A. J. Acosta. I would like to thank Professors Street and \( \text{La} \)rock for presenting a very interesting and useful summary of their recent work in cavity flow theory. As mentioned by them, a number of different models for the nonlinear free streamline flow past hydrofoil shapes have been developed in the past; the present summary brings attention to newer models for the termination of the constant pressure cavity since this evidently leads to certain analytical simplifications. I wonder if it would be possible for the authors to describe somewhat more fully than is evident in Fig. 1 what the physical differences are between the two spiral models. Perhaps it would be of interest also to hear why these models are preferable to some of the other schemes proposed in the past.

The calculated lift coefficient as a function of spray sheet thickness shows an impressive agreement with the experimental results of reference [15]. It would be interesting to know for the various conditions of Fig. 3 the lift slope value, as this is a useful design parameter. In connection with the spray sheet thickness, there is a point upon which I am not quite clear; namely, the definition of submergence and spray sheet thickness. As the authors clearly point out, the two are not the same nor is it necessary that they be assumed to be the same even in the linear theory. It was used in a perturbation theory with the acceleration potential applied to an unsteady counterpart of the problems presented in this paper. Since, for a steady flow, the complex acceleration potential is equivalent to the linearized version of \( \omega \) as defined by equation (6), the solution shown by equation (8) is equivalent to the solution given by Song. It should also be noted that the solution (8) can be obtained by invoking the principle of minimum singularity without considering the "double-spiral model."

It can readily be observed that the "double-spiral model" leads to a logarithmically singular solution at the tail of the cavity, while the "single-spiral model" admits simple poles at the tail of the cavity. Therefore, it is not a coincidence that the "double-spiral model" is simpler and, perhaps, more accurate than the "single-spiral model."

Authors' Closure

The authors are most appreciative of the comments by Drs. Acosta and Song.

For a complete description of the physical aspects and mathematical bases of the cavity models that have been used, as well as justification for and useful mathematical properties of the models, we refer the reader to Tulin's original and most complete paper on supercavitating flows delivered to the International Symposium on the Applications of Analytic Functions in Continuum Mechanics, Tbilisi, Georgia, USSR, in September 1963 (available as Hydraulics, Inc. TR 121-3, September 5, 1963, or reference [13]). In this work, Tulin also establishes the relations between the nonlinear and linear versions of cavity models, the importance of which Dr. Song has emphasized in his discussion.

Unfortunately, the authors have not had available the computer time required to calculate the lift slope as suggested by Dr. Acosta. Because the lift slope is a nonlinear function of cavitation number, angle of attack, and spray sheet thickness, a large number of computations would be required to construct a representative plot. Indeed, it may be more profitable at this time to pursue the suggestion of Dr. Acosta regarding a redefinition of submergence. As the authors noted in reference [6], the present definition of submergence, which is arbitrary, was made to give some idea of the relationship between spray sheet thickness, submergence, and the other parameters of the problem. In the present formulation the vertical location of the free surface of the fluid for negative \( z \) is asymptotically proportional to the product of the lift coefficient and \( |x| \) for large negative \( x \). If, as Dr. Acosta suggests, the potentials at points \( D \) and \( E \) were allowed to be different, or perhaps additional singularities were introduced at \( D \) and \( E \), the extra parameters thereby made available could be used in the solution to make \( |x| \) finite. The submergence could then be defined unambiguously. An investigation of this idea would be worthwhile.

C. S. Song. It may be interesting to point out that the author's "double-spiral model," initially named so by Tulin, is equivalent to one of the two models proposed by Song and called "solution with constant wake pressure." The model was justified by the result of a pressure survey in the wake. It was used in a perturbation theory with the acceleration potential applied to an unsteady counterpart of the problems presented in this paper. Since, for a steady flow, the complex acceleration potential is equivalent to the linearized version of \( \omega \) as defined by equation (6), the solution shown by equation (8) is equivalent to the solution given by Song. It should also be noted that the solution (8) can be obtained by invoking the principle of minimum singularity without considering the "double-spiral model."

It can readily be observed that the "double-spiral model" leads to a logarithmically singular solution at the tail of the cavity, while the "single-spiral model" admits simple poles at the tail of the cavity.

Computerized Method of Characteristics Calculations for Unsteady Pneumatic Line Flows

Michael A. Stoner. The author has presented a method of analysis for unsteady gas flow which is certain to receive much attention in the future as a design and analysis tool for pneumatic lines. The method of characteristics' ability to handle the nonlinear terms of the gas dynamics equations and the ease with which the resulting equations can be programmed on a digital computer lends one to this conclusion.

The author has chosen to go to a rectangular grid or a method of specified time intervals rather than use a free grid or characteristic grid for the \( x,t \) solution scheme of the characteristics equations. His reasons for doing so are acknowledged; however, it is possible that a characteristics grid scheme would give better results. In problems such as the author's where the

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1 By E. E. Street and B. E. Larock, published in the June issue of the *Journal of Basic Engineering*, pp. 269-274.
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