Magnetic Moments of Composite Leptons and Quarks
in a Dynamical Subquark Model

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We propose a dynamical subquark model where the composite leptons and quarks have
almost vanishing anomalous magnetic moments, and look pointlike. The fundamental in­
teraction is of the six-fermion type. The apparent tremendous divergences are entirely manageable.

Nowadays so many leptons and quarks
are found or conjectured to explain various
aspects of elementary particle phenomena.
The increasing numbers of weak doublets
and color triplets tempt us to presume
more fundamental matters, i.e., sub­
quarks, which compose leptons and
quarks. A wide spread criticisms for such
models are the questions why leptons and
quarks, especially the electron and muon
look pointlike, and why their magnetic
moments are so close to the Dirac
due to the remarkable binding forces.
It is the purpose of this paper
to propose a dynamical model which renders
a possible answer to these questions.

Various types of subquark models have
been proposed by many authors mainly to
classify so many leptons and
quarks. The essential idea is to factorize lepton
or quark states into more fundamental ones
bearing each single quantum number.
In some models, leptons and quarks are
composed of two subquarks, while in other
models, of three subquarks. For example,
in the unified spinor-subquark model in
Ref. 6), quarks and leptons are bound
states of three spinor subquarks, \( \omega_i, h_j \)
and \( \epsilon_k \), bearing weak isospin, heaviness
and color, respectively. We consider here
such a three-spinor-model. From the view­
points of classification, the subquarks are
introduced very naturally, and even neces­
sarily. From the dynamical points of view,
however, the leptons and quarks look like
elementary. For example, they have almost
vanishing anomalous magnetic moments, in
conflict with the predictions of the ordinary
loosely-bound state model. This indicates
that the binding forces should be effective
at very short distances. So we work with
the following abstract model of the sub­
quark dynamics.

Suppose the quark or lepton is a bound
state of three spinor subquarks (denoted by \( \phi^{(i)} (i=1, 2, 3) \)) interacting with each
other through six-fermion interactions.

The relevant Lagrangian is given by

\[
L = \sum_{\mathbf{i} \in \{1, 2, 3\}} \bar{\psi}^{(i)} \left( i \gamma_5 \mathbb{D} - m \right) \psi^{(i)}
+ f P^{(x)}(\psi^{(1)} \psi^{(2)} \psi^{(3)}) P^{x \dagger}(\phi^{(1)} \phi^{(2)} \phi^{(3)})
\]

(1)

where \( m \) is the mass of the subquarks, \( f \)
is the coupling constant and \( P^{(x)} \) is the
projection operator of the product of the
three spinor fields to a state with spin 1/2.
In the product there are three independent
spin-1/2 states, which we denote by \( x=a, b, c \).
The projection operator \( P^{(x)} \) is explicitly written as

\[
(P^{(x)}(\psi^{(1)} \psi^{(2)} \psi^{(3)}))_x = \sum_{\alpha, \beta, \gamma} P^{(x)}_{\alpha \beta \gamma}
\times \phi^{(1)}_\alpha \phi^{(2)}_\beta \phi^{(3)}_\gamma,
\]

(2)

For leptons, see Ref. 4). If quarks have
significant anomalous magnetic moments, deep
inelastic scattering does not scale. See, for ex­
ample, Ref. 5).
Table 1. Values of $\langle \sigma | \alpha \beta \gamma \rangle_{\omega}.$

<table>
<thead>
<tr>
<th>$\omega$</th>
<th>1</th>
<th>1</th>
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<th>1</th>
</tr>
</thead>
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<td>$\langle \sigma</td>
<td>\alpha \beta \gamma \rangle_{\omega}$</td>
<td>1</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\omega$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-1</td>
<td>-2</td>
<td>2</td>
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<td></td>
</tr>
</tbody>
</table>

where the coefficients $P^{(i)}_{a, \alpha, \beta, \gamma}$ are given by

$$P^{(i)}_{a, \alpha, \beta, \gamma} = \frac{\langle \sigma | \alpha \beta \gamma \rangle_{\omega}}{(4 \pi)^{3} \int_{0}^{1} \int_{0}^{1-x} d \psi \psi \phi \sigma},$$

(3)

and otherwise

$$P^{(i)}_{a, \alpha, \beta, \gamma} = 0,$$

(4)

with the values of $\langle \sigma | \alpha \beta \gamma \rangle_{\omega}$ in Table 1. Introducing the auxiliary field $\Phi$ to describe the bound state (i.e., quark or lepton), we rewrite the Lagrangian $L$ into the equivalent form,

$$L' = \sum \tilde{\psi}^{(i)} (i \frac{\partial}{\partial \rho} - m) \psi^{(i)} + g \tilde{\Phi}^{(i)} \psi^{(i)} \psi^{(3)} + \mathrm{h.c.}$$

(6)

where $g$ is a coupling constant to be adjusted by the compositeness condition of $\Phi.$ The Lagrangians $L$ and $L'$ are equivalent because an Euler equation of $L'$ gives the constraint relation,

$$\Phi = \frac{\tilde{f}}{g} P^{(i)}_{a, \alpha, \beta, \gamma} \langle \chi | \psi^{(i)} | \psi^{(3)} \rangle,$$

(7)

and this leads to

$$L = L'.$$

(8)

Furthermore, we can display one-to-one correspondence between the Feynman diagrams of $L$ and $L'.$

The Feynman amplitudes in this model contain many badly divergent integrals, and they are not renormalizable in the ordinary sense. For example, the self-energy part to lowest order (Fig. 1) is calculated to be

$$\sum (p) = -g^2 \left( \frac{\partial p_{1} + m_{2} I_{2}}{p} \right) + \mathrm{less \ divergent \ terms},$$

(9)

where

$$I_{1} = \frac{9}{2} A^{4} \left( \int_{0}^{1} dx \int_{0}^{1-x} d \psi \psi \sigma \right)^{2},$$

(10)

and

$$I_{2} = 3 A^{5} \left( \int_{0}^{1-x} d \psi \right)^{2} \left( \int_{0}^{1-x} d \psi \psi \sigma \right)^{3},$$

(11)

with

$$J = xy + xz + yz$$

(12)

In Eq. (9), we have introduced the momentum cutoff $A$ by multiplying the suppression factor

$$\frac{A^{4} - \rho^{2}}{A^{4} - \rho^{2}},$$

(13)

to each propagator. The order of divergence increases without limit, as the number of loops increases. They, however, are entirely manageable and calculable, as follows. We first sum up the chains of the leading divergent parts of $\Sigma (p)$ (Fig. 1), which gives the propagator of the bound state

$$\frac{1}{\tau \delta - M},$$

(14)

where we have put

$$g^2 = \frac{1}{I_{1}}.$$
and

\[ M = - \left( \frac{1}{\omega} + \ln m^2 \right) / \omega. \]  

(16)

In order to have nonvanishing \( g \), we assume that the cutoff is very large, but finite. Such a finite cutoff may cause various difficulties at very high energy region. We consider, however, Eq. (13) is only an approximation in today's energy region. If we take account of the suppression due to \( g \), simple power counting shows that any skeleton diagram behaves like

\[ A^{\left( \pi_0 - 1 \right) n_s} \]  

(17)
as \( A \rightarrow \infty \), where \( n_s \) is the number of the external lines, and \( \pi_0 \) is the largest even number \( \leq x \). Then, the only primitive divergents are the self-energy parts for \( \phi \) and \( \bar{\Psi} \), which can be removed by redefinition of masses \( m \) and \( M \) and rescaling of \( \phi \) and the compositeness condition of \( \bar{\Psi} \). The last is the condition that the renormalization constant for \( \bar{\Psi} \) is vanishing, and it relates the coupling constant \( g \) to the cutoff \( A \). We further assume that the momenta flowing through the bound state propagator are cut off at \( A' \) much smaller than \( A \). Then, the higher order diagrams are suppressed by extra powers of \( A' / A \), so that we can calculate any physical quantities order by order in \( A' / A \).

Now we turn to investigation of the anomalous magnetic moment. We introduce the electromagnetism by replacing every \( i\partial_\mu \) (or \( p_\mu \) in the momentum space) in the Lagrangian and the suppression factors like (13) by \( i\partial_\mu - eQA_\mu \), where \( A_\mu \) is the electromagnetic field, and \( eQ \) is the electric charge of the matter field to which the \( \theta_\mu \) operates. The matrix elements of the electromagnetic current \( J_\mu \) with respect to the bound states \( \langle \rho \rangle \) have the form

\[
\langle p + q | J_\mu | \rho \rangle = \bar{u}(p + q) (\gamma_\mu Q F_i(q^2) + i\sigma_\mu q^i A F_i(q^2)) u(p),
\]

(18)

where \( Q, A \) and \( F_i(q^2) (i=1,2) \) are the electric charge, the anomalous magnetic moment and the form factors of the bound state, respectively. Because of gauge invariance, the total charge \( Q \) is precisely the sum of the constituent charges, \( Q_i \) (\( i=1,2,3 \)). According to the argument above and dimensional analysis, the anomalous magnetic moment and the form factors should have the forms

\[
\Delta \mu = n \sum_{n=1}^m a_n \left( \frac{A'}{A} \right)^n + O\left( \frac{1}{A'^4} \right)
\]

and

\[
F_i(q^2) = 1 + \frac{q^2}{A^2} \sum_{n=1}^m f_n \left( \frac{A'}{A} \right)^n + O\left( \frac{1}{A'^4} \right)
\]

respectively, where \( a_n \) and \( f_n \) are calculable coefficients. Actual calculation to lowest order (Fig. 2) gives

<table>
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<tr>
<th>Type</th>
<th>( c_1 )</th>
<th>( c_1' )</th>
<th>( c_2 )</th>
<th>( c_2' )</th>
<th>( c_3 )</th>
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<tbody>
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<tr>
<td>b</td>
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<tr>
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<td>1</td>
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</tr>
</tbody>
</table>

Fig. 2. The electromagnetic vertex part.
\[ \Delta \mu = m^3 \sum_{i=1}^{3} Q_i (c_i I_0 - c_i' I_0') / I_s \]
\[ \sim O \left( \frac{m^3 (\ln A^5)^2}{A^4} \right), \]  \hspace{1cm} (21)

where \( c_i \) are the coefficients given in Table II, and
\[ I_0 = \frac{(\ln A^5)^2}{(4\pi)^4} \quad \text{and} \]
\[ I_0' = \frac{\ln A^5}{(4\pi)^4} \int_0^1 dx \int_0^{1-x} dy \]
\[ \times \frac{x y (1-x-y)}{(x+y-xy-x^2-y^2)^3}. \] \hspace{1cm} (22)

In this order, the term of \( O(1/A^3) \) vanishes, and only the terms of \( \approx O(1/A^4) \) remain. In principle the relation (19) can be checked by experiment, if the cutoff is not too large. If, as in the unified model of the Nambu-Jona-Lasinio type, \( \frac{1}{A} \) the cutoff is of order of the Planck mass, the anomalous magnetic moment is much smaller than the electromagnetic Schwinger correction, and may hardly be observed. Furthermore, Eq. (20) shows that the \( q^2 \)-dependence of the form factors of \( \mathcal{F} \) is \( O(q^2/A^4) \). Therefore, the bound state, i.e., the lepton or quark, looks almost pointlike, as is consistent with the observation. What makes such differences from the ordinary bound state models which were so successful in atoms and nuclei, and in the quark model of hadrons? In the latter case, the binding is very loose and each constituent incoherently interacts with the photon, so that the magnetic moments are definite combinations of those of constituents with Clebsch-Gordan coefficients, while in the former case, because of the short range property of the binding force, the size of the bound state is very small, looking almost like a Dirac particle.\footnote{C. K. Chang, Phys. Rev. D5 (1971), 950.}

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For a brief review, see H. Terazawa, INS-Rep. 351 (1979), chap. II. See the other references therein.

2) J. C. Pati, A. Salam and J. Strathdee, Phys. Letters 58B (1975), 265.

6) K. Akama and H. Terazawa, Ref. 4.

8) For a review, see M. J. Lipkin, Phys. Reports 8C (1973), 173.
9) A similar argument was made by Novak et al., in their model.