$$M = \int_0^c p_L(x)xdx + \int_0^c f_L(x)f_L'(x)p_L(x)dx - \int_0^c p_u(x)xdx - \int_0^c f_u(x)f_u'(x)p_u(x)dx \quad \text{(A3)}$$

The coordinates x_0 and y_0 are calculated as follows:

$$x_0 = \frac{\int_0^c p_L(x)xdx - \int_0^c p_u(x)xdx}{F_L} \quad \text{(A4)}$$

$$y_0 = \frac{-\int_0^c p_L(x)f_L(x)f_L'(x)dx + \int_0^c p_u(x)f_u(x)f_u'(x)dx}{F_D} \quad \text{(A5)}$$

The total aerodynamic forces and moment for the blade group are calculated differently for the various modes of vibration. The best way to find the total aerodynamic force on the blade group is by summing up the actual forces for each blade in the group. The aerodynamic flow around each blade is different because the adjacent blade motions are different for each blade. For example, in the first vibration mode, all of the blades of the middle blade group move together, but the side blade groups are clamped. Therefore, the actual aerodynamic force on the first blade is somewhat different from the forces on the second, third and fourth blades. In the twist vibration mode, each blade in the vibrating group has a different vibration amplitude and, therefore, a different relative flow incidence. Extensive experimental evidence shows that the unsteady aerodynamic force depends strongly on the relative flow incidence (Fig. 16), and that this force varies nearly linearly with the vibration amplitude. The twist mode aerodynamic moment for the blade group is calculated as follows:

$$F_\theta = -\sum_{n=1}^{n_t} \{F_{Dn}[r_n \cos(\theta_s) - d + y_0] + F_{Ln}[r_n \sin(\theta_s) + e - x_0]\} \quad \text{(A6)}$$

where F_{Dn} and F_{Ln} are determined as functions of time from interferograms for each blade while the blade group is vibrating in the twist mode.

The instantaneous force center is assumed to be stationary with respect to the blade during a small time increment $t_2 - t_1$, but the blade CG is not stationary relative to a stationary observer when the blades are vibrating. The incremental displacements of the instantaneous force center in the directions of forces F_D and F_L (in the x and y directions, respectively) in a time interval $(t_2 - t_1)$ are calculated as:

$$\Delta x_n = [-R_n\theta_0 \sin(\gamma_n) + h_A \cos(\theta_s) - h_T \sin(\theta_s)] \times [\sin \omega t_2 - \sin \omega t_1] \quad \text{(A7)}$$

$$\Delta y_n = [-R_n\theta_0 \cos(\gamma_n) + h_A \sin(\theta_s) + h_T \cos(\theta_s)] \times [\sin \omega t_2 - \sin \omega t_1] \quad \text{(A8)}$$

$$R_n = \frac{r_n}{|r_n|}\sqrt{[r_n \cos(\theta_s) - d + y_0]^2 + [r_n \sin(\theta_s) + e - x_0]^2} \quad \text{(A9)}$$

$$\gamma_n = \tan^{-1}\left(\frac{r_n \cos(\theta_s) - d + y_0}{r_n \sin(\theta_s) + e - x_0}\right) \quad \text{(A10)}$$

The energy input for each blade during a time increment $(t_{i+1} - t_i)$ by F_{Dn} and F_{Ln} forces is calculated. The energy input to a blade by the lift and drag forces are:

$$E_{Ln} = F_{Ln}\Delta y \qquad E_{Dn} = F_{Dn}\Delta x \quad \text{(A11)}$$

where F_{Ln} and F_{Dn} are functions of ωt.

The computer program also calculates the instantaneous drag, lift and moment coefficients per unit span for a blade and the moment coefficient per unit span for the blade group. Each force coefficient per unit length is defined in terms of the dynamic pressure $\rho v_1{}^2/2$ such that $C = 2F/\rho V_1^2 c$. By using the relation $a^2 = \gamma p/\rho$ and $\gamma = 1.40$, the various force and moment coefficients of the blade become:

$$C_L = \frac{F_L}{0.70\, p_0 M_1^2 c} \qquad C_D = \frac{F_D}{0.70\, p_0 M_1^2 c} \qquad C_M = \frac{F_M}{0.70\, p_0 M_1^2 c^2} \quad \text{(A12)}$$

The moment coefficient of a blade group is:

$$C_\theta = \frac{F_\theta}{0.70\, p_0 M_1^2 c^2} \quad \text{(A13)}$$

The instantaneous inlet flow angle β_1 relative to the blade CG is calculated for each blade in the group from:

$$\beta_i = \frac{180}{\pi}\left\{\pi + \theta_0 \sin\phi - \tan^{-1}\left[\frac{M_1 a \sin\frac{\pi}{180}(180 - \beta) - h_A\,\omega\cos\phi + r_n\theta_0\omega\cos\phi}{M_1 a \cos\frac{\pi}{180}(180 - \beta) + h_T\,\omega\cos\phi}\right]\right\} \quad \text{(A14)}$$

DISCUSSION

F. Sisto[1]

This paper, and its companion paper describing the experimental apparatus (79-WA/GT-6 "Dynamic Cascade Facility and Methods for Investigating Flow-Excited Vibration and Aerodynamic Damping of Model Low Pressure Blade Groups" by Z. Kovats) are an interesting and informative introduction to the engineering community of an intriguing new program for the acquisition of unsteady cascade flow data. A similar program was initiated years ago at the Gas Turbine Laboratory at M.I.T. using a single moving airfoil in a cascade of fixed neighbors. The present program is much more ambitious and results of practical interest, i.e., with moving half-group neighbors, are anticipated with interest.

Stemming from the uncertainty of cycle-to-cycle repeatability of the nonsteady components, whether due to physical flow variability or unavoidable random measurement errors, additional information on this question might be available from presenting the range of values of, say, the lift coefficient, in addition to the average obtained from 5 determinations. Presenting only the averaged data does not give all the information that is available.

As a middle ground towards obtaining more data with the present apparatus, short of oscillating the two end half-groups of blades one wonders whether the cascade blading could be reversed, or the interferometric window moved to the other side of the tunnel, so that data could be obtained for blade #4 in the lashed packet (assuming the data already obtained is for the blade labelled #1).

All in all, however, one has to recognize that the data received to date are very preliminary. Presuming the reliability to be good, one can look forward to the experimental results from parametric variations of the governing variables as useful and eagerly awaited information from this program.

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Author's Closure

I wish to thank Dr. Sisto for his discussion and welcome the opportunity to provide further clarification.

To find the cycle-to-cycle repeatability of the nonsteady components, the standard deviation of pressure was calculated at each point on the blade surface from the following equation

$$\sigma = \left[\frac{\sum_{i=1}^{n} (p_i - \overline{p})^2}{n - 1} \right]^{1/2}$$

where $\overline{p}$ is the average of n sample values. The standard deviation ranged between 0.7 and 3kPa for the five pressure values obtained at each vibration position. Typical values for $\overline{p}$ along the blade surfaces are shown in Fig. 4 of the paper.

Interferometric data for Blade 4 have not yet been obtained, but will be in the future. The test apparatus is built so that the observation windows can be moved to the proper position for measuring the gas density field around Blade 4, as shown in Fig. 2 of companion paper 79-WA/GT-6.