mathbf{r}_m - \frac{2}{3} \mathbf{r}_2 \right) \phi_{NL} (r, \nu_2). \]

(2.7)

In this expression \( \mathcal{V} (r) \) is a radial part of proton-proton interaction. The multipole expansion of the form factor (2.7) can be easily done, the technique given by Ref. 10 being employed.

Equations (2.3) \( \sim (2.7) \) lead to the spectroscopic amplitude for the proton-proton interaction,

\[ \mathcal{A}_{NL} = - (1 / \sqrt{2}) \mathcal{O}_{NN} \mathcal{V} \left( \rho \rho \right) \]

\[ \times \sum \langle B | \left[ j_n, j_j, j_j (J_n) \right] | A \rangle \]

\[ \times \langle \left( I, I_2, J_n \right) \frac{1}{2} \frac{1}{2} | 0 \rangle \frac{1}{2} \frac{1}{2} (j_1, j_1) \frac{1}{2} \frac{1}{2} (j_2, J_n) \].
where the sum is over \( j_0, j_1, j_2 \) and \( J_n \). The symbol \( O_{\alpha \beta} \) is the overlap integral of \( \psi (\mathbf{r}_{12}, \frac{1}{2} \mathbf{p}_n) \) and \( \psi (\mathbf{r}_{12}, \frac{1}{2} \mathbf{p}_n) \), and \( V^{2s+1}(\mathbf{p}\mathbf{p}) \) stands for the strength of the proton-proton central interaction with the total spin \( S \).

Next we treat the parts of proton-proton interaction, \( V_{p1} \) and \( V_{p2} \). Owing to antisymmetry of two neutrons, \( V_{p1} \) and \( V_{p2} \) give the same nuclear matrix element. If the interactions, \( V_{p1}, V_{p2} \) and \( V_{p3} \), have the same radial dependence, all the nuclear matrix elements take the same form factor (2.7). This can be seen by cyclic permutation of the labels “0, 1 and 2” in the triton.*

The spectroscopic amplitude for the proton-neutron interaction can be given in the form

\[
A_{J_n}^{2s+1} = (1/\sqrt{2}) O_{\alpha \beta} \sum_{\delta \epsilon} \langle B | J_n, j_0 j_1 j_2 | J_B \rangle \langle J_B | J_n, j_0 j_1 j_2 | B \rangle \times \sum_{\delta \epsilon} V^{2s+1}(\mathbf{p}\mathbf{p}) \sqrt{2S_\delta + 1} \times \langle L_d | L_d \rangle \frac{1}{2} \langle S_\delta | S_\delta \rangle \frac{1}{2} \langle J_B | J_B \rangle \times \langle L_d | L_d \rangle \frac{1}{2} \langle S_\delta | S_\delta \rangle \frac{1}{2} \langle J_B | J_B \rangle \times \langle N_d L_d | N_d L_d \rangle \times \langle N_d L_d | N_d L_d \rangle \times \langle N_d L_d | N_d L_d \rangle .
\]

In this expression the factor \( \sqrt{2S_\delta + 1} \) comes from the rearrangement of the spin wave function in Eq. (2.4) to the form

\[
[\chi (0) \chi (1)]_{2x} [\chi (2) \chi (\rho)]_{2x}^{0},
\]

and

\[
\langle J_n, j_0 j_1 j_2 | J_B | J_B \rangle
\]

is the coefficient to transform \( \Phi_{J_n}^{j_0, j_1 j_2} \) in Eq. (2.3) to \( \Phi_{J_B}^{j_0, j_1 j_2} \). The total spectroscopic amplitude is the sum of Eqs. (2.8) and (2.9).

At the end of this section we discuss what kinds of internal motions of the triton there are for the \( 3/2^+ \) state \( (J_B=3/2) \). We assume that one proton is

* This was pointed out by Professor R. C. Johnson.
picked up from the sd-shell state \((2n_1 + l_1 = 2)\) and two neutrons from the pf-shell states \((2n_1 + l_1 = 2n_2 + l_2 = 3)\). The properties of the Talmi coefficients and the Clebsch-Gordan coefficients give the relations

\[
2N + L + 2n + A = 8
\]  
\[
|L - A| \leq l \leq L + A.
\]

Among the sets \((NLnLl)\) which satisfy these relations, there exist \((3200, 2), (3101, 1), (3101, 2), (2301, 2), (1402, 2), (3002, 2)\) and so on. The set \((3200, 2)\) is the main component and corresponds to the form factor used in the zero-range DWBA analysis. As will be shown in §4, the necessary condition that the internal motions of the triton give some visible effects on the analyzing power means that the spectroscopic amplitude of the set \((3200, 2)\) is much smaller than those of the other sets.

§ 3. Treatment of surface distortion

In order to calculate the expansion coefficient \(\langle B|j_o j_j j_o j_{J_2}\rangle |A\rangle\) for \(J_2 = \frac{3}{2}\), it is convenient to use the second quantization formalism. We express it in terms of the operator \(b_{j_1}^+\) creating a proton sd-hole and the operator \(a_{j_2}^+\) creating a pf-particle:

\[
\langle B|j_o j_j j_o j_{J_2}\rangle |A\rangle = \langle A|b_{j_1}^+ \left[a_{j_1}^+ a_{j_2}^+\right]_{J_2} j_o j_j j_{J_2} |B\rangle^*,
\]

where the state \(\widetilde{j\bar{m}}\) is the time reversed state,

\[
\widetilde{j\bar{m}} = (-1)^{j+m} |j, -m\rangle.
\]

We assume that \(^{40}\)Ca is an inert core, noted by \(|0\rangle\), and pf-shell states have a \(Y_\pi\) deformation; the operator creating a particle in a Nilsson state \(|\alpha\rangle\) is noted by \(a_{\alpha}^+\). The wave functions,

\[
|A\rangle = N_a \int d\Omega \tilde{R}(\Omega) (a_{\alpha_1}^+ \cdots a_{\alpha_n}^+ a_{\alpha}^+ \tilde{R}^{-1}(\Omega) |0\rangle,
\]

\[
|B\rangle = N_b \int d\Omega D_{\theta_2 \phi_2 \gamma_2}^m(\Omega) \tilde{R}(\Omega) (a_{\alpha_1}^+ \cdots a_{\alpha_n}^+ a_{\alpha}^+ b_{j_1}^+ |0\rangle,
\]

are employed for the target and residual nuclei. Here the pf-nucleons are assumed to occupy the lowest Nilsson states \(\alpha_1, \alpha_2, \cdots\), and the pf-intrinsic states are time reversal symmetric. Symbols \(\tilde{R}(\Omega)\) and \(D^m_{\theta_2 \phi_2 \gamma_2}(\Omega)\) stand for the rotation operator and \(D\)-function.

From straightforward calculation we obtain
\[ \langle A | [b_j, [a_j, a_j^+]]_{j_n} | J_{n} m_n | B \rangle = N (j_0) (2J_n + 1) (j_\text{m}_0 J_n, 0 | J_{\text{m}} m_0) \\
\times \langle (A - 2) J_n | (A - 2) J_n \rangle^{1/2} \frac{\langle A | [a_j^+, a_j^+]_{j_n} | (A - 2) J_n \rangle}{\langle (A - 2) J_n | (A - 2) J_n \rangle^{1/2}}, \]

where

\[ N^{-1}(j_0) = \sum_{j_\text{m}} (2J_n + 1)^{1/2} (j_\text{m}_0 J_n, 0 | J_{\text{m}} m_0) \langle (A - 2) J_n | (A - 2) J_n \rangle \]

is a normalization factor; \( \langle A | [a_j^+, a_j^+]_{j_n} | (A - 2) J_n \rangle \) is the reduced matrix element of the spectroscopic amplitude for two-nucleon transfer reactions and is calculated by using the computer code.\(^\text{10}\) The spin \( J_n \) of the state \( | (A - 2) J_n \rangle \) takes only even values because of the time reversal property of the \( jJ \)-intrinsic state. The effect of the sd-hole appears here through the Clebsch-Gordan coefficient \( (j_0 m_0 J_n, 0 | J_{\text{m}} m_0) \); for \( J_n = 0 \) we have only 1\( d_{1/2} \) as \( j_0 \), and for \( J_n = 2 \) we take 2\( s_{1/2} \) as well as 1\( d_{1/2} \). For \( J_n = 4 \) we have only 1\( d_{1/2} \), but the excitation energy of this hole state is much higher than those of 2\( s_{1/2} \) and 1\( d_{1/2} \) hole states. Therefore we describe the 3/2\(^+ \) state of Co isotopes as a linear combination of the states generated from 1\( d_{1/2} \) hole state and 2\( s_{1/2} \) hole state and represent the mixing amplitude of the 2\( s_{1/2} \) hole state by \( \eta \). The value of the Clebsch-Gordan coefficient \( (j_\text{m}_0 J_n, 0 | J_{\text{m}} m_0) \) changes its sign as \( m_0 \) varies, but this \( m_\text{m} \)-dependence can be included by \( \eta \).

## § 4. Numerical analysis

Most of our calculations were carried out using a \( \delta \)-type interaction for \( V(r) \) in Eq. (2-7). We have used also a Gaussian interaction for \( V(r) \), but this interaction makes little change in our conclusions obtained by the \( \delta \)-type interaction. The \( \delta \)-type interaction seems to take out the essence of the finite-range DWBA. An example of the comparison between two interactions is shown in Fig. 3. The optical-model parameters used to calculate out theoretical curves are listed in Table I. The parameters for the proton were taken from Ref. 14) and for the \( \alpha \) particle the parameters of Ref. 15) were a little modified to give a better fitting to the analyzing power of the \( (p, \alpha) \) reaction to the ground state.\(^\text{13}\) A deep optical potential for the \( \alpha \) particle is required to get a good fitting to the analyzing

| Table I. Optical-model parameters for the proton and the \( \alpha \) particle. The potentials are of Woods-Saxon type with volume absorption \( (W) \) and surface derivative absorption \( (W_s) \). |
|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| \( V \) (MeV) | \( W \) (MeV) | \( W_s \) (MeV) | \( r_v \) (fm) | \( a_v \) (fm) | \( r_w \) (fm) | \( a_w \) (fm) | \( r_c \) (fm) | \( V_{ss} \) (MeV) | \( r_{ss} \) (fm) | \( a_{ss} \) (fm) |
| \( p \) | 51.4 | 2.14 | 24.8 | 1.17 | 0.75 | 1.32 | 0.56 | 1.25 | 24.8 | 1.01 | 0.75 |
| \( \alpha \) | 206.8 | 17.0 | 0.0 | 1.41 | 0.519 | 1.41 | 0.519 | 1.40 | — | — | — |
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Fig. 3. Comparison between δ-type and Gaussian interactions. The cross section (a) and analyzing power (b) of the 3/2⁺ state are shown for the set (3200, 2). The solid line represents the results obtained from the δ-type interaction and the broken one for the Gaussian interaction with a range 1.7 fm. The absolute value of the cross section for the latter is 6.8 times as large as that of the former.

power of the (p, α) reaction.

Employing the δ-type interaction and setting $A_{1 \frac{3}{2} \frac{1}{2}}^{Nl_{n}A} = 1$ for all the sets, we calculated the cross section for each set $(Nl_{n}A, l)$, which is shown in Fig. 4. It indicates the following: (1) The set (3200, 2) has the largest cross section. (2) Both sets (2301, 2) and (3101, 2) give the second largest cross sections, which are approximately one tenth that of (3200, 2). (3) The cross sections of sets (3101, 1) and (3002, 2) are of the same order as that of the set (3101, 2) or a little less. (4) The other sets give very small cross sections. Roughly

Fig. 4. Cross sections for each set $(Nl_{n}A, l)$. Spectroscopic amplitudes are supposed to be one. The different symbols identify $(Nl_{n}A, l)$: —— (3200, 2); —— (3101, 2); —— (3101, 1); —— (2301, 2); —— (3002, 2); —— (1401, 2).
speaking, the cross section for a set \((NLnA, l)\) decreases by one order as \(2N + L\) decreases by one. Therefore it is reasonable to neglect the sets belonging to \(2N + L \leq 6\), except for \((3002, 2)\). (5) If the value of the spectroscopic amplitude for \((3200, 2)\) is of the same order as those for the other sets, the cross section and the analyzing power of the \(3/2^+\) state are similar to the results obtained from the zero-range DWBA calculation (see Fig. 1).

Spectroscopic amplitudes are now estimated for \(^{60}\text{Ni}(p, \alpha)^{57}\text{Co}\) following the treatment described in the last section. Our results are shown in Table II for the case of a deformation parameter \(\beta = -0.3\), \(\nu_a = -0.229\) and \(V'(pp) = -3V'(pn)\); this ratio is obtained from mixing character \((\sigma \cdot \sigma) (\tau \cdot \tau)\). Here we have used the values of 0, -1.42, -0.37 and -5.91 MeV for relative energies of single-particle states \(2p_{13/2}, 2p_{3/2}, 1f_{5/2}\) and \(1f_{7/2}\).

From Table II we find that the values of spectroscopic amplitudes for the proton-neutron interaction depend strongly on mixing character of the interaction. For such kinds of mixing character as Rosenfeld mixture and \((\sigma \cdot \sigma) (\tau \cdot \tau)\), most of the contributions from \(S_d = 0\) and \(S_d = 1\) cancel each other for \(l = 2\), and so the proton-neutron interaction parts of these spectroscopic amplitudes become rather small. For the \(l = 1\) transition \((3101, 1)\), \(S_d = 0\) and \(S_d = 1\) parts are additive and then the proton-neutron interaction part gives the same order as the proton-proton interaction part, having an opposite sign.

Table II also shows that for the \(1d_{3/2}\) hole state the contributions of \(J_a = 2\)

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<th>((NLnA, l))</th>
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</table>
are much small, compared with those of \( J_n = 0 \), but that the contributions from the \( 2s_{1/2} \) hole state (\( J_n = 2 \)) are of the same order as those from \( J_n = 0 \) of the \( 1d_{5/2} \) hole state for \((3200, 2)\), \((3101, 2)\) and \((3101, 1)\).

The spectroscopic amplitudes depend on the mixing amplitude \( \eta \) and here we choose the sign of \( \eta \) such that the contributions from \( 1d_{5/2} \) and \( 2s_{1/2} \) hole states are destructive for \((3200, 2)\). Then the contributions from the two hole states are constructive for \((3101, 2)\) and destructive for \((3101, 1)\). The dependence on \( \eta \) of our theoretical analyzing power is shown in Fig. 5. The zero-range DWBA calculation gives a large negative value to the analyzing power around \( \theta_{\text{CM}} \sim 50^\circ \), but this value changes to be positive when the components \((2301, 2)\) and \((3101, 2)\) begin to give effective contributions. Around \( \eta \sim 0.8 \) our spectroscopic amplitudes for the main three components give constructive phase relations to make larger the analyzing power around \( \theta_{\text{CM}} \sim 50^\circ \). The components \((3101, 1)\) plays an important role at forward angles. As \( \eta \) increases from 0.6 to 0.8, the ratio of the spectroscopic amplitude for \((3101, 1)\) to that for \((3200, 2)\) increases rapidly from 0.24 to 3.5. This leads to translation of a nodal angle of the analyzing power. The component \((3101, 1)\) is also necessary to make the analyzing power negative at forward angles (see Fig. 5). The contributions of \((3002, 2)\) are almost negligible.

In Fig. 6 the best results (\( \eta = 0.8 \)) are compared with the experimental data. Then the spectroscopic amplitudes take the values given in Table III. The theoretical curve fairly well explains qualitative features of the observed anomalous analyzing power.

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**Fig. 5.** Dependence on \( \eta \) of the theoretical analyzing power. All the sets with \( 2N+L \geq 7 \) are here taken into account, because \((3002, 2)\) yields only minor changes. (a) \( \eta = 0.0 \); \( \eta = 0.3 \); \( \eta = 0.6 \); (b) \( \eta = 0.8 \); \( \eta = 1.0 \). The broken line in (b) shows the result obtained when \((3101, 1)\) is omitted at \( \eta = 0.8 \).
§ 5. Discussion and summary

We have investigated a possibility to explain the anomalous analyzing power of the $3/2^+$ state within the framework of DWBA. Our main assumption is the destructive interference between the states generated from a $1d_{3/2}$ hole and a $2s_{1/2}$ hole. Under this assumption we have shown that the anomaly can be explained by the internal motions of the transferred three nucleons. However we have taken no notice of the absolute value of the cross section by reason of employment of harmonic oscillator basis and $\delta$-type interaction. We here estimate only the ratio of the cross section for the $3/2^+$ state to that for the ground $7/2^-$ state; the experimental value of this ratio is about $1/7$ at $\theta_{CM}\sim 50^\circ$. For the ground state we have chosen the set $(3300,3)$ and calculated its spectroscopic amplitude in a manner similar to the one given in § 3. If the $3/2^+$ state is purely generated from a $1d_{3/2}$, $(2s_{1/2})$ hole state and the triton is transferred through the only set $(3200,2)$, the theoretical value of this ratio is approximately equal to one (one half). This seems to support the assumption of the destructive interference, too. In the case of Fig. 6 the value of this ratio is approximately one-third of the experimental value. At forward angles the discrepancy between our theoretical cross

Table III. Theoretical values with $\eta=0.8$ of spectroscopic amplitudes multiplied by $(-210^\dagger)^{(pp)}$.

<table>
<thead>
<tr>
<th></th>
<th>$(3200, 2)$</th>
<th>$(2301, 2)$</th>
<th>$(3101, 2)$</th>
<th>$(3101, 1)$</th>
<th>$(3002, 2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\eta H)$</td>
<td>-0.342</td>
<td>5.849</td>
<td>-5.059</td>
<td>1.527</td>
<td>1.645</td>
</tr>
</tbody>
</table>

Fig. 6. Comparison of the results ($\eta=0.8$) with the experimental data. The absolute value of the theoretical curve in (a) is adjusted arbitrarily.
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section and the experimental one becomes larger. These disagreements seem to be partially originated from the wrong tail of harmonic oscillator wave functions used in the form factor.

We also estimated spectroscopic amplitudes for the ${}^{58}\text{Ni}(t,a){}^{57}\text{Co}$ reaction using the wave functions described in § 3 and the value $\gamma = 0.8$ of the mixing amplitude. The zero-range DWBA analysis for this reaction yields the ratio of the spectroscopic amplitude for the $3/2^+$ state to that for the ground $7/2^-$ state. We calculated this ratio and obtained the theoretical value 0.85 which is close to the experimental value 0.92. The sign of the mixing amplitude cannot be determined by one proton pick-up reactions.

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