

Discussion: “Three-Dimensional Vortex Method for Gas-Particle Two-Phase Compound Round Jet” (Uchiyama, T., and Fukase, A., 2005, ASME J. Fluids Eng., 127, pp. 32–40)

L. A. Barba

Department of Mathematics,
University of Bristol,
Bristol BS8 1TW, UK

The paper by Uchiyama and Fukase [1] proposes a three-dimensional vortex method for particulate flows, which is applied to a circular jet of air laden with glass particles. The numerical method presented has the following characteristics: (1) it includes the two-way coupling of the particles and the fluid, (2) it discretizes the vorticity in the gas phase in the usual manner when vortex blob methods are used, and (3) it accounts for the effects of viscosity in the fluid by means of the less-than-usual core spreading method.

The method presented in the paper under discussion is claimed to be a 3D extension of the two-dimensional vortex method for particle-laden flows proposed by Uchiyama and Naruse [2]. Upon reading this previous work, however, one notes important differences in the core spreading viscous scheme. There is an apparent inconsistency in the paper in relation to the implementation of the vortex method, which will be discussed below. There are three issues that should be made clear.

1. The core spreading formula presented is not the same as used in the previous work by Uchiyama and Naruse [2] (and several subsequent works by Uchiyama and others). The extension to 3D of [1] refers to the particle-fluid coupling only.
2. The core spreading scheme in [1] corresponds to that presented by Leonard [3]; that scheme is derived for simple Gaussian vortex blobs, whereas in [1] a *cubic* Gaussian is used. This is a severe inconsistency, and readers should be made aware that core spreading is derived for a *specific* blob distribution.
3. The core spreading method used in [1] (Leonard's) is a two-dimensional scheme, derived from an exact solution to the 2D Navier-Stokes equation. It is unclear how the authors are applying the scheme for 3D flows; in fact, the mere application of this scheme to 3D is questionable.

The 2D antecessor [2] of the method presented in [1] incorporated the two-way coupling for particles and fluid, by means of

turning the force of particles acting on the fluid into a source term in the vorticity transport equation. This technique was well known at the time. The generation of vorticity due to the presence of the particles and by the action of the drag force was pointed out in the review by Crowe et al. [4]. The concept was implemented successfully in a vortex-based method in [5].

The salient feature of the 2D method of Uchiyama and Naruse [2] is the use of the core spreading method for viscous effects in the gas phase. The core spreading method has not been used often, due to it falling out of favor with vortex methods workers when consistency objections were raised [6]. The scheme was presented by Leonard [3], and consists of the idea that elementary vortex blobs can be made to grow in time to simulate viscosity.

The consistency issues with the core spreading vortex method stem from the fact that the vortex blobs are advected without deformation, and thus they cannot be allowed to grow larger and larger in time. Basically, the size of the blobs represents the smallest scales that the vortex blob method can resolve. If the size of the vortex blobs is σ , then the truncation error of the vortex method grows with σ^2 , as proved in [3]. Moreover, since the vorticity is advected with an average velocity and not with the actual local velocity, the convection of vorticity is incorrect even in the limit of infinitely many particles, as proved by [6]. These issues were successfully addressed by Rossi [7], who proposed a vortex splitting technique to control the core sizes, and proved it to be convergent. This result prompted new recent work with the core spreading method [8,9]. Most recently, a new formulation for the vortex method with core spreading has been developed, in which the core size control necessary for consistency and convergence is provided (without splitting) in a spatial adaption scheme using radial basis function interpolation [10].

In the paper under discussion, and the previous work of Uchiyama and co-workers, starting with [2], no correction is provided for the consistency problem of the core spreading method. As far as can be inferred from their publications, they allow the vortex blobs to grow uncontrolled throughout a time marching calculation. This is acceptable if the Reynolds number is large and the simulations short, but it is important to underscore that the algorithm is inconsistent if applied to another flow situation.

Going into more detail, in [2] the authors cite as their source for the core spreading method the scheme proposed by Nakanishi and Kamemoto [11], and thus the core sizes are grown in time following the rule:

$$\frac{d\sigma}{dt} = \frac{vc^2}{2\sigma} \quad (1)$$

where the constant c takes the numeric value of 2.242; this is Eq. 15 of [2], taken from Eq. 10 of [11] with changes in notation.

The cited work [11] contains a rather scant derivation, and is easy to question. They claim to obtain the above formula for the change in blob radius due to viscosity in three dimensions by *approximating it by two-dimensional viscous diffusion* “for convenience.” They seem to take the 3D vortex blobs, with spherical symmetry, and approximate these with a cylinder of equal volume, then use 2D core spreading to diffuse the (finite-length!)

Contributed by the Fluids Engineering Division of ASME for publication in the JOURNAL OF FLUIDS ENGINEERING. Manuscript received July 22, 2005; final manuscript received October 25, 2005. Assoc. Editor: Georges L. Chahine.

cylindrical vortex. Apparently they then switch back to a spherical blob of volume equal to the “diffused” cylindrical one.

The first question that comes to mind is, what may be the accuracy of the approximation of spherical blobs with cylinders, and of using a 2D viscous scheme in a 3D method? Secondly, and more importantly, how is the 2D core spreading formula given by (1) obtained, and what is its accuracy? The authors of [11] claim that the numeric value of the constant c in the formula is “obtained using an exact solution of Navier-Stokes equation for a straight and infinitely long vortex filament of fluid.” Indeed, this is the concept behind the core spreading method that is well known by the rest of the community. In [11], strangely, the blob distribution function is an *algebraic* cutoff, given by:

$$\zeta(\rho) = \frac{15}{8\pi} \frac{1}{(\rho^2 + 1)^{7/2}} \quad (2)$$

where $\rho = |\mathbf{x} - \mathbf{x}_i| / \sigma^2$, and the vorticity distribution of each blob is $(1/\sigma^2)\zeta(\rho)$. The above cutoff function is certainly not an exact solution of the Navier-Stokes equations. How this spherical distribution of vorticity is turned into a “straight and infinitely long filament” to obtain the core spreading formula is simply not explained.

Turning our attention back to the work by Uchiyama and co-workers, we note that in [2] they have used the core spreading formula given by (1), but they utilize a *different cutoff function* (Chorin’s). This is disconcerting, as the core spreading method is explicitly tied to the cutoff function. In subsequent papers [12–14], the authors continue to give the same core spreading formula as above, and continue to use Chorin’s cutoff function. This cutoff was introduced in [15] and is the following:

$$\zeta(\rho) = \begin{cases} \frac{1}{2\pi\rho}, & \rho \leq 1 \\ 0, & \rho > 1 \end{cases} \quad (3)$$

The above is a 2D blob function and indeed the work of [2,12–14] is always in two dimensions. However, recall that the method of [11] is a *3D method*, where an algebraic cutoff is used, and some transformations between spherical and cylindrical symmetry are applied to obtain the formula given by (1).

Subsequently, in [16], the authors no longer reference [11] but instead cite [3] for the core spreading method. They now use a Gaussian blob distribution function, and present the following formula for core spreading:

$$\frac{d\sigma}{dt} = \frac{2\nu}{\sigma} \quad (4)$$

One can see that the formula above is equivalent to (1) if one takes $c=2$, and it is also equivalent to the core spreading formula more familiar to the community, as presented by [3]:

$$\frac{d\sigma^2}{dt} = 4\nu \quad (5)$$

The factor 4 in the above equation is sometimes replaced by 2, or even 1, depending on the normalization of the Gaussian cutoff used.

In deriving the core spreading scheme, the viscous term of the vorticity equation is satisfied identically, using the classical exact solution of the Navier-Stokes equation known as “spreading line vortex” ([17], p. 204). This exact solution is given by

$$\omega(\mathbf{x}, t) = \frac{\Gamma}{4\pi\nu t} \exp(-|\mathbf{x}|^2/4\nu t) \quad (6)$$

With the 2D Gaussian cutoff used by Leonard,

$$\zeta(\rho) = \frac{1}{\pi} \exp(-\rho^2) \quad (7)$$

clearly the diffusion equation is satisfied *exactly* by each blob

when making σ^2 grow linearly according to (5). Since the diffusion equation is linear, the discretized vorticity field in 2D can be seen as a superposition of spreading line vortices of different strengths (circulation). This is the essence of the core spreading vortex method.

In the three-dimensional extension of the the method presented in the paper under discussion, the authors give the core spreading formula as in (5), citing Leonard. This time, however, they use a cubic Gaussian cutoff function, given by:

$$\zeta(\rho) = \frac{3}{4\pi} \exp(-\rho^3) \quad (8)$$

It is not explained how the authors use the two-dimensional core spreading formula with a three-dimensional cutoff function. It is also not clear why the standard core spreading formula is given, which applies only to simple Gaussian blobs, like (7).

The compilation of articles by Uchiyama and co-workers, from [2] to the paper under discussion, seems to indicate that the core spreading method has been misinterpreted. I tend to believe that the method that has been implemented is rather like a point vortex model, regularized. It seems quite probable that viscous effects are incorrectly modeled; the method seems to be mathematically inconsistent.

It is important, in my opinion, to discuss and acknowledge the problems just examined. The vortex method has sometimes suffered the reputation of being only a modeling approach for unsteady flows. It is a fact that the vortex method can produce direct numerical simulation results, and many researchers have shown accuracy in competition with finite difference methods and even spectral methods (see, for example, [18–22]). One should not, under the cover of considering the “application to engineering problems,” fail to have concern for numerical consistency and accuracy.

Acknowledgment

For reading an early draft of this discussion, thanks to A. Leonard, C. Meneveau, and L. F. Rossi.

References

- [1] Uchiyama, T., and Fukase, A., 2005, “Three-dimensional Vortex Method for Gas-Particle Two-Phase Compound Round Jet,” *J. Fluids Eng.*, **127**(1), pp. 32–40.
- [2] Uchiyama, T., and Naruse, M., 2001, “A Numerical Method for Gas-Solid Two-Phase Free Turbulent Flow Using a Vortex Method,” *Powder Technol.*, **119**(2–3), pp. 206–214.
- [3] Leonard, A., 1980, “Vortex Methods for Flow Simulation,” *J. Comput. Phys.*, **37**, pp. 289–335.
- [4] Crowe, C. T., Trout, T. R., and Chung, J. N., 1996, “Numerical Models for Two-Phase Turbulent Flows,” *Annu. Rev. Fluid Mech.*, **28**, pp. 11–43.
- [5] Chen, H., and Marshall, J. S., 1999, “A Lagrangian Vorticity Method for Two-Phase Particulate Flows With Two-Way Phase Coupling,” *J. Comput. Phys.*, **148**(1), pp. 169–198.
- [6] Greengard, C., 1985, “The Core Spreading Vortex Method Approximates the Wrong Equation,” *J. Comput. Phys.*, **61**, pp. 345–348.
- [7] Rossi, L. F., 1996, “Resurrecting Core Spreading Vortex Methods: A New Scheme That is Both Deterministic and Convergent,” *SIAM J. Sci. Comput.*, **17**, pp. 370–397.
- [8] Shiels, D., and Leonard, A., 2001, “Investigation of a Drag Reduction on a Circular Cylinder in Rotary Oscillation,” *J. Fluid Mech.*, **431**, pp. 297–322.
- [9] Moeleker, P., and Leonard, A., 2001, “Lagrangian Methods for the Tensor-Diffusivity Subgrid Model,” *J. Comput. Phys.*, **167**(1), pp. 1–21.
- [10] Barba, L. A., Leonard, A., and Allen, C. B., 2005, “Advances in Viscous Vortex Methods—Meshless Spatial Adaption Based on Radial Basis Function Interpolation,” *Int. J. Numer. Methods Fluids*, **47**(5), pp. 387–421.
- [11] Nakanishi, Y., and Kamemoto, K., 1993, “Numerical Simulation of Flow Around a Sphere With Vortex Blobs,” *J. Wind. Eng. Ind. Aerodyn.*, **46–47**, pp. 363–369.
- [12] Uchiyama, T., and Naruse, M., 2002, “Numerical Simulation of Gas-Particle Two-Phase Mixing Layer by Vortex Method,” *Powder Technol.*, **125**(2–3), pp. 111–121.
- [13] Uchiyama, T., and Okita, T., 2003, “Numerical Prediction of a Plume Diffusion Field Around a Circular Cylinder by the Particle Method,” *Adv. Environ. Res.*, **7**(2), pp. 573–581.
- [14] Uchiyama, T., and Naruse, M., 2003, “Vortex Simulation of Slit Nozzle Gas-Particle Two-Phase Jet,” *Powder Technol.*, **131**(2–3), pp. 156–165.

- [15] Chorin, A. J., 1973, "Numerical Study of Slightly Viscous Flow," *J. Fluid Mech.*, **57**, pp. 785–796.
- [16] Uchiyama, T., and Naruse, M., 2004, "Numerical Simulation for Gas-Particle Two-Phase Free Turbulent Flow Based on Vortex in Cell Method," *Powder Technol.*, **142**(2–3), pp. 193–208.
- [17] Batchelor, G. K., 1967, *An Introduction to Fluid Dynamics*, Cambridge University Press, Cambridge, UK.
- [18] Cottet, G.-H., and Poncet, P., 2004, "Advances in Direct Numerical Simulations of 3D Wall-Bounded Flows by Vortex-in-Cell Methods," *J. Comput. Phys.*, **193**(1), pp. 136–158.
- [19] Cottet, G.-H., Michaux, B., Ossia, S., and VanderLinden, G., 2002, "A Comparison of Spectral and Vortex Methods in Three-dimensional Incompressible Flows," *J. Comput. Phys.*, **175**, pp. 702–712.
- [20] Ploumhans, P., Winckelmans, G. S., Salmon, J. K., Leonard, A., and Warren, M. S., 2002, "Vortex Methods for Direct Numerical Simulation of Three-Dimensional Bluff Body Flows: Application to the Sphere at $Re=300, 500$ and 1000 ," *J. Comput. Phys.*, **178**, pp. 427–463.
- [21] Cottet, G.-H., and Koumoutsakos, P., 2000, *Vortex Methods. Theory and Practice*, Cambridge University Press, Cambridge, UK.
- [22] Ploumhans, P., and Winckelmans, G. S., 2000, "Vortex Methods for High-Resolution Simulations of Viscous Flow Past Bluff Bodies of General Geometry," *J. Comput. Phys.*, **165**, pp. 354–406.