Antisymmetric Tensor Gauge Field in General Covariant Gauges

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By introducing Lagrangian multiplier fields, the covariant quantization of antisymmetric tensor gauge field is carried out in a general covariant gauge. The confinement of unphysical components is examined from the viewpoint of the "quartet mechanism" of Kugo and Ojima.

Recently, Townsend has shown that there exist similarity and dissimilarity between the mode counting of spin 3/2 field and that of antisymmetric tensor gauge field. Especially, one cannot find a mechanism of introducing two extra modes in contrast to the case of spin 3/2 theory where an additional extra Majorana spinor ghost has been introduced by Nielsen. Nielsen adopted the ordinary form of Faddeev-Popov (FP) ghosts \( c^* \) and \( c \). On the other hand, Hata and Kugo have proposed a new formalism of supergravity by adopting the unconventional form of FP ghost \( i c^* \). Since the ghosts \( c \) and \( c^* \) of Hata-Kugo obey the second order differential equations, all their components are independent and give rise to eight modes of FP ghosts. There should then exist eight unphysical modes of spin 3/2 field sector which compensate eight modes of FP ghosts. Such an unphysical sector is constructed by introducing the Lagrangian multiplier spinor field \( B \) which plays a role of fixing the gauge. The unphysical modes consist of four longitudinal components of \( c^* \) and four components of \( B \), and their number is equal to that of FP ghosts. Fronsdal and Hata have further shown that such a type of formalism is necessary to construct the interacting theory of higher spin fields so that the unitarity of the physical S-matrix is assured. The present author has also shown that the axial anomaly for the gravitino in the background curved space is obtained by \( B \) field not by ghost fields in Hata-Kugo formalism. It is, therefore, desirable to discuss the mode counting of the quantized antisymmetric tensor gauge field by introducing the Lagrangian multiplier fields.

We shall start with the following Lagrangian:

\[
L = -\frac{1}{4} \partial_{\mu} B_{\rho\sigma} \partial^{\mu} B^{\rho\sigma} + \frac{1}{2} \partial_{\mu} B_{\rho\sigma} \partial^{\mu} B^{\rho\sigma}
- \frac{1}{2} B^{\mu\rho} \partial_{\rho} B_{\mu} - \frac{i}{2} c_{\mu} \partial_{\mu} c^* + d^* \partial_d d - B^\mu \partial_\mu \eta
+ i \partial_d \psi - \partial_\mu \partial^\mu \psi,
\]

(1)

where \( \alpha \) and \( \beta \) are real parameters, \( c_{\mu} = \partial_{\mu} C - \partial_C \partial_{\mu}, \ c_{\mu} = \partial_{\mu} C + \partial_C \partial_{\mu} \) and the metric is \( g_{\mu\nu} = (-1, 1, 1, 1) \). We postulate that all field variables are Hermitian. The fields \( B_{\mu\nu}, B_{\mu}, B_{\eta}, \phi, \) and \( \psi \) are subject to Bose statistics, while \( c_{\mu}, c_{\mu}, d, \) and \( d^* \) subject to Fermi statistics. Interactions can be added, either with other fields or in a non-Abelian generalization. We tentatively regard the above Lagrangian as that of the asymptotic fields. The Lagrangian density (1) is invariant under the following BRS transformations:

\[
\delta B_{\mu\nu} = \lambda (\partial_{\mu} \phi - \partial_{\nu} \phi), \ \delta c_{\mu} = -i \lambda \partial_\mu \phi, \ \delta d = -i \partial_\mu \phi, \ \delta \psi = \lambda d, \ \delta c_{\phi} = -i \lambda \partial_\phi \psi.
\]

(2)

in which the \( x \)-independent \( \lambda \) anti-com
mutes with all the Fermi fields and commutes with all the Bose fields. The introduction of \( B_\alpha \) etc. assures the off-shell nilpotency of BRS transformation. In the formalism of Townsend, however, one obtains 
\[
\delta'(\delta \phi_\alpha) = -2\lambda \lambda' \delta_x (\delta \phi_\alpha - \partial \phi_\alpha) \neq 0
\]
and \( \delta' \phi_\alpha = 0 \) only when one uses the equation of motion \( \Box \phi = 0 \).

To simplify the discussion we take \( \beta = 1 \) (Feynman gauge) by which all fields are simple pole fields except for \( B_\alpha \). The equations of motion derived from (1) are
\[
\delta \phi_\alpha = 0, \\
\delta \psi_\alpha + \lambda = 0, \\
\delta B_\alpha + \alpha B_\alpha - \partial \phi_\alpha = 0, \\
\Box B_\alpha = (1-\alpha) (\partial_\alpha B_\alpha - \partial \phi_\alpha), \\
\Box \Phi' - 0, \\
\Box \phi = 0,
\]
where \( \Phi' \) stands for field variables except for \( B_\alpha \). The field \( B_\alpha \) is dipole fields except for the case \( \alpha = 1 \) (Feynman gauge).

There appear four 'pairs of the second class constraints among field variables and their conjugate momenta. To get a consistent theory we employ the new Dirac bracket method following the procedure of Ref. 6). Using the equal-time (anti-) commutators obtained by this procedure, we have the four-dimensional (anti-) commutators among the field variables
\[
[B_\alpha (x), B_\beta (y)] = i \alpha \partial_\alpha D(x-y), \\
[B_\alpha (x), B_\beta (y)] = -i \partial_\beta D(x-y), \\
[B_\alpha (x), B_\beta (y)] = -i \partial_\alpha D(x-y), \\
[B_\alpha (x), B_\beta (y)] = i \partial_\beta D(x-y),
\]
all the other (anti-) commutators vanish. Here, \( D(x) \) and \( \bar{D}(x) \) are defined by
\[
D(x) = -i (2\pi)^{-1} \int d^4 k \delta (k) e^{ik x}, \\
\bar{D}(x) = -i (2\pi)^{-1} \int d^4 k \delta (k) e^{ik x},
\]
and \( \delta \phi \) satisfies \( \Box \phi = 0 \). The corresponding conserved charge is
\[
Q_B = \int d^4 x J_0 \\
= \int d^4 x [\partial_\alpha B_\alpha + \partial \phi_\alpha]
\]
which is obtained by the help of equations of motion. The \( Q_B \) generates the BRS transformation
\[
\delta \Phi = [i \alpha Q_B, \Phi (x)].
\]

We shall now consider the mode counting in the four-dimensional momentum space. We refer to a Lorentz frame such that \( p_1 = p_2 = 0 \) and \( p_3 > 0 \). Equation (5) implies that \( B_{11} \) and \( B_{12} \) are not independent variables and
\[
-p_3 B_{03} - i \alpha B_{03} - p_3 \gamma = 0, \\
-p_3 B_{03} - i \alpha B_{03} - p_3 \gamma = 0
\]
which lead to Eq. (3) when \( \alpha \neq 0 \). We can regard that \( \gamma \) is not an independent component. By the help of Eq. (12) it is written as a linear combination of two independent components \( -i B_{03}/p_3 \) and \( B_1 \). From the four-dimensional Fourier transforms of (8), we have the following (anti-) commutators among independent field variables (we can put \( p_3 = p_\gamma \) for simple pole
fields:

| $B_1^a(p)$ | 1 | 0 |
| $\gamma_1(p)$ | $\omega_{kl}$ | $\delta_{kl}$ |
| $\beta_1(p)$ | 0 | 0 |
| $\gamma_{stk}(p)$ | 0 | 0 | $-i\delta_{kl}$ |

where $\gamma_k = (-iB_0/p_0, -iB_2/p_0, -iB_3/p_0, \phi_0)$, $\beta_k = (B_1, B_2, B_3, \phi_0)$, $\gamma_{stk} = (c_1, c_2, id/p_0, d_s)$, $\gamma_{stk} = (c_{B_1}, c_{B_2}, c_{B_3}, ic_0/p_0)$. In the above, the common factor $i\omega_{ij}$ is omitted and $\omega_{kl}$ needs not to be specified for the present purpose. The term $id/p_0$ in $\gamma_k$ is equal to $ic_03/j\lambda_0$ in our Lorentz frame. The creation and annihilation operators $\Phi_i(p), \Phi_i(p)$ are defined by

$$\Phi_i(x) = (2\pi)^{-3/2} \int d^4p (\Phi_i(p) e^{ipx} + \Phi_i^*(p) e^{-ipx}).$$

The BRS transformation properties are

$$[iQ_B, \gamma] = \gamma_1, \quad [iQ_B, \beta] = i\delta_1,$$

$$[Q_B, \beta] = [Q_B, \gamma] = [Q_B, B_1] = 0.$$ (14)

The above forms of (13) and (14) are the same as in the theory of Kugo and Ojima where the “quartet mechanism” works and then unphysical particles are confined into the physically invisible world by imposing the condition

$$Q_B\langle \text{phys} \rangle = 0.$$ (15)

The unitarity of the $S$-matrix can be proved in the same way as in Kugo-Ojima formalism. In our procedure, the introduction of Lagrangian multiplier fields has played an essential role. With the aid of these fields the structure of mode counting is clarified.

At first sight, it seems natural to take a more general BRS transformation law for $c_{\alpha}$,

$$\delta c_{\alpha} = i\lambda (B_\alpha + \partial_\alpha \xi): \partial \xi = 0,$$ (16)

which may be presumed as our version of Townsend’s one, i.e., $\delta c_{\alpha} = i\lambda (-\partial^\beta B_\beta + \partial_\beta \xi)$. The quantization is carried out by taking the Lagrangian density as

$$L' = L - \frac{1}{2} (1 - \alpha) B^\alpha \partial_\alpha \xi - \partial_\beta \xi \partial^\beta \xi,$$ (1')

where $L$ is given by (1). The component $\xi$, however, cannot enter into a member of quartet mechanism, and therefore should be inhibited by

$$\xi \langle \text{phys} \rangle = 0.$$ (17)

The above condition is consistent only when $\xi$ satisfies a free field equation, but there is no such guarantee if the interaction is introduced either with other fields or in a non-Abelian generalization. Thus it is not adequate to adopt the subsidiary condition (17). On the other hand, taking account of the equation of motion (5), we can see that our BRS transformation law $\delta c_{\alpha} = i\lambda B_\alpha$ already has a form of Townsend’s one ($\alpha \neq 0$). Furthermore, $\xi$ and $\gamma$ have no FP ghost charge, since there are no scale transformations for $\xi$ and $\gamma$ under which the Lagrangian density (1') is invariant. Owing to these facts, we have no necessary reason to adopt (16) instead of $\delta c_{\alpha} = i\lambda B_\alpha$. The adoption of $\delta c_{\alpha} = i\lambda B_\alpha$ enables us to confine all unphysical components by (15). There is no need to introduce additional ghosts contrary to the case of Townsend. The quantization can be proceeded in parallel with the case of gravitino in the supergravity. We may conclude that Townsend’s difficulty originates from the procedures that he did not use the Lagrangian multiplier fields and took an unsuitable BRS transformation law for $c_{\alpha}$.

Finally, we add a comment concerning the decoupling of longitudinal components $\partial B_\alpha$, mentioned by Townsend. From the
commutation relation (8), we have \[ [\partial x B_{\mu}(x), \partial y B_{\rho}(y)] = -i\alpha \delta_{\mu\rho} D(x-y). \] Therefore, the decoupling of Townsend occurs only when we take the Landau gauge (\(\alpha = 0\)). Since Townsend adopted Feynman gauge (\(\alpha = 1\)), the decoupling did not occur from the first. The preferable property of the Landau gauge has already appeared in the case of the gravitino in the theory of supergravity. The longitudinal part of \(\phi_{\mu}\) (essentially, \(\langle r \phi \rangle\)) is inhibited only in the Landau gauge even in the ordinary formalism with conventional form of FP ghost.\(^{3}\)