Structure Functions of Electron-Deuteron Elastic Scattering and Deuteron Magnetic Moment in Relativistic Quark Model

Yoshiki KIZUKURI, Mikio NAMIKI, Keisuke OKANO and Noriyuki OSHIMO

Department of Physics, Waseda University, Tokyo 160

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Elastic electromagnetic form factors and magnetic moment of the deuteron are discussed within the framework of a relativistic harmonic oscillator quark model.

In a previous paper,1) we showed that the experimental penta-pole behaviour of the deuteron electromagnetic form factor at large momentum transfers can be reproduced with a theoretical formula given by the relativistic harmonic oscillator model (R. H. O. M.), assuming that the deuteron wave function has a small six-quark configuration in addition to the ordinary proton-neutron one, correspondingly, the deuteron form factor is decomposed as

\[ F_n = F_0 + F_6 \sin^2 \theta \]

where \( F_0 \) becomes dominant at large momentum transfers. There \( \sin^2 \theta \), which is the probability of finding the six-quark bound state \((6q)_b\) in the deuteron or of transition \( pn \rightarrow (6q)_b \), was estimated to be 0.07. We have also pointed out that di-baryon resonances (DB's) recently observed \( \lambda \) and the \( (6q)_b\) part of the deuteron originate in the same origin, by estimating the order of elastic width of the resonances through \( \Gamma_{DD}^{\text{pp}} = \tau^{-1} \text{Prob} \ (DB \rightarrow pp) = \tau^{-1} \text{Prob} \ (pn \rightarrow (6q)_b) = \tau^{-1} \sin^2 \theta \) on the basis of the reciprocity theorem \( \tau^{-1} \) being the collision frequency. In the previous work, however, we discarded completely the spin-unitary spin dependence of the \( (6q)_b\) wave function in the deuteron. In this short note we develop our model by taking it into account to obtain the charge form factor \( G_q \) magnetic one \( G_M \) and quadrupole one \( G_Q \) and analyse again the experimental plot of the structure functions \( A \) and \( B \) in the Rosenbluth formula

\[ \frac{d\sigma}{dQ} = \frac{d\sigma}{dQ}\text{m.n.}[A + B \tan^2(\theta_e/2)], \quad \theta_e \text{ being the scattering angle in } e-d \text{ elastic scattering.} \]

The form factors \( G_e, G_M \) and \( G_Q \) are defined by \( G_e = G_1 - \frac{q^2}{6Mn^2}G_Q \) and \( G_M = G_2 - G_3 + \frac{1-q^2}{4Mn^2}G_3 \), in which the \( G_i \)'s (\( i = 1, 2, 3 \)) are given through the current vertex function

\[ \langle T_f | J_\mu(0) | T_i \rangle = \frac{1}{\sqrt{2E_f V}} \left[ -G_1 P_\mu(D^\ast \cdot D) \right. \\
+ G_2 \left( D_\mu^\ast D_\nu - D_\nu^\ast D_\mu \right) q^\nu \\
+ G_3 \left( D^\ast \cdot q \right) \left( D\cdot q \right) P_\mu \right] \frac{1}{2Mz^2}, \tag{1} \]

where we have followed the conventional notation: \( P_\mu = P_{1\mu} + P_{2\mu}, q_\mu = P_{3\mu} - P_{4\mu} \) and \( D_\mu \) and \( D_\nu \) are initial and final deuteron polarization vectors.

Our model starts with the assumption that the six-quark wave function is represented by a product of internal orbital part \( \phi_{6q} \) and spin-unitary spin part \( u_{6q}(P) \). The detailed form of \( \phi_{6q} \) was already given in the Appendix of the previous paper.\(^{15}\)

Note that the mass parameter \( M \) contained in \( \phi_{6q} \) is in principle different from the deuteron physical mass \( M_D \).\(^{15}\) Our task here is to construct \( u_{6q}(P) \) for the deuteron with 4 momentum \( P_\mu \) by boosting the SU(6) wave function prepared in the deuteron rest frame. As was already formulated,\(^{15}\) we have two methods of boosting a rest frame wave function to a...
\( \bar{U}(12) \) wave function and a \( SU(6)_W \) wave function. The \( \bar{U}(12) \) wave function obeying the Bargmann-Wigner equation is obtained by boosting separately each constituent as a Dirac particle, while the \( SU(6)_W \) wave function is to be constructed by boosting the composite particle as a whole. Roughly speaking, both the spin wave functions give almost the same \( q^2 \) dependence to the form factors except that the \( \bar{U}(12) \) method yields an extra factor \((1-q^2/2M^2)^n\), \( n \) being equal to "1" for the nucleon case and "2" for the deuteron case. The extra factor breaks the relation between the asymptotic power of the form factor and the internal degrees of freedom predicted by our naive R. H. O. M., and then prevents us from reproducing the experimental form factors by theoretical ones. It has been verified that the \( SU(6)_W \) method never gives the extra factor in the pion and nucleon cases, and it is strongly suggested that this is true for the six-quark system. However, the \( \bar{U}(12) \) method is very much simpler than the \( SU(6)_W \) method. For simplicity in this letter, we adopt the \( \bar{U}(12) \) method supplemented by dropping out the extra factor \((1-q^2/2M^2)^n\). The details of the \( SU(6)_W \) method will be given in a forthcoming paper.

The rest frame wave function \( \psi_{6q}(P=0) \) is so designed as to have the deuteron quantum number \((J=0, S=1 \text{ and color-singlet}) \) in the \( SU_s(3) \otimes SU_f(2) \otimes U_c(2) \) symmetry scheme and as to be totally antisymmetric. Then we get

\[
\psi_{6q}(P=0) = (1 - \sum_{j=1,2,3} \mathcal{D}_j) \psi_{\bar{U}(12) SU(6)_W}(P=0),
\]

where the color state is represented by \( \epsilon_{ij} \) and \( \epsilon_{i,m} \), the isospin state by the \( 2 \times 2 \) charge conjugation matrices \( c \) and the spin state by the symmetric spin wave functions \( S \) and the antisymmetric ones \( A \). Following the prescription of the \( \bar{U}(12) \) method, \( S \) and \( A \) should be replaced by

\[
S_{ab} = \frac{1}{2} \left( \frac{\tau \cdot p + M}{2M} \right) \epsilon_{ad} D_{db},
\]

\[
A_{ab} = \frac{1}{2} \left( \frac{\tau \cdot p + M}{2M} \right) c_{db},
\]

where \( \epsilon = i\gamma^5 \). Constructing a current vertex through the method given in Ref. 5 and using the above wave function, we can obtain

\[
\langle \Phi_f | J_\mu | \Phi_i \rangle = \frac{1}{\sqrt{2E_f N}} \sqrt{2E_f V(R_f) \tilde{Q}} \epsilon_{\mu
u} \left[ (1 - 3q^2/2M^2 + 6i\epsilon_{\alpha\beta\gamma}q^\gamma) \right] \psi_{6q}(q^2).\]

Equations (2), (3) and (4) yield

\[
G_c(q^2) = (1 - 7q^2/4M^2) \times (1 - q^2/2M^2)^{-1} I_{6q}(q^2),
\]

\[
G_q(q^2) = 2I_{6q}(q^2),\]

where we have dropped out the factor \( \frac{1}{2} \).
We have the structure functions

\[ A(Q^2) = \sin^4 \theta \left[ G_s^2 + \left( \frac{Q^2}{18M_p^2} \right) G_s^2 \right. \]
\[ + \left. \left( \frac{Q^2}{6M_p^2} \right) (1 + Q^2/4M_p^2) G_n^2 \right], \]

\[ B(Q^2) = \sin^4 \theta \left( \frac{Q^2}{3M_p^2} \right) \]
\[ \times \left( 1 + Q^2/4M_p^2 \right) G_n^2 \]

at large \( Q^2 = -q^2 \). We can put \( \alpha_{3q} = 1.1 \) (GeV/c)\(^2\) because of the formula \( \alpha_{3q} = (6/3)^{3/2} \alpha_{3q} \) and the numerical value of \( \alpha_{3q} \) determined by the best fit of the nucleon form factors. We have still free parameters \( M \) and \( \sin^2 \theta \) to be determined through the best fit of \( A(Q^2) \) to the experimental plot. As is shown in Fig. 1, the best fit is obtained by putting \( M = 1.3 \) GeV and \( \sin^2 \theta = 0.05 \).

Now we can completely fix the \( Q^2 \) dependence of \( B(Q^2) \) coming exclusively from the magnetic form factor \( G_m \). Although only one experimental point of \( B(q^2) \) is available at rather large momentum transfers, we can say that our result is consistent with the existing experiment — see Fig. 2. Finally it is shown that theoretical deuteron magnetic moment can also be fairly well improved by our model. Ordinary nuclear physics gives us the theoretical formula

\[ \mu_N = \mu_p + \mu_n \]
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for the deuteron magnetic moment, \( P_D \) being the D-state probability. For \( P_D = 0.065 \) accepted in nuclear physics, we have \( \mu_{np} = 0.8428 \) nuclear magneton which is smaller than the experimental value \( \mu_d(\exp) = 0.8574 \) nuclear magneton. In our scheme the deuteron magnetic moment must be written as

\[ \mu_d = \mu_{np} \cos^2 \theta + \mu_{3q} \sin^2 \theta, \]

in which we can put \( \mu_{3q} = (1/2)G_s(0) = 1 \) nuclear magneton by (5) and \( \sin^2 \theta = 0.05 \) by the above analysis. Hence we get our theoretical prediction \( \mu_d = 0.8506 \) nuclear magneton. This is really a considerable improvement. It is, however, well known that the discrepancy between \( \mu_{np} \) and \( \mu_d(\exp) \) has been explained in terms of exchange current effects in ordinary nuclear physics. Here we have to
recall a similar situation around the structure of nucleon isobars, for example, \( J_{13}(1236) \). Twenty years ago many physicists believed that a nucleon isobar was an excited state of pion cloud or a sort of bound state of virtual pion and nucleon to be described by the Chew-Low equation. Nowadays, however, we consider it to be a three quark bound state in the sense of hadron physics. The exchange current explanation just corresponds to the old aspect of nucleon isobars which was already replaced by the modern quark model. The present authors assert that the six-quark approach to the deuteron magnetic moment is correct.

Finally we should comment on similar works performed by Mitra\(^{71}\) and Kobyshikin\(^{81}\) in comparison with ours. Mitra has presented the deuteron form factors with the \( \langle Q^2 \rangle^{-
frac{3}{2}} \) behaviour within the framework of the non-relativistic quark model supplemented by the so-called relativization procedure, by considering the deuteron to be a pure six-quark system quite differently from ours. Kobyshikin has also given similar form factors by applying the relativistic three dimensional quark model to the deuteron considered to be a superposed state of the ordinary \( n-p \) configuration and the six-quark one similarly as ours, even though his estimation of \( \sin^2 \theta \) is much smaller than ours. As far as practical calculations leading to the \( q^2 \) dependence are concerned however, both the theories have used the essentially same nonrelativistic calculations supplemented by the ad hoc replacement: \( q^2 \rightarrow (1 - q^2/4M^2)^{-1} (-q^2) \), and multiplying each form factor by \( (1 - q^2/4M^2)^{-3} \) in order to achieve dimensional scaling. On the contrary, the penta-pole behaviour of the deuteron form factor in our model can naturally be derived in a frame independent manner from the four dimensional overlap integral of the initial and final wave functions given by the fully relativistic harmonic oscillator model, without resort to any ad hoc replacement.

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B. Sakita and K. C. Wali, Phys. Rev. 139 (1965), B1355.
8) A. P. Kobyshikin, Yadern Fiz. 28 (1978), 486.