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Discussions

Discussion: ''Response Bounds for Linear Damped Systems'' (Hu, B. and Eberhard, P., 1999, ASME J. Appl. Mech., 66, pp. 997–1003)

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In a recent paper, the authors consider the dynamics of an n-degree-of-freedom linear system represented by

$$M\ddot{y}(t)+D\dot{y}(t)+Ky(t)=f(t), \quad y(0)=y_0, \quad \dot{y}(0)=\dot{y}_0, \tag{1}$$

for all $t\geqslant 0$, where the vector of displacements $y(t)\in\mathbb{R}^n$ and the vector of applied forces $f(t)\in\mathbb{R}^n$. They obtain upper bounds on the norms of responses of the system (1) for the cases of free and forced vibration.

The authors obtain an upper bound on $\|y(t)\|:=[y(t)^Ty(t)]^{1/2}$ for all $t\geqslant 0$. Although, it is useful to know the time evolution of an upper bound on the function $t\mapsto\|y(t)\|$, it is more important to have a tight upper bound on

$$\|y\|:=\max_{t\geqslant 0}\|y(t)\|, \tag{2}$$

which is an indication of the largest displacement (strain) of the system (1). A tight bound upper bound on $\|y\|$, which is desirable for the worst-case scenario studies, results in less conservative designs.

The authors write that [1] ''In comparison to the response bounds available in the literature, the ones presented here are not only closer to the exact responses, but are also simpler to compute.'' This statement is evaluated in the following discussion.

According to [1], an upper bound on $\|y\|$ for the case of free vibration is obtained as follows:

(i) Compute

$$D^*=M^{-1/2}DM^{-1/2}, \quad K^*=M^{-1/2}KM^{-1/2}. \tag{3}$$

(ii) Compute

$$\mu=\begin{cases}\lambda_{\min}(D^*)/2, & \text{for } \lambda^2_{\max}(D^*)\leqslant 4\lambda_{\min}(K^*),\\ \min\left\{\frac{1}{2}\left(\lambda_{\max}(D^*)-\sqrt{\lambda^2_{\max}(D^*)-4\lambda_{\min}(K^*)}\right),\right. & \\ \left.\frac{1}{2}\lambda_{\min}(D^*)\right\} & \text{otherwise.}\end{cases} \tag{4}$$

(iii) Compute

$$D'=D-2\mu M, \quad K'=K-\mu D+\mu^2 M, \tag{5}$$

$$E_0=\frac{1}{2}\dot{y}_0^TM\dot{y}_0+\frac{1}{2}y_0^TKy_0, \tag{6a}$$

$$E_0^*=E_0+\mu^2y_0^TMy_0+\mu y_0^TM\dot{y}_0-\mu y_0^TDy_0/2. \tag{6b}$$

(iv) An upper bound on $\|y\|$ is

$$\|y\|\leqslant\min\{\sqrt{2\lambda^{-1}_{\min}(K)E_0},\sqrt{2\lambda^{-1}_{\min}(K')E_0^*}\}. \tag{7}$$

The computation of the upper bound in (7) via steps (i)–(iv) is not as easy as that of some bounds in the literature. For instance, according to Shahruz and Mahavamana [2] an upper bound on $\|y\|$, when the matrix $DM^{-1}K+KM^{-1}D$ is positive definite (such as in classically damped systems), is

$$\|y\|\leqslant([\lambda_{\max}(M)/\lambda_{\min}(M)](y_0^Ty_0+\dot{y}_0^T\dot{y}_0/\omega_1^2))^{1/2}, \tag{8}$$

where ω_1 is the smallest undamped natural frequency of the system (1). It is evident that the computation of the upper bound in (8) is much simpler than that in (7) via steps (i)–(iv).

Now, it is determined how conservative the upper bounds in (7) and (8) are. In [1], the system (1) with the following coefficient matrices is considered:

$$M=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, \quad K=\begin{bmatrix}5 & -1\\ -1 & 1\end{bmatrix}, \quad D=(M+K)/2. \tag{9}$$

For different initial conditions the following upper bounds are obtained:

(B1) $y_1(0)=1$, $y_2(0)=0$, $\dot{y}_1(0)=\dot{y}_2(0)=0$:

From numerical simulation: $\|y\|=1$, (10a)
According to (7): $\|y\|\leqslant 2.51$, (10b)
According to (8): $\|y\|\leqslant 1$. (10c)

(B2) $y_1(0)=0$, $y_2(0)=1$, $\dot{y}_1(0)=\dot{y}_2(0)=0$:

From numerical simulation: $\|y\|=1$, (11a)
According to (7): $\|y\|\leqslant 1.15$, (11b)
According to (8): $\|y\|\leqslant 1$. (11c)

Results in (10) and (11) show that the upper bounds computed by (7) are *much more* conservative than those obtained by (8).

Next, upper bounds are computed for another system whose coefficient matrices are given in (92) of [1]. For the coefficient $\xi=0.1$ (see [1] for details), $y_0=[0\ 0\ 0\ 0]^T$, and $\dot{y}_0=[1\ 1\ 1\ 1]^T$, it is concluded that

From numerical simulation: $\|y\|=1.64$, (12a)
According to (7): $\|y\|\leqslant 4.14$, (12b)
According to (8): $\|y\|\leqslant 4.11$. (12c)

This example shows that both (7) and (8) yield conservative upper bounds on $\|y\|$, even though (8) resulted in a tight bound for the system whose coefficients matrices are given in (9).

In summary, it is shown that the upper bounds on responses of the system (1) derived in [1] are neither easily computable nor are tight, as it is evident from (10) and (11). Also, it is shown that no upper bound can be expected to be tight for all systems, as it is apparent from (12).

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Closure to "Discussion of 'Response Bounds for Linear Damped Systems'" (2000, ASME J. Appl. Mech., 67, p. 636)

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We thank Mr. S. Shahruz for his interest in our paper and welcome his comment. In our paper we consider a response bound as a bound with time evolution. Instead of Eq. (7) in Mr. Shahruz's discussion, our original response bound (Eq. (49) in [1]) reads

$$\|\mathbf{y}(t)\| \leq \min\{\sqrt{2\lambda_m^{-1}(\mathbf{K})E_0},\ e^{-ut}\sqrt{2\lambda_m^{-1}(\mathbf{K}')E_0^*}\}. \tag{1}$$

The term e^{-ut} appearing in this response bound plays an important role. For the maximum amplitude of the response $\max_t\|\mathbf{y}(t)\|$, we prefer to call it amplitude bound. S. Shahruz and P. Mahavamana's results in paper ([2]) and some results from W. Schiehlen and the first author of this closure in papers ([3,4]) are for the amplitude bounds. Here we would like to point out that the procedure listed in S. Shahruz's discussion should be to compute our response bound given in Eq. (1) above. It may not be meaningful for the amplitude bounds. In Eq. (29) of our paper, we gave an amplitude bound

$$\max_t\|\mathbf{y}(t)\| \leq \sqrt{2\lambda_m^{-1}(\mathbf{K})E_0} \tag{2}$$

which also follows directly from Eq. (1) in this closure. We can see that for the computation of this amplitude bound, most operations in the procedure listed in S. Shahruz's discussion are not necessary. Compared with Mr. Shahruz and Mr. Mahavamana's amplitude bound given in Eq. (8) of Mr. Shahruz's discussion, we have the opinion that our amplitude bound is not harder to compute since either the computation of the smallest undamped frequency ω_1 or the determination whether the matrix $\mathbf{DM}^{-1}\mathbf{K}+\mathbf{KM}^{-1}\mathbf{D}$ is positive semi-definite costs extra time. Though he showed their amplitude bounds are tighter than ours for two examples, we do not think this conclusion holds in general. Let us choose a simple example to explain this point. If we change the mass matrix in Eq. (9) of Mr. Shahruz's discussion to

$$\mathbf{M}=\begin{bmatrix}10 & 0\\ 0 & 1\end{bmatrix} \quad \text{or} \quad \mathbf{M}=\begin{bmatrix}0.1 & 0\\ 0 & 1\end{bmatrix} \tag{3}$$

and the numerical values of the damping matrix and the stiffness matrix remain unchanged, then Mr. Shahruz and Mr. Mahavamana's amplitude bounds for both cases B1 and B2 become 10. However, our amplitude bounds remain unchanged. They are still 2.51 for the case B1 and 1.15 for the case B2. It is not difficult to find examples which show neither method to be superior.

Besides, we would like to state that although Mr. Shahruz and Mr. Mahavamana's paper about amplitude bounds for some nonclassically damped systems was published in December 1998 in the *Journal of Sound and Vibration*, their results were not known to the authors since our paper was received by the ASME Applied Mechanics Division on Aug. 24, 1998 and the final revision of the paper was received on Jan. 19, 1999. Therefore, a comparison with their results was not possible (and maybe not even reasonable since in our paper we discussed mainly response bounds with time evolution and not amplitude bounds).

In conclusion, we agree with Mr. Shahruz that no upper bound can be expected to be tight for all systems. In fact, in our paper we also stated that K. Yae and D. Inman's response bounds given in paper ([5]) are in some cases better than ours. But in contrary to Mr. Shahruz we think that improvements on the *response bounds* are meaningful and do not consider only the *amplitude bounds* to be important. Mr. Shahruz stated that our response bounds are neither easily computable nor are tight. We hope that we have been able to contribute to this interesting field of research and that in the future more easily computable and tighter response bounds will be developed.

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Discussion: "An Energy Method for Analyzing Magnetoelastic Buckling and Bending of Ferromagnetic Plates in Static Magnetic Fields" (Yang, W., Pan, H., Zheng, D., and Cai, Z., 1999, ASME J. Appl. Mech., 66, pp. 913–917)

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The authors gave an energy method to analyze the magnetoelastic buckling and bending of ferromagnetic plates in different static magnetic fields. The elastic strain energy of Eq. (2) employed in this paper is for the bending of the beam-type plate. And in the derivation of magnetic energy of Eq. (5), the effect of end edges on magnetic fields is not taken into account. After the longitudinal and transverse demagnetizing factor N_l and N_h are calculated by Eqs. (12)–(13), respectively, the expressions of critical field B_{cr} and bending deformation δ at free end are formulated by Eqs. (14) and (17), respectively. In this approach, the effect of width, denoted by w here, is considered only in the demagnetizing factors but not in the deformation. If a rectangular ferromagnetic plate under consideration is constrained by simple or clamped supports along the edges normal to the direction of width, it is possible that the same results for the magnetoelastic interaction will be obtained since N_l and N_h are independent on the boundary conditions. In other words, the results given in this paper are independent upon the support conditions of the edges along the longitudinal direction, which is obviously in contradiction to the practical problems. When the width of a rectangular plate increases to infinite, from the theory of plates, we know that the deflection of the plate approaches to that of a correspondingbeamtype plate. When χ is very large, e.g., 10^3 order in Moon and Pao [1] to this case, the condition $1/\chi \ll N_l = 1$ is satisfied ($N_l = 1$ may be got by Eq. (12) when $w \to \infty$). According to Eq. (14a), however, it is found that the critical magnetic field $\bar{B}_{cr}$ for this case of cantilevered plates in transverse magnetic fields approaches to infinite. This results in contradiction to the finite critical magnetic fields given in literature to the same problem, e.g., Moon and Pao [1], Zhou et al. [2], and Zhou and Zheng [3] which are in agreement with the experimental data ([1,4]). For the prediction of bending of the plate in this paper, it is found by Eq. (17) and Fig. 3 that the incident angle α of the magnetic field does not influence the critical magnetic field B_{cr} of the magnetoelastic instability. This result is also in contradiction to the conclusion given in the literature using the imperfect sensitive analysis in Popelar [5] and the numerical analysis in Zhou et al. [2]. In fact, both the experimental measurement ([1,6]) and theoretical research display a fact that the critical magnetic field of a cantilevered ferromagnetic plate in transverse magnetic field is sensitive to the imperfect of incident angle of misalignment or oblique magnetic field. That is one of reasons why the theoretical predictions for the perfect case of the cantilevered plate in transverse magnetic field ([1,3,7] for example) are almost higher than their experimental data ([2]). For the case of a ferromagnetic plate in longitudinal magnetic field, the authors gave a differential Eq. (20) which indicates that there is neither bend nor buckle. The authors did not give a comparison of their theoretical prediction and the experimental data to the increasing of natural frequency of the considered plate ([8]). Zhou and Miya [9] successfully gave a theoretical prediction of this problem. For the general model of magnetoelastic interaction for ferromagnetic plate structures and bodies in arbitrary magnetic fields, by which the experimental phenomena of magnetoelastic buckling, bending and increasing of natural frequency can be described, it can be found in Zhou and Zheng [10,11]. It is obvious that these recent researches of magnetoelastic interaction do not support the opinion of authors: ''It seems that no further progress has been made in theoretical analysis since the Moon-Pao theory was presented.''

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