Inclusive Vector Meson Production in Fragmentation Regions of Meson at Small $P_T$

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Inclusive vector meson spectra in fragmentation regions of meson at small $P_T$ are studied from a two-component model (quark fragmentation and triple Regge contributions). It is shown that our model can explain successfully various inclusive vector meson spectra. The analyses of this paper show that quark decay functions into vector mesons are nearly the same as those into pseudo-scalar mesons and the strange quark suppression factor $\lambda$ is nearly the same as in pseudo-scalar meson production.

§ 1. Introduction

In a previous paper (we call I hereafter), we studied inclusive pseudo-scalar meson and octet baryon production in fragmentation regions at small $P_T$. The key points of our model are the co-existence of quark fragmentation and triple Regge contributions and the momentum distribution of constituent quark predicted by our model.\(^a\) In order to test the model, many other processes should be investigated. In this paper, we shall study inclusive vector meson production in fragmentation regions at small $P_T$.

From analyses of inclusive vector meson spectra we can know fragmentation functions of quark into vector mesons. Large $P_T$ hadron production experiments show that they are nearly equal to those into pseudo-scalar meson, e.g., $g_u = g_s$.\(^a\) If our model succeeds in explaining inclusive vector meson spectra at small $P_T$ using quark decay functions consistent with large $P_T$ experiments, it will be a strong support to our model.

The strange quark suppression factor ($\lambda$) was determined to be 0.22 in I. It is, however, premature to conclude that the true value of $\lambda$ in an elementary process is 0.22 since there is a possibility that resonance decay contributions affect the $K/\pi$ ratio, although they are expected to be small in fragmentation regions. In vector meson production in fragmentation regions, resonance decay contributions can be safely neglected since cross sections of higher mass mesons (e.g., tensor mesons) are known to be small.\(^a\) In this sense a vector meson production is adequate to determine $\lambda$.

Inclusive vector meson spectra have been studied since many years ago.\(^a,\!^b\)
Anisovich and Shekhter investigated them assuming that cross sections of vector meson production are three times those of pseudo-scalar mesons. Kinoshita et al. attempted to describe inclusive vector meson spectra on the basis of quark fragmentation model and obtained \( g_{v} \approx 1.67 g_{s} \). However, quark fragmentation model alone cannot explain the interesting \( P_{T} \) dependence of invariant cross sections as shown typically in \( \pi^{\pm} \rightarrow \pi^{0} \). Inclusive vector meson spectra should be re-analyzed by a model which can explain main features of inclusive hadron productions at small \( P_{T} \). In this paper we shall perform this task on the basis of a two-component model with quark fragmentation contributions predicted by our model.\(^{11}\)

In the next section our model is briefly summarized to the extent relevant to the purpose of this paper. In §3, we analyze inclusive vector meson spectra in fragmentation regions. Section 4 is devoted to discussions.

\section{Model}

The invariant cross section \((E d\sigma/dP^{a})\) of inclusive hadron production at small \( P_{T} \) is described by the sum of quark fragmentation contribution \((QFC)\) and triple Regge contribution \((TRC)\).\(^{11}\)

\[ E \frac{d\sigma}{dP^{a}} = QFC + TRC. \]  

QFC has the following form\(^{11}\) in the fragmentation of hadron \( b \) into hadron \( c \) in hadron \( a\)-hadron \( b \) scattering (we denote this process \( b \rightarrow c \)).

\[ QFC = \sigma_{R}(ab) \frac{\hat{s}}{\pi} e^{-s/F} F(x_{F}), \]  

where \( \sigma_{R}(ab) \) is the total cross section of hadron \( a\)-hadron \( b \) scattering and \( x_{F} \) is the Feynman variable. Using the momentum distribution function of constituent quark in hadron \( b (f_{q}^{b}) \) and the decay function of quark into hadron \( c (g_{q}^{c}) \), \( F(x_{F}) \) is described as follows:

\[ F(x_{F}) = x_{F} \int f_{q}^{b}(y) g_{q}^{c}(x) \delta(yz - x_{F}) dy dz. \]  

\( \beta \) is taken to be \( 1.5 \text{(GeV/c)}^{-2} \) in meson fragmentations.\(^{11}\) If \( b \) is a meson, we have\(^{11}\)

\[ f_{q}^{b}(x) = \frac{8}{\pi} \sqrt{x (1-x) \beta^{2}}. \]  

Quark decay functions into vector mesons are assumed to be \( \alpha \) times those into pseudo-scalar mesons which were determined in I. \( g_{q}^{c} \) is as follows in the case that \( c \) is a vector meson:
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\[ g_4^e = \frac{1}{4} \alpha \frac{\sqrt{1-z}}{z} \] if \( c = q\bar{u} \) or \( q\bar{d} \),
\[ g_5^e = \frac{1}{4} \alpha \frac{\sqrt{1-z}}{z} \] if \( c = q\bar{s} \),
\[ g_6^e = 1.14\alpha \frac{(1-z)^{1.5}}{z} \] if \( c \) does not contain \( q \) and is composed of non-strange quarks only,
\[ g_7^e = 1.14\alpha \lambda \frac{(1-z)^{1.5}}{z} \] if \( c \) does not contain \( q \) and contain one strange quark

and
\[ g_8^e = 1.14\alpha \lambda z \frac{(1-z)^{1.5}}{z} \] if \( c \) does not contain \( q \) and \( c = \bar{s}s \),

where \( \lambda \) is the strange quark suppression factor.

If precise data on \( E\sigma/dP^3 \) exist over a wide \( P_T \) range, we can determine TRC completely. Since unfortunately this is not the case, we take a leading trajectory dominance assumption. TRC is large only at \( x_F \) near unity and leading trajectories are dominant in this region. Therefore this assumption is expected to be not so bad. We use the following form for TRC,

\[ \text{TRC} = A e^{\beta} (1 - x_F)^{1-2\alpha(t)} \] (6)

The form \( e^\alpha \) was obtained in I. We use this form also for all meson \( \rightarrow \) vector meson processes.

As for the relation between \( t \) and \( P_T \), we take

\[ t = - \frac{P_T^2}{x_F} + m_b^2(1 - x_F) + m_c^2(1 - x_F^{-1}) \] (7)

where \( m_b(m_c) \) is the mass of hadron \( b \) (\( c \)). In this paper we use \( \sigma_T(\pi N) = 24 \text{mb} \), \( \sigma_T(K^- p)/\sigma_T(\pi N) = 0.9 \) and \( \sigma_T(K^+ p)/\sigma_T(\pi N) = 0.7 \). Since the data are not so precise, we take an approximation that \( \int E(d\sigma/dP^3)d^3P_T = x_P d\sigma/dx_P \).

§ 3. Analyses of inclusive vector meson production

(1) \( \pi^+ p \rightarrow \rho^0 \)

In this process QFC and TRC are as follows using \( \alpha_A = 0.8t + 0.5 \):

\[ \text{QFC} = 2.9\alpha e^{-1.5t} \frac{1}{x_F} \frac{1}{1-\rho^0} \text{mb/GeV} \] (8)

and

\[ \text{TRC} = A e^\beta (1 - x_F)^{-1-\alpha} \] (9)
At not so small \( p_T \) (e.g., \( p_T \sim 0.8 \text{ GeV/c} \)), Eq. (8) is dominant. Therefore, if precise data on \( E\sigma / dP^2 \) exist at such \( P_T \), we can determine \( \alpha \) directly. However, since we have good data only on \( x_P d\sigma / dx_P \nu \) we try to determine \( \alpha \) and \( A \) by the data on \( x_P d\sigma / dx_P \).

From Eqs. (8) and (9), we obtain for \( (x_P/\pi) (d\sigma / dx_P) \) as follows:

\[
\frac{x_F}{\pi} \frac{d\sigma}{dx_F} = 1.9\alpha (1-x_F)^{1+\frac{1}{2}} + \frac{Ax_F}{u} e^{u_0}, \quad (10)
\]

where \( u = 3 - 1.6 \ln (1-x_F) \) and \( u_0 = m_0^2 (1-x_F^{-1}) \). The first term of Eq. (10) has a behavior versus \( x_F \) very different from the second one. Therefore we can determine \( \alpha \) and \( A \) considerably precisely. As shown in Fig. 1, we obtain a good fit in the case \( \alpha = 1 \) and \( A = 10 \text{ mb/GeV}^2 \). Figure 1 shows that the cases \( \alpha > 2 \) and \( \alpha < 0.5 \) are excluded. Hereafter we take \( \alpha = 1 \). \( d\sigma / dx_F dP_T \) with \( \alpha = 1 \) and \( A = 10 \text{ mb/GeV}^2 \) is plotted and compared with the data in Fig. 2. In order to confirm a \( (1-x_F)^{1+\frac{1}{2}} \) behavior of QFC, more precise data are needed.

Using Eq. (5) and taking \( \alpha_{x_F} = 0.8t + 0.36 \), we have \( x_F d\sigma / dx_F \) for this process as follows:

\[
x_F \frac{d\sigma}{dx_F} (p^{+}p^{+} \rightarrow K^{+}K^{-}) = 6\lambda (1-x_F)^{1+\frac{1}{2}} \frac{\pi x_F}{u} A (p^{+}p^{+} \rightarrow K^{+}K^{-}) (1-x_F)^{0.18} e^{u_0}, \quad (11)
\]
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In the cases $\lambda=0.2$ and $A(\pi^+\to K^{*+})=2\text{mb/GeV}^2$ (solid line), $\lambda=0.15$ and $A(\pi^+\to K^{*+})=2\text{mb/GeV}^2$ (dashed line), and $\lambda=0.5$ and $A(\pi^+\to K^{*+})=1.84\text{mb/GeV}^2$ (dash-dotted line).

where $t_1=m_{K^*}^2(1-x_F)$. The experimental data show that $x_Fd\sigma/dx_F(\pi^+\to K^{*+})=0.25\text{mb}$ for $x_F>0.5$. From this information we can determine $\lambda$ and $A(\pi^+\to K^{*+})$. As shown in Fig. 3, we can fit the data in the case that $\lambda$ is near 0.2. The case $\lambda=0.5$ is definitely excluded. The best fit is obtained in the case $\lambda=0.15$. The process $K^+\phi\phi$ has a feature similar to the process $\pi^+\to K^{*+}$. SU(3) flavour symmetry tells us that TRC of both processes are common. QFC of both processes are also common except for the difference between $\sigma_T(\pi\rho)$ and $\sigma_T(K\phi)$. Although the data on $K^+\phi\phi$ are very scarce, the data seem to require a slightly larger value for $\lambda$ (e.g., $\lambda=0.25$). Therefore we take $\lambda=0.2$ since it gives reasonable fits to both processes. In this case, $A(\pi^+\to K^{*+})=2\text{mb/GeV}^2$.

For this process we can determine cross sections without any adjustable parameter from the previous analyses. From SU(3) we have the relation $A(K^+\phi\phi)=\frac{1}{4} A(\pi^+\to K^{*+})$. Therefore we obtain using $\alpha=1$ and $A(\pi^+\to K^{*+})=2\text{mb/GeV}^2$,

$$x_F\frac{d\sigma}{dx_F}(K^+\phi\phi)=2.1(1-x_F)^2+\frac{\pi_x}{u}(1-x_F)^{3.3}e^{V_1},$$

Equation (12) is plotted and compared
with the data\(^9\) in Fig. 4. As shown in Fig. 4, our model can explain nicely both the magnitude and shape of \(K^+\bar{\rho}^0\). This gives a strong support to our model.

(4) \(\pi^+\bar{\varphi}^0\)

Since \(\alpha = 1\), QFC(\(\pi^+\rightarrow\rho^-\)) is the same as QFC(\(\pi^+\rightarrow\pi^-\)) obtained in I. Therefore we have

\[
x_F \frac{d\sigma}{dx_F} (\pi^+\bar{\varphi}^0) = 4.52 (1-x_F)^4 r(x_F) \text{mb},
\]

where \(r(x_F) = \int_{1-x_F}^{x_F} (1+x_F)/(2-x_F)^t/2 \, dt\). Equation (13) is very small at large \(x_F\) compared with the experimental data\(^9\). However, in this process QFC is very small compared with that in \(\pi^-\bar{\rho}^0\) and TRC does not exist. Therefore a resonance decay contribution cannot be neglected at large \(x_F\).

Here we assume the dominance of \(\pi^+\rightarrow A_1^0\rightarrow\varphi^0\) for a resonance decay contribution and take an approximation that \(A_1\) is produced with \(P_F = 0\) only and decays isotropically at the rest system of \(A_1\). Since the cross section of tensor meson production is \(\sim 20\%\) of that of vector meson production according to the experiments\(^3\), we assume that \(x_F d\sigma/dx_F(\pi^+\bar{\varphi}^0) = 0.2 \times x_F d\sigma/dx_F(\pi^+\bar{\rho}^0) = 0.5 \text{mb}\). As seen in Fig. 5, the sum of Eq. (13) and \(A_1^0\) decay contribution agrees reasonably with the data\(^9\).

(5) \(\pi^-\rightarrow\phi\)

\(\varphi\) meson has a small \(u\bar{u}+d\bar{d}\) component and large \(s\bar{s}\) one. Therefore QFC of \(\pi^+\rightarrow\phi\) has two components. Using the octet-singlet mixing angle \(\theta_s\), QFC of \(\pi^+\bar{\varphi}^0\) is expressed as follows:

\[
x_F \frac{d\sigma}{dx_F} (\pi^+\bar{\varphi}^0) = 2 \left( \frac{1}{\sqrt{6}} \cos \theta_s - \frac{1}{\sqrt{3}} \sin \theta_s \right)^2 \frac{1}{4} (1-x_F)^4
\]

\[
+ (0.2)^2 \left( -\frac{2}{\sqrt{6}} \cos \theta_s - \frac{1}{\sqrt{3}} \sin \theta_s \right) \frac{x_F}{\sigma_F} \frac{d\sigma}{dx_F} (\pi^+\bar{\rho}^0)
\]

where the first (second) term corresponds to the \(u\bar{u}+d\bar{d} (s\bar{s})\) component.

As for TRC, \(\rho\) and \(A_1\) exchange contributions exist. However, we neglect TRC terms since they are very small due to the OZI rule. This approximation is valid except for \(x_F\) very close to unity, where TRC can contribute significantly due to the small exponent of \((1-x_F)\). Since a resonance decay contribution to \(\varphi\) spectrum can be safely neglected, we try to fit the experimental data by Eq. (14). In doing this, the following point should be taken into account.

The connected diagram rule tells us that a jet produced through the fragmentation of \(\pi^-\) has a \(\phi\bar{K}K\) state in the case that \(\varphi = s\bar{s}\). Therefore, the second term of Eq. (14) works only in very high energy where \(\sqrt{s}/2 \gg (m_x^2/4 + m_x)^{1/2} + 2m_x\). In the data available at present \((P_T = 16 \text{GeV}/c)\),\(^{10}\) this condition is not satisfied.
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for \( x_F > 0.5 \). Hence we should compare only the first term of Eq. (14) with the data for \( x_F > 0.5 \).

Equation (14) is sensitive to the value of \( \theta_v \). As shown in Fig. 6, a good fit is obtained in the case \( \theta_v = 40.5^\circ \), which is in good agreement with the one required by the mass formula. In Fig. 6, our prediction for \( x_F d\sigma / dx_F \) in very high energy (e.g., \( P_L \sim 100 \text{ GeV/c} \)) is also plotted in the case \( \theta_v = 40.5^\circ \).

(6) Other processes
Since \( \alpha \) and \( \lambda \) are determined \((\alpha = 1 \text{ and } \lambda = 0.2)\), we can calculate QFC of any vector meson production. For example, we have for the proton target,

\[
\text{QFC}(\pi^+ \to \rho^+) = 2\text{QFC}(\pi^+ \to \omega) = 2r \text{QFC}(K^- \to K^{*0}) \\
= 1.67r \text{QFC}(K^- \to K^{*-}) = 4r \text{QFC}(K^- \to \phi) \\
= 2\text{QFC}(\pi^+ \to \rho^+) = 5.8e^{-1.19x} (1 - x_F)^4 \text{mb/GeV}^2 ,
\]

where \( r = \delta_T(\pi p) / \delta_T(K^- p) \). The leading trajectory dominance, vector-tensor exchange degeneracy and \( SU(3) \) tell us that TRC of \( K^- \to K^{*0} \) and \( \pi^+ \to \rho^+ \) are common. Therefore the cross section of \( K^- \to K^{*0} \) is roughly equal to that of \( \pi^+ \to \rho^+ \) in our model. The experiments show that this is the case. The experiments also show that \( \omega / \phi \sim 1 \).
§ 4. Discussion

Main results obtained in this paper are the following.

(1) The two-component model with QFC predicted by our model can explain successfully inclusive vector meson productions at small $P_T$.

(2) Fragmentation functions of quark into vector mesons are nearly equal to those into pseudo-scalar mesons, i.e., $\alpha \approx 1$.

(3) The strange quark suppression factor $\lambda$ in vector meson production is roughly the same as in pseudo-scalar meson production ($\lambda \approx 0.2$). The behavior of $x_T d\sigma/dx_T$ of $K^\pm \to \rho^0$ is very different from those of $\pi^\pm \to \rho^0$ and $p^+ \to K^{*+}$. It seems impossible to explain these data by a quark fragmentation model alone. The two-component model can describe successfully these features. Especially, the successful explanation of both $\pi^+ \to K^{*+}$ and $K^+ \to \rho^0$ implies that our QFC is correct, since TRC of both processes are intimately related.

The fact that $\alpha \approx 1$ implies that small $P_T$ jets have features similar to large $P_T$ jets except for triple Regge contributions. This gives a support to the treatment of small $P_T$ inclusive spectra by a quark fragmentation model.

In the recombination model, the cross section of vector meson production is expected to be three times that of pseudo-scalar meson production, if the so-called sea components are spin singlet. This is because in a state $|\uparrow\downarrow\rangle$ spin triplet and singlet states exist fifty-fifty and $|\uparrow\rangle|\downarrow\rangle = |\text{spin triplet}\rangle$.

As for $\lambda$, we obtain a value consistent with the one in I. This strongly suggests that $0.2$ is the true value of $\lambda$ in an elementary process. Field and Feynman considered that the true value of $\lambda$ is $0.5$ and $\lambda$ approaches $0.2$ as $z \to 0$ due to the resonance decay contributions. We cannot take this idea, because it predicts that $u \to K^{*+} / u \to \rho^+ \approx 0.5$. Considering the ratio $K/\pi$ in both small and large $P_T$ experiments, it seems to be forced to conclude that the true value of $\lambda$ itself has a $z$ dependence at $z$ very close to unity.

Assuming $\alpha = 3$, Anisovich and Shekhter derived $\lambda = 1/3$ from the ratio $K/\pi$ at $x_T \approx 0 (\approx 0.1)$. If we take $\alpha = 1$ and only vector meson decay products, their method tells us that $\lambda = 0.2$ corresponds to $K/\pi (x_T \approx 0) \approx 0.09$, which is consistent with the data.

References

   See also, Prog. Theor. Phys. 46 (1971), 550.