Nucleon-Nucleon Tensor Force Effect on the A-Separation Energy $B_A$ in s-Shell Hypernuclei

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(Received January 19, 1980)

The realistic $N-N$ interaction including a strong tensor force component gives rise to a sizable rearrangement effect on the $A$-separation energy $B_A$. The effect is shown to be more than twice larger for $^6$He than for $^6$He ($^6$H) and their difference is found to amount to $\sim 1$ MeV which is considerably larger than other effects so far examined. This is able to resolve a half of the overbinding difficulty encountered in previous calculations.

Theoretical attempts to explain the observed $A$-particle separation energy $B_A$ in s-shell hypernuclei have encountered an "overbinding difficulty". In the calculation by Dalitz, Herndon and Tang, reproduction of $B_A$ for $^3$H, $^4$H and $^4$He, 0.13, 2.04 and 2.39 MeV respectively, leads to $B_A(^4$He) $\sim 5.5$ MeV in comparison with the experimental value 3.12 MeV, with an overbinding of $\sim 2.4$ MeV. Possible effects such as the tensor force suppression in $^4$He, the repulsive $ANN$ three-body force effect and the suppression of the $A-\Sigma$ channel coupling, have been examined so far but found to be too small to avoid the overbinding difficulty. It seems that little attention has been paid to the reality of the nucleon-nucleon interaction ($v_{NN}$) which determines the property of nuclear core of $A$-nuclei. In Ref. 1 for example, central potentials are employed for both $N-N$ and $A-N$ interactions. The realistic $N-N$ interaction is characterized by a strong tensor force component which is known to be of vital importance in various aspects of nuclear system. In this short note we point out an effect arising from the use of the strong $N-N$ tensor force, which can be helpful to resolve the overbinding problem of $B_A$.

A typical manifestation of the strong $N-N$ tensor force correlation in nuclei is the repulsive rearrangement effect on the single particle separation energy. The same rearrangement effect, actually the so-called $\omega$-rearrangement, should sizably contribute to reducing the $A$-separation energy $B_A$ although the Pauli rearrangement does not exist in this case. It is noted that the rearrangement effect originates entirely from the $N-N$ correlation in the nuclear core part, without relevance to what the $A-N$ interaction $v_{AN}$ is.

In terms of the effective interaction theory, $B_A$ is obtained by diagonalizing the effective Hamiltonian $H^e$ in a chosen model space $P_A$ for $A$, which is given by a series of diagrams:

$$H^e_{\alpha\beta}(\varepsilon) = \varepsilon_{\alpha}^{\Lambda} \alpha^\Lambda \otimes \phi^N \otimes (\epsilon_A^{\Lambda} \alpha^A \otimes \phi^N)$$

where the first three terms in the r.h.s. indicate the unperturbed Hamiltonian $H^e_{\alpha\beta} = T^e + U^e$, the Hartree type contribution $\sum_{N} G_{AN} - U^e_{\beta\alpha}$ and the $\omega$-rearrangement effect, respectively, with the wavy line denoting a $G$-matrix for $A-N(G_{AN})$ or

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\textsuperscript{80} Importance of using the realistic $v_{NN}$ is emphasized by one of the authors (I.S.) in Ref. 6.)
The diagram starting energy $\epsilon_A$ is to be determined by solving the Bloch-Horowitz equation

$$P_A[H_A^{\text{eff}}(\epsilon_A) - \epsilon_A]P_A\Phi_A = 0$$  \hspace{1cm} (2)

and the $A$-separation energy is given by $B_A = -\epsilon_A$. By taking only the above three terms in Eq. (1), an approximate $H_A^{\text{eff}}$ is expressed as

$$H_A^{\text{eff}}(\epsilon_A) = T_A + U_A(\epsilon_A) + U_A^N(\epsilon_A),$$

where $\epsilon_N$ is the single nucleon energy in the core nucleus and $\kappa_N$ is the famous wound-integral parameter for which the tensor force correlation is principally responsible.

We focus the present discussion on the rearrangement effect ($U_{AN}$) contribution to $B_A$ for $^3$He($^4$He) and $^4$He in connection with the overbinding problem. We only look at the average of $B_A$ for $^4$He and $^4$He, neglecting the charge symmetry breaking interaction. A simple consideration leads to the following qualitative features of $U_{AN}$ for $^3$He($^4$He) and $^4$He: (i) $|U_{AN}(^3\text{He})| > (4/3)|U_{AN}(^4\text{He})|$, (ii) $\kappa_N(^3\text{He}) > (3/2)\kappa_N(^4\text{He})$, (iii) Size effect, i.e., the $^4$He core is more compact than the $^3$He core. The first two features simply reflect a difference of bond numbers, the inequality coming from the size effect of (iii). Thus we see that

$$U_{AN}(^3\text{He}) > 2U_{AN}(^4\text{He}).$$  \hspace{1cm} (4)

This relation implies that reduction of $B_A$ value due to the rearrangement effect is more than twice larger for $^3$He than for $^4$He($^4$He), qualitatively with desirable nature of saving the overbinding difficulty.

We now make a quantitative evaluation. We employ the Reid-SC potential\(^9\) for $v_{NN}$ and the following s-state central potential for $v_{AN}$,

$$v_{AN}(r) = \begin{cases} V_0, & r < r_c, \\ V_0 e^{-\beta(r-r_c)}, & r > r_c, \end{cases}$$

$$V_0 = 7 \text{ GeV},$$

This $v_{AN}$ gives the scattering lengths $-2.11$ and $-1.71$ fm and the effective ranges $3.22$ and $3.55$ fm for the singlet and triplet $A$-$N$ scatterings, respectively, which are fitted well to experiments. We define the G-matrix with plane wave intermediate spectrum\(^10\) by

$$G_{AN}(\omega) = v_{AN} + v_{AN} \frac{Q_{AN}}{\omega - Q_{AN}(T_A + T_N) Q_{AN}} G_{NN}(\omega),$$  \hspace{1cm} (6)

and $G_{NN}$ in a standard way. We use harmonic oscillators (H.O.) for the unperturbed Hamiltonian and hence H.O. wave functions to describe the model spaces for nucleons and $A$, which have a common oscillator frequency $\Omega = \sqrt{\frac{E}{\hbar M}}$, where $b$ is the size parameter of H.O. and chosen to be appropriate for the core nucleus of each $A$-nucleus. The model space rejection operator $Q_{AN}$ in Eq. (6) is taken here to be 0 for $\{A,N\} = \{0s \sim 3s-0i, 0s\}$ and otherwise, 1. Correspondingly, $P_A$ in Eq. (2) is now of $\{0s \sim 3s\}$ states. Essentially exact ways to solve the G-matrix equation like Eq. (6) are summarized in Ref. 11).

The table displays the result obtained by solving a set of equations (2), (3) and (6). It is evident in the 3rd–5th rows of the table that the qualitative three points (i)–(iii) and Eq. (4) are actually the
case. The difference between $U^R$ contributions to $B_A$ for $^4\text{He}(^3\text{H})$ and $^3\text{He}$ amounts to an order of 1 MeV. Note that this amount is considerably bigger than other effects so far examined and is about one half of the overbinding magnitude found in previous calculations. The last fact is also seen in the last three rows of the table. It is emphasized again that the large $\kappa_N$ value used in the present calculation arises predominantly from the strong $N-N$ tensor force correlation. Some spurious center of mass excitations can be mixed in diagonalizing Eq. (2), but the basic feature and the order of magnitude of the rearrangement effect presented above should not essentially change.

In conclusion, the rearrangement effect due to the strong $N-N$ tensor force correlation is able to resolve a half of the overbinding difficulty of $B_A$ in s-shell hypernuclei. The other half might be overcome by incorporating the effects of $A-\Sigma$ coupling, $ANN$ three-body force, $A-N$ tensor force and so forth.

We would like to thank Professor Y. Shôno and Professor K. Hasegawa for valuable discussions and encouragement and also Mr. K. Shirakami for his assistance in the computer calculations.

9) I. Shimodaya, Soryushiron Kenkyu (Kyoto) 55 (1977), A38.