Stresses and Displacements in an Elastic-Plastic Wedge[1]

D. C. Drucker.[2] So few elastic-plastic solutions have been found that the additional one provided by the author is a very substantial contribution. It is worth noting, perhaps, that two of the interesting features of this plane-strain solution can be established directly without finding the solution itself. They are: the lack of dependence on the radial co-ordinate r, and the symmetry of the regions of plastic action, $\varphi_1 = \beta - \varphi_2$, Fig. 1($a$) of this discussion.

The independent variables of the problem are the co-ordinates r, θ, the elastic constants, say E and ν, the yield stress k, the uniform load p, and the wedge angle β. Dimensional analysis then shows that any component of stress σ, and any component of displacement u must be given by the following functional forms or their equivalents

$$\frac{\sigma}{k} = f\left(\frac{p}{k}, \frac{k}{E}, \nu, \beta, \theta\right)$$

$$\frac{u}{r} = F\left(\frac{p}{k}, \frac{k}{E}, \nu, \beta, \theta\right)$$

The co-ordinate r cannot appear in f or F.

The symmetry of the plastic regions may be established for the Mises yield criterion or any of the wide class of symmetric yield or loading functions which are independent of the mean normal stress. With such a criterion, a hydrostatic pressure may simply be superposed on any elastic-plastic solution. The answer to the original problem of Fig. 1(a) is then exactly the same as the sum of the hydrostatic pressure $p/2$ of Fig. 1(b), and the antisymmetric problem of Fig. 1(c), $p/2$ compression applied at $\theta = 0$ and $p/2$ tension at $\theta = \beta$. As hydrostatic compression is assumed to be without influence on the plastic action, the plastic zones are the symmetrically located ones of the antisymmetric case.

[1] By P. M. Naghdi, published in the March, 1957, issue of the Journal of Applied Mechanics, Trans. ASME, vol. 79, pp. 98–104.

[2] Chairman, Division of Engineering, Brown University, Providence, R. I. Mem. ASME.

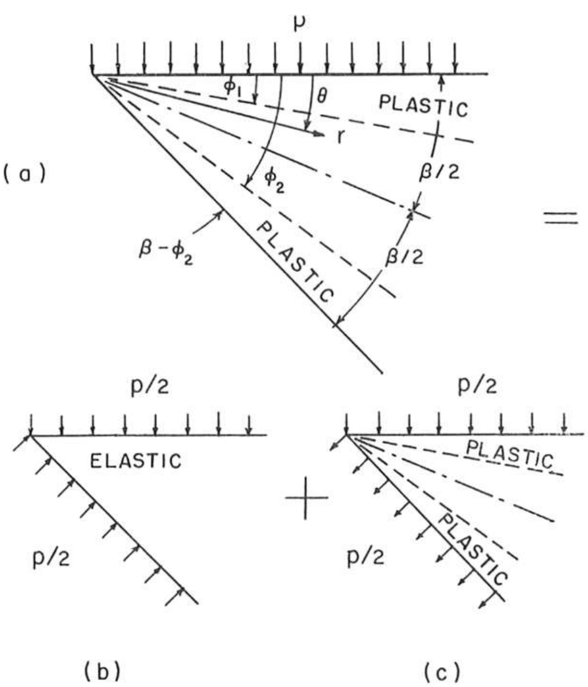

Fig. 1 Symmetry of Plastic Regions, $\varphi_1 = \beta - \phi_2$, ($a$) = ($b$) + ($c$)

Author's Closure

The author is very appreciative of Professor Drucker's interesting observations on two aspects of the solution.

New Method to Measure Prandtl Number and Thermal Conductivity of Fluids[1]

J. Kestin.[2] Most methods which are used for the measurement of thermodynamic and transport properties may be called static, as they are based on the principle of effecting the smallest possible departure from equilibrium in the fluid to be measured. The authors have demonstrated that a dynamic method involving high velocities of flow is also feasible. The ingenious concept leads to a very simple experimental setup in which the major sources of error can be eliminated. In principle, as the experimental data determine the Prandtl number, Equation [1] of the paper, any one of the three quantities μ, c_p, or k can be evaluated if the other two are sufficiently well known. The authors rightly point out that, from the point of view of existing accuracy, the determination of thermal conductivity k will benefit most from the introduction of the new method, as its direct measurement constitutes a very difficult experimental task. It is to be hoped that the authors will continue to perform measurements with an improved and more elaborate setup, as they clearly intend to do.

Personally the writer always finds it regrettable that limitations of space in contemporary scientific journals make it impossible to quote tables of experimental results. Nevertheless, he would like to ask the authors to indicate some numerical values concerning the following quantities:

1 Temperature fluctuation in the main chamber (as measured by thermocouple f).

2 Maximum and minimum values of the pressure ratio p_s/p_t.

3 Range of exit velocities used.

4 Maximum and minimum temperature difference $t_t - t_s$ used in the measurements.

5 The values of γ chosen for insertion into Equation [6].

It is believed that publication of these numerical values will permit a more accurate appraisal of the potentialities of this excellent new idea.

Authors' Closure

In response to Professor Kestin's questions, we are pleased to submit the following information:

1 The heat capacity of the storage-type heat exchanger was such that the air temperature as indicated by the total temperature thermocouple f was constant within ±0.1 F for approximately 40 sec after the flow was started. During this period the time fluctuations of the indicated temperature were within 0.05 F.

2 to 4 The choice of the ratio static to total pressure which determines the exit velocity is dictated by the following con-

[1] By E. R. G. Eckert and T. F. Irvine, Jr., published in the March, 1957, issue of the Journal of Applied Mechanics, Trans. ASME, vol. 79, pp. 25–28.

[2] Professor of Engineering, Brown University, Providence, R. I. Mem. ASME.

siderations: The larger the velocity the larger is the temperature variation throughout the boundary layer. On the magnitude of the temperature variation depends the error connected with the use of the constant property relation, Equation [3], for the recovery factor. This error can well be calculated from available solutions of the laminar boundary-layer equations for variable property fluids. A decrease of the velocity, on the other hand, decreases the difference between total and recovery temperature and increases the errors connected with the measurement of this value. It is advisable to optimize conditions with respect to these errors for each investigation. In the reported measurements the pressure ratio was adjusted on the basis of these considerations to values between $p_t/p_s = 1.27$ and 1.82. Occasionally, because of air supply limitations, the pressure ratio was lower than optimum. Correspondingly, the exit velocity varied from 661 to 1135 ft/sec, the Mach number from 0.60 to 0.96, and the difference between total and recovery temperature from 5.67 F to 22.4 F. The temperature with which a measured Prandtl number was associated and the temperature at which the property values were introduced were the "reference temperature," familiar in boundary-layer analysis.[3] In these particular data, choosing the extremes of either the total or free-stream static temperature as a reference produced a data shift within the accuracy of the measurements. In Fig. 2, the Prandtl number is plotted over the reference temperature as calculated from Eckert.[3]

5 The heat capacity ratio required for evaluation of Equation [6] was obtained from Keenan and Kaye[4] and introduced at the total temperature.

[3] "Engineering Relations for Heat Transfer and Fluid Friction in High-Velocity Laminar and Turbulent Boundary-Layer Flow Over Surfaces With Constant Pressure and Temperature," by E. R. G. Eckert, Trans. ASME, vol. 78, 1956, pp. 1273–1283.

[4] "Gas Tables," by J. Keenan and J. Kaye, John Wiley and Sons, Inc., New York, N. Y., 1949, p. 34.

Buckling of Initially Imperfect Cylindrical Shells Subject to Torsion[1]

T. T. Loo.[2] The author is to be commended for his continuous interest and effort in this problem, which is one of considerable technical importance. Experiments have shown that the buckling strengths of thin-shell structures are usually sensitive to initial imperfections; thus the need for large-deflection theories in such a class of problems is self-evident.

In 1952, the writer, as a student of Prof. L. H. Donnell, established a large-deflection theory for both perfect and imperfect thin cylindrical shells under pure torsion.[3] In developing the theory, some approximations were made to simplify algebraic computations that otherwise would be very tedious. In the expression for radial displacement w, there appear four parameters. Two of these, γ and n, were assumed to take values as given by small-deflection theory. This was done before making the first variation of the total potential energy of the system stationary. The fact that close agreement was obtained in the buckling strengths of this simplified theory with those from Donnell's more accurate method[4] has warranted such a simplification. Even though it may be expected that there will be somewhat less reduction of resistance during the postbuckling stage due to such a simplification, it seems to have little effect on the values of critical loads, which is all we are really interested in.

The author has made a valuable contribution in his discussion of the condition for periodicity of the circumferential displacement v. This condition will lead to a relationship between the parameters, making three rather than four independent parameters actually involved in the total potential-energy expression. Thus the computational difficulties might be lessened to the extent that a more accurate solution could be found readily without the writer's simplification. But the author obviates this saving by introducing a new and unnecessary parameter.

In the energy method of solution of torsion loading, the boundary condition of vanishing radial displacement w at both ends is usually considered to be more important than other conditions required for either clamped or simply supported shells. By introducing a uniform radial displacement, which is in effect a fifth parameter, in his assumed expression for w, Equation [6] and Equation [7], the author complicates the computation and, by violating the major rather than the secondary boundary condition, he has thus treated a problem with even less rigorous boundary conditions. The result of such an introduction, instead of "obtain(ing) the minimum theoretical buckling load," results in values which are much too high in comparison with those of the existing theories.[3,4]

In previous publications,[3,5] attempts were made to express the inherent imperfections of the specimens in terms of their physical dimensions. It is obvious that the imperfections in any specimen are independent of the type of loading; with this in mind the writer has suggested a rational formula[3] which seems to be reasonably satisfactory in the light of its adaptability to both buckling under axial compression and under pure torsion. This formula without any change of parameters accounts for the widely contrasting phenomena which have been observed from tests in these cases. However, in the present paper, a similar formula, but with a different value for the unknown parameter U_0, was used for the case of pure torsion alone. When the same parameter is used for the case of axial compression, it is found that the theoretical curve is much higher than the experimental result. It is always possible with the two parameters in Equation [21] to fit the experimental curve for one type of loading. However, the usefulness of the expression becomes evident only if several types of loading are approximated.

Author's Closure

The author would like to thank Dr. Loo for his constructive comments on the topic of torsional buckling of initially imperfect cylindrical shells. As stated in the author's paper, the current work is an extension of earlier work done by Dr. Loo. In an effort to refine the process, the present author rendered the total potential energy stationary with respect to four parameters, whereas Dr. Loo had assumed in his approximate analysis that two of these parameters had the values given by small-deflection theory.

The boundary conditions investigated by the author were chosen in an effort to investigate the effects of various possible buckling displacements and did not necessarily correspond to

[1] By W. A. Nash, published in the March, 1957, issue of the Journal of Applied Mechanics, Trans. ASME, vol. 79, pp. 125–130.

[2] Associate Professor of Mechanics, Rensselaer Polytechnic Institute, Troy, N. Y.

[3] "Effects of Large Deflections and Imperfections on the Elastic Buckling of Cylinders under Torsion and Axial Compression," by T. T. Loo, Ph.D. dissertation, Illinois Institute of Technology, 1952; Proceedings of the Second U. S. National Congress for Applied Mechanics, 1954.

[4] "Stability of Thin-Walled Tubes Under Torsion," by L. H. Donnell, NACA Report No. 479, 1934.

[5] "Effect of Imperfections on Buckling of Thin Cylinders and Columns Under Axial Compression," by L. H. Donnell and C. C. Wan, Journal of Applied Mechanics, Trans. ASME, vol. 72, 1950, pp. 73–83.