



# Discussion

## Free Vibrations of Thick Hollow Circular Cylinders From Three-Dimensional Analysis<sup>1</sup>

**J. S. Popovics and K. V. Subramaniam<sup>2</sup>** In a recent paper, So and Leissa (1997) present free vibration frequency data for hollow, elastic cylinder structures as calculated by the 3-D Ritz method using both global and local coordinates. We feel that the overall approach of So and Leissa has merit and their findings are of interest to the technical community. In addition, we feel that the presented frequency data is correct. However, we have found several typographical errors in the mathematics of the Ritz formulation presented by So and Leissa which should be noted by the readers:

(a) The second term of Eq. (4) is presented as

<sup>1</sup> By J. So and A. W. Leissa, published in the January 1997 issue of the JOURNAL OF VIBRATION AND ACOUSTICS, Vol. 119, pp. 89–95.

<sup>2</sup> Department of Civil Engineering, Northwestern University, Evanston, IL 60208

$$2 \left[ (U_{,\xi})^2 + \left( \frac{U}{\xi} + \frac{n}{\xi} V \right)^2 + \left( \frac{R_o}{L} W_{,\eta} \right)^2 \right] \Gamma_1 \quad (1a)$$

but we feel the correct form is

$$2 \left[ (U_{,\xi})^2 + \left( \frac{U}{\xi} + \frac{n}{\xi} V \right)^2 + \left( \frac{R_o}{L} W_{,\xi} \right)^2 \right] \Gamma_1. \quad (1b)$$

That is,  $W_{,\eta}$  should be replaced by  $W_{,\xi}$ .

(b) The third term of Eq. (4) is presented as

$$\left[ \left( -\frac{n}{\xi} U + V_{,\xi} - \frac{V}{\xi} \right)^2 + \left( \frac{R_o}{L} V_{,\xi} - \frac{n}{\xi} W \right)^2 \right] \Gamma_2 \quad (2a)$$

but we feel the correct form is

$$\left[ \left( -\frac{n}{\xi} U + V_{,\xi} - \frac{V}{\xi} \right)^2 + \left( \frac{R_o}{L} V_{,\xi} - \frac{n}{\xi} W \right)^2 \right] \Gamma_2. \quad (2b)$$

That is,  $V_{,\xi}$  should be replaced by  $V_{,\zeta}$ .

(c) When using local coordinates in place of global, So and Leissa state that, among other changes, the energy functionals should be modified such that  $\xi d\xi$  be replaced by  $\gamma d\xi$ . In addition, we feel that  $\xi$  should be replaced by  $\gamma$  in the functional expression.