$$F_\mu = \frac{\Delta R}{2} \int Hp \Big|_{\varphi_i}^{\varphi_e} dz + \iint_S \frac{\Delta R \epsilon}{2} p \sin \varphi \, d\varphi dz$$

$$+ \frac{\mu U R}{\Delta R} \iint_S \frac{\theta}{H} d\varphi dz = \frac{\Delta R}{2} \int Hp \Big|_{\varphi_i}^{\varphi_e} dz - \frac{\epsilon \Delta R}{2R} F_\varphi$$

$$+ \iint_S \frac{\mu U}{\Delta R} \frac{\theta}{H} R \, d\varphi dz \quad \text{(A7)}$$

All integrations were performed using a Simpson's rule for single as well as double integrations.

Power Loss. The power loss can be obtained from the shear force; i.e.,

$$P_L = F_\mu U \quad \text{(A8)}$$

Bearing Torque. The bearing torque is given as

$$M = F_\mu R \quad \text{(A9)}$$

Pseudo-Gümbel Boundary Conditions. The Gümbel boundary conditions are often used as a substitute for the more rigorous Swift-Stieber rupture boundary conditions for reasons of computational economy. Normally the Gümbel conditions are implemented by first solving the Reynolds equation without any regard to cavitation. All subambient pressures are disallowed by setting them equal to the ambient pressure. This serves as the condition for cavitation. Note that the specification of the pressure gradient at the boundary is neglected. In this study, the nonconservative theory was not strictly in accord with Gümbel; that is, subambient pressures were allowed, and the vapor bubble was determined by disallowing pressures less than the cavitation pressure.

The numerical procedure used to implement the Gümbel conditions was the commonly used Gauss-Seidel iterative technique. This was done to provide a point of reference in assessing the penalty (if any) of invoking a more accurate theory.

DISCUSSION

R. W. Jakeman[1]

The author has presented a most illuminating picture of the behavior of the cavitation region in a dynamically loaded journal bearing, and of analysis techniques for modelling this. It is assumed that reference to "conserving mass" is equivalent to: "satisfying flow continuity within any control volume." Flow continuity within both the full film and cavitation regions must clearly be satisfied under steady operating conditions, and an alternative analysis method in which this is featured is described in reference [22]. However, continuity does not have to be satisfied within the cavitation region in a dynamically loaded situation, since the oil content of any control volume may be increasing or decreasing at any instant. This is a fundamental part of the oil film history concept [23, 24].

Oil film history tends to be more significant where the journal orbit is large in relation to the clearance circle, and where the oil feed arrangements are less efficient. In the case used by the author, the orbit could be classified as moderate in size. Furthermore, while no oil groove was assumed, the clearance to diameter ratio was about ten times that for normal practice, therefore the oil feed should have been reasonably efficient. Having regard to these factors, oil film history effects are unlikely to be important in this case, consequently discrepancies due to the application of continuity to the cavitation region should be insignificant. Little work has been done on oil film history to date, therefore the conditions under which it should be taken into account are not clearly defined.

Additional References

22 Jakeman, R. W., "A Numerical Analysis Method Based on Flow Continuity for Hydrodynamic Journal Bearings," *Tribology International*, Vol. 17, No. 6, 1984, pp. 325–333.

23 Jones, G. J., "Crankshaft Bearings: Oil Film History," 9th Leeds-Lyon Tribol. Symp. Sept., 1982, pp. 83–88.

24 Jakeman, R. W., "Journal Orbit Analysis Taking Account of Oil Film History and Journal Mass," 4th Int. Conf. Numerical Methods in Laminar and Turbulent Flow, Swansea, July 1985, pp. 199–210.

[1]Senior Surveyor – Lloyds' Register of Shipping, London, England.

A. O. Lebeck[2]

The author has performed an interesting and useful piece of work in applying a proper cavitation algorithm to the solution of a dynamic journal bearing problem. Having used the "Elrod Cavitation Algorithm" myself to solve for cavitation and pressure distributions in wavy face seals,[3] several comments and questions come to mind.

First, it is somewhat surprising that there is so little difference between the load support predicted using the non-conservative Gümbel solution and the more precise Elrod Algorithm. Perhaps in other geometries the difference would be more significant. For example in wavy seal analysis, it was found that distribution of pressure had a different shape using the Elrod algorithm compared to the non-conservative method used previously on this problem. Because the hydrodynamic problem was coupled to the elastic deformation of the seal, this more exacting pressure distribution had a large effect on the final result.

Second, in my work referenced above, the Elrod algorithm was used to find a two dimensional steady state solution. It was found that in many cases a fixed pattern of switching function values could not be obtained. In fact the switching function patterns were found to cyclically repeat themselves with a period of from one to five or so iterations. That is, the cavity boundary moved about. The numerical difficulties which resulted from having to iterate upon functions of these moving patterns were solved by averaging over the cycle. In the authors' dynamic solution, were similar problems encountered?

Finally, the variable θ is discontinuous in its derivatives as one goes from vapor cavitation to a full liquid film. What numerical or mathematical difficulties does this create particularly in this dynamic solution where $\partial\theta/\partial t$ forms the time solution itself?

[2]Professor, Mechanical Engineering Dept., University of New Mexico, Albuquerque, N.M. 87131.

[3]Lebeck, A. O., and Young, L. A., "The Wavy Mechanical Face Seal – Theoretical and Experimental Results," Annual Report ME-111(81))ONR-414-1, prepared for ONR under contract N00014-76-C-0071, Bureau of Engineering Research, University of New Mexico, Albuquerque, NM, Jan. 1981.

Authors' Closure

In response to Dr. Jakeman's question concerning the oil film history, the increase or decrease of the oil film content of any control volume within the cavitated region at any instant in time is accounted for in the time march. In this sense the oil film history is taken into account. The role of the switch function in the algorithm accomplishes this automatically. For example, equation (8) governs the transport of the fluid within the cavitated region as a result of switching out the pressure flow term in the modified Reynolds equation (i.e., equation (9)). The mass content within the cavity is thus changing in time and conforming to requirements of continuity and the JFO theory.

Dr. Lebeck's observation that differences between the conservative and nonconservative theory were greater using an ehl theory was very interestng. It is certainly a point that I will keep in mind for future investigations when deformation effects have been included in this particular algorithm. I agree with Dr. Lebeck that it is quite likely the differences could have been more significant for other geometries.

The other question raised by Dr. Lebeck concerning the cyclic behavior of the switch function was not an inherent problem. This was due to the fact that the method of solution in this investigation was direct rather than iterative. No numerical difficulties were encountered other than that mentioned in the text regarding "Switch Function." That difficulty was very troublesome to say the least.