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Discussion

Discussion: "Heat Transfer Measurement and Analysis for Sintered Porous Channels" (Hwang, G. J., and Chao, C. H., 1994, ASME J. Heat Transfer, 116, pp. 456–464)

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Our purpose in this discussion is to demonstrate that some of the results stated in Hwang and Chao's paper appear to be in error. We can show that a recalculation of Hwang and Chao's equations leads to different results that differ from those put forth in their paper. Specifically, some of the calculated values of $\mathrm{Nu}_{f\infty}$ for $d_p=1.59$ mm in Fig. 6 of their paper are incorrect. This can be proven by comparing their numerical results with the exact $\mathrm{Nu}_{f\infty}$, which is obtained from analytical solutions for velocity and temperature distributions in the case of no dispersion. The momentum Eq. (9) in their paper can be analytically solved to yield the closed-form solution for the velocity distribution as follows ([1]):

$$U=\left(\frac{z+1}{z-1}\right)^2\left(3U_\infty+\frac{3}{2F_s}\right)-2U_\infty-\frac{3}{2F_s}, \tag{1}$$

where

$$F_s=\frac{\mathrm{Re}_d}{d_p}F\sqrt{K},\quad Da=\frac{K}{\varepsilon H^2},\quad U_\infty=\frac{-1+\sqrt{1+4F_sC}}{2F_s}$$

$$z(Y)=\begin{cases}\dfrac{\sqrt{2U_\infty+\frac{3}{2F_s}}+\sqrt{3U_\infty+\frac{3}{2F_s}}}{\sqrt{2U_\infty+\frac{3}{2F_s}}-\sqrt{3U_\infty+\frac{3}{2F_s}}}\exp\left[Y\sqrt{\frac{2F_s}{Da}U_\infty+\frac{1}{Da}}\right] & \text{for } 0\leqslant Y\leqslant\frac{1}{2}\\[2ex] \dfrac{\sqrt{2U_\infty+\frac{3}{2F_s}}+\sqrt{3U_\infty+\frac{3}{2F_s}}}{\sqrt{2U_\infty+\frac{3}{2F_s}}-\sqrt{3U_\infty+\frac{3}{2F_s}}}\exp\left[(1-Y)\sqrt{\frac{2F_s}{Da}U_\infty+\frac{1}{Da}}\right] & \text{for } \frac{1}{2}\leqslant Y\leqslant 1.\end{cases}$$

In the case of no dispersion, energy Eqs. (10) and (11) in their paper can be reduced to

$$0=\mathrm{Bi}(\theta_f-\theta_s)+\frac{d^2\theta_s}{dY^2} \tag{2}$$

$$U=\theta_s-\theta_f. \tag{3}$$

Equations (2) and (3) can be merged into Eq. (4).

$$\frac{d^2\theta_s}{dY^2}=\mathrm{Bi}U \tag{4}$$

By integrating Eq. (4), the analytical solution for the solid temperature distribution can be obtained as follows:
for $0\leqslant Y\leqslant 1/2$,

$$\theta_s=\frac{\mathrm{Bi}}{B}\left[12\left(U_\infty+\frac{1}{2F_s}\right)\left\{\frac{2}{A\exp\left(\frac{1}{2}B\right)-1}-\frac{1}{A-1}\right\}-U_\infty(\ln A+B)\right]Y+\frac{\mathrm{Bi}}{B}\left[\frac{U_\infty}{2}\{(\ln z)^2-(\ln A)^2\}-12\left(U_\infty+\frac{1}{2F_s}\right)\ln\frac{A(z-1)}{(A-1)z}\right], \tag{5a}$$

for $1/2\leqslant Y\leqslant 1$,

$$\theta_s=\frac{\mathrm{Bi}}{B}\left[U_\infty\ln A-12\left(U_\infty+\frac{1}{2F_s}\right)\frac{1}{A-1}\right]Y+\frac{\mathrm{Bi}}{B}\left[\frac{U_\infty}{2}\{(\ln z)^2-(\ln A+B)^2\}-12\left(U_\infty+\frac{1}{2F_s}\right)\ln\frac{A\exp(B)(z-1)}{(A\exp(B)-1)z}\right], \tag{5b}$$

where

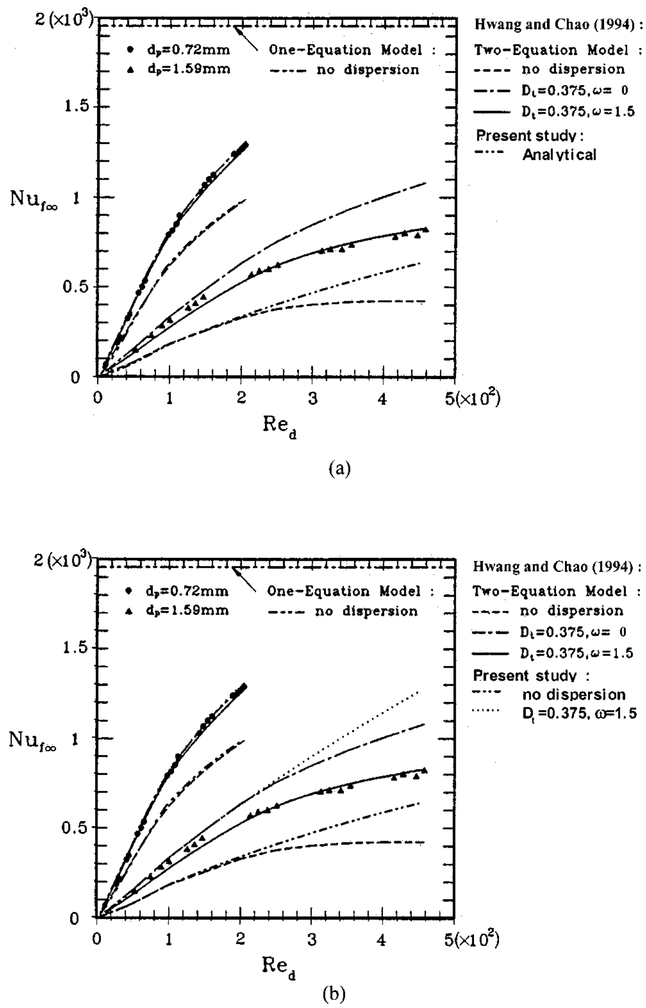

Fig. 1 Comparisons of the fully developed Nusselt number; (*a*) analytical result for no dispersion, (*b*) numerical results for limiting cases

$$A=\frac{\sqrt{2U_\infty+\dfrac{3}{2F_s}}+\sqrt{3U_\infty+\dfrac{3}{2F_s}}}{\sqrt{2U_\infty+\dfrac{3}{2F_s}}-\sqrt{3U_\infty+\dfrac{3}{2F_s}}},\quad B=\sqrt{\frac{2F_s}{Da}U_\infty+\frac{1}{Da}}.$$

Now the fluid temperature distribution can be obtained by rearranging Eq. (3) as

$$\theta_f=\theta_s-U. \tag{6}$$

From the definition of the overall Nusselt number, the analytical form of $\mathrm{Nu}_{f\infty}$ can be obtained as

$$\mathrm{Nu}_{f\infty}=-\frac{5k_s^*\mathrm{Bi}}{3k_f\theta_{fb}}, \tag{7}$$

where $\theta_{fb}=\int_0^1 U\theta_f dY$.

Figure 1 shows the relation between $\mathrm{Nu}_{f\infty}$ and Re_d. For the case of $w=\infty$ (no dispersion), the numerical results (broken line) of Hwang and Chao start deviating from the analytical results (double-dotted broken line) of Eq. (7) when $\mathrm{Re}_d>200$, as shown in Fig. 1(*a*). In contrast with the results of Hwang and Chao, results from our numerical simulation (double-dotted broken line in Fig. 1(*b*)) for the same problem are in excellent agreement with the exact $\mathrm{Nu}_{f\infty}$ of Eq. (7) (double-dotted broken line in Fig. 1(*a*)). For another limiting case where $D_t=0.375$, $w=0$ (maximum dispersion), similar deviations between our results (dotted line) and the results of Hwang and Chao (single-dotted broken line) exist for $\mathrm{Re}_d>200$, as shown in Fig. 1(*b*). Note that ''$w=\infty$'' in Fig. 6 of their paper should be replaced by ''$w=0$.''

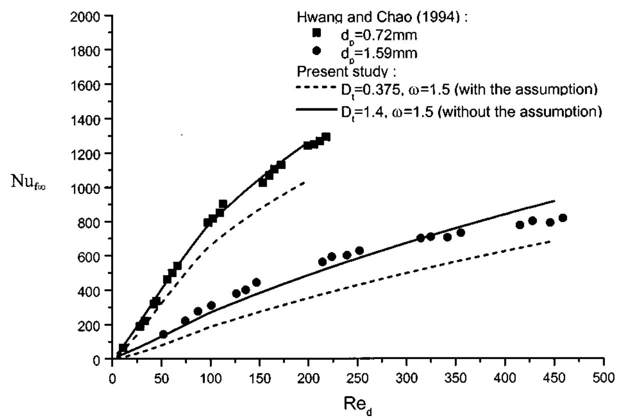

Fig. 2 Comparison of the fully developed Nusselt number

In addition, it is worth mentioning that Hwang and Chao employ an inappropriate assumption in their paper. They assumed that the effective thermal conductivity of the fluid is negligible. However, the effective thermal conductivity of the fluid is not negligible in comparison with the thermal dispersion conductivity when Re_d is small, because the thermal dispersion conductivity decreases with Re_d. Re_d in their paper ranges from 0 to 500 which is not large enough to neglect the effective thermal conductivity. Hence in their case the neglect of the effective thermal conductivity of the fluid can result in more than a ten percent error.

Hwang and Chao proposed that $D_t=0.375$ and $w=1.5$ in order to match their numerical results to the experimental results by using the inappropriate assumption and their simulation code. Now we tried to obtain the more appropriate values of D_t and w for which our numerical results, without neglecting effective thermal conductivity, can match the experimental data of Hwang and Chao. From Eqs. (11) and (12) in their paper, it can be clearly seen that increasing D_t or decreasing w has the same effect on the thermal dispersion conductivity. Therefore, by adjusting either D_t or w, it is possible to match the numerical results to the experimental results. In our numerical simulation, D_t is selected as a variable for adjustment, since D_t is proportional to the thermal dispersion conductivity, as shown in Eq. (11) in their paper. On the other hand, w is fixed at 1.5 which has been consistently used in previous studies ([2,3]). From our numerical simulation for the condition that Hwang and Chao proposed, it can be shown that $D_t=0.375$ and $w=1.5$ are not appropriate, as denoted by broken lines in Fig. 2. Without using the assumption of Hwang and Chao, which neglects the effective thermal conductivity of the fluid, our numerical results are shown to be in good agreement with the experimental results of Hwang and Chao when $D_t=1.4$ and $w=1.5$, as denoted by solid lines in Fig. 2.

References

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Note from the Editor: Professor G. J. Hwang passed away last year, and despite repeated attempts, the Editor was unable to locate or contact the second author, C. H. Chao. Any response from Dr. Chao will be published in a later issue.