as in the discussion, and not on the equilibrium relations (1). Second, rather than saying that the stress rates which are not relative are inadmissible, it would be more precise to say that they are inconvenient. They can be used correctly (as has been done in Timoshenko's works) if one is aware of the special precautions needed in this case, e.g., that the matrix of elastic moduli is nonsymmetric when potential energy exists. Finally, the author does not share the view that the questions raised are mainly academic, as might seem from the uncertainty governing the exact stress-strain relations of "soft" materials, mentioned by the discussor. Although a direct measurement of the incremental elastic moduli may be difficult, these moduli could be determined unequivocally, at least in theory, by carrying out the exact stress analysis of the test specimen in finite strain. Also, the differences between formulations are important not only for soft materials but also for continua with very low shear modulus, such as some fiberglass composites (see Fig. 2 in the paper) or in continua which are used to approximate large regular rectangular frameworks without diagonals. The author wishes to thank Dr. Masur for pointing out several misprints, which have been corrected in an Errata which appeared in the March, 1972, issue of the Journal of Applied Mechanics.


Nonlinear Analysis of Steep, Compressible Arches of Any Shape

R. Schmidt and D. A. DaDeppo. The value of $Q$ corresponding to the first bifurcation point on the load-deflection plot must be considered as the critical value $Q_c$ at which the arch begins to deflect sideways, i.e., buckle by sidesway. For the semicircular arch of radius $a = L/2$, $Q_c = 25.6$, from the table on page 945, so that the critical value of the downward point load at the crown of the arch is $P_{cr} = 6AET/\alpha^2$ for the large value of the compressibility parameter $P/4A\epsilon^2 = 0.01$ defining a very thick arch rib. From reference [4] in the paper, $P_{cr} = 5.86Eh^3/\alpha^2$ for a two-hinged semicircular arch with inextensible (incompressible) centroidal line. This value has been verified by a different computational method in [1].

What is noteworthy about these two critical values of the load $P$ is the fact that the critical value for the arch with extensible centroidal line is larger than the critical value for the more rigid arch with inextensible centroidal line. This is contrary to our experiences with the theoretical buckling of straight columns, flat plates, and circular rings, even though these types of structures display the bifurcation type of buckling. Moreover, the observed fact that the approximate Ritz method yields higher critical values than the exact ones in the familiar buckling problems characterized by bifurcations is commonly explained by pointing out that the assumed finite set of functions in effect makes the (mathematical) model more rigid than the actual structure.

The vertical deflection of the crown, corresponding to the first bifurcation, is given as $\Delta = 0.206L = 0.412a$ in the table on page 945 and in Fig. 2(a). In view of the value $\Delta = 0.195a$ in reference [4] of the paper and reference [1] of this discussion, for the inextensible centroidal line, the author's value seems too large by a factor of two. This is also indicated by the approximate results in [2]. However, this error does not seem to invalidate all other results; perhaps DELV should simply mean $2\Delta/L$ instead of $\Delta/L$ as stated on page 944.

References

Author's Closure
The definition for DELV printed on page 944 is the one that was used; the value DELV = 0.206 printed on page 945 is the result obtained from the computations, so the question raised of whether there is an error must be answered in the negative. It would seem likely, however, that the large compressibility (0.01) used in the numerical example could account for the much larger crown deflection exhibited by the arch considered than by a similar incompressible arch. The author is grateful to Professors Schmidt and DaDeppo for their comments.

The Loading-Frequency Relationship in Multiple Eigenvalue Problems

H. H. E. Leipholz. The paper deals with multiple eigenvalue problems, a subject of great practical importance. Although some predecessors like Papkovich and Collatz, as mentioned in the reference list of the paper, and Schaefer investigated already certain aspects of multiple eigenvalue problems, only this paper tackles the problem consistently and in full generality. Compared with Papkovich and Schaefer, the paper goes much further taking dynamic systems into account; compared with Collatz, this paper formulates and solves the problem in the language of engineers, while Collatz remains in the domain of mathematical abstraction. Hence, the merit of this excellent paper consists in presenting a general solution for multiple eigenvalue problems of dynamic systems in the form of loading-frequency relationships which enable the engineer to draw a very important conclusion: the fundamental characteristic surface in the frequency-load-space cannot have convexity toward the fundamental region.

This theorem can be used for (lower) bound theorems on the fundamental characteristic surface, thus giving the engineer a very welcome possibility to estimate eigenvalues in a simple way.

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3 Numbers in brackets designate References at end of Discussion.