F. W. FRENCH.[4] The results obtained by the authors for the final rotation angle and duration of motion suggest an examination of the accuracy of a very elementary type of analysis. If it is assumed that the motion of the cantilever occurs as a rotation about a stationary plastic hinge at the base with no deformation of the remainder of the beam, application of the impulse-momentum relation then leads to

$$\dot{\theta} = \frac{(IL - M_0 t)}{mL^3(1 + 3K)} \qquad (1)$$

From equation (1), it is seen that motion ceases when

$$t_f = \frac{IL}{M_0} \qquad (2)$$

a result that is the same as that predicted by the rigid, perfectly plastic, rate-independent analysis discussed in the paper. Value of time calculated from this relation agree fairly well with the measured experimental values shown in Tables 3 and 4 of the paper.

The final rotation angle may be found, by integration of equation (1), to be

$$\theta_f = \frac{3}{2(1 + 3K)} \frac{I^2}{mLM_0} \qquad (3)$$

The ratio of the final rotation angle predicted by the foregoing rigid-plastic rotation approach to that resulting from the rigid, perfectly plastic, rate-independent analysis [given by equation (5) of the paper] is

$$\frac{(\theta_f)_{r-pr}}{(\theta_f)_{\text{rate indep}}} = \left[\frac{3(1 + 2K)}{2(1 + 3K)}\right]^2 \qquad (4)$$

The authors later discuss the agreement between their results for final rotation angle, those obtained by Ting[5] in an approximate solution, and those obtained as experimental results. The agreement between the two theoretical results and the experimental results, as shown in Tables 3 and 4 of the paper, is found to be good, with the results of Ting agreeing somewhat better with the test results than those of the authors. The authors then present, in their equation (17), the ratio between final rotation angle as predicted by Ting and that predicted by the rate-independent theory. This expression may be written in the form

$$\frac{(\theta_f)_{\text{rate dep}}}{(\theta_f)_{\text{rate indep}}} = \left[\frac{3(1 + 2K)}{2(1 + 3K)}\right]^2 \times \left(\frac{1}{1 + A_0}\right) \qquad (5)$$

Comparison of equations (4) and (5) shows that the relation between θ_f as predicted by the elementary rigid-plastic rotation analysis and as predicted by the rate-dependent analysis (which agrees well with experiment) is

$$\frac{(\theta_f)_{r-pr}}{(\theta_f)_{\text{rate dep}}} = 1 + A_0 \qquad (6)$$

The foregoing results suggest calculating the final rotation angle by using the elementary theory to calculate θ_f from equation (3), and then applying the correction factor $1/(1 + A_0)$ to account for the strain-rate effects.

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[5] T. C. T. Ting, "The Plastic Deformation of a Cantilever Beam With Strain Rate Sensitivity Under Impulsive Loading," Technical Report 70 from Brown University to ONR under Contract Nonr-562(10), July, 1962.

A Swirling Round Turbulent Jet, 1—Mean-Flow Measurements[1]

A. L. KISTLER.[2] The problem of the swirling jet is an interesting and important one since, for a specified jet mass flow and for given geometric constraints, the introduction of swirl to a jet changes the rate of fluid entrainment by the jet. One would expect that the properties of a swirling jet are strongly dependent on the ratio of the tangential velocity to the axial velocity at the jet origin, and, in fact, if this ratio is sufficiently large it has been observed that the axial velocity can reverse its direction near the jet axis. The work reported here is concerned only with the case where this ratio is quite small and the emphasis is on obtaining detailed measurements of the flow field of the jet. This case is most probably the only one where such measurements can be obtained and this work should serve as a secure stepping stone to a fuller understanding of the properties of swirling jet motion.

Since the easiest way to produce a swirling jet is either by injection of the fluid tangentially into the pipe or by placing fixed guide vanes in the pipe, it is to be expected that the most frequently occurring flows in technology will have an initial angular velocity distribution between a rigid-body motion and an irrotational motion. It would be interesting to see this work extended, therefore, perhaps in a more qualitative fashion, to such initial conditions as well as over a wider range of angular velocities.

[1] By W. G. Rose, published in the December, 1962, issue of the Journal of Applied Mechanics, vol. 29, Trans. ASME, vol. 84, Series E, pp. 615–625.

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