J. G. Lenard.[1] The authors are to be commended for their careful analysis of the rolling problem. It is especially good to see an account of nonhomogeneous compression, velocity distribution, and steady as well as nonsteady states in the roll gap. Further, comparison to the experimentally determined roll pressure distribution of Al-Salehi et al. [25] is very interesting; the multiple pressure peaks reported in those experiments are reproduced. Indeed, the true test of a mathematical model of flat rolling is its ability to predict roll pressures, roll separating forces, and roll torques – Li and Kobayashi's paper is among the few publications attempting just that.

The theory of rolling that is currently widely accepted as the most comprehensive is that attributed to Orowan [26]. During the last twenty years several shortcomings of his theory have been identified in a large number of publications. When cold rolling of thin, hard strips is analyzed, Orowan's equations fail to give good predictions of roll torque. As the roll/strip thickness ratio is increased, the iterative calculations even fail to converge. I believe that the causes include the following: use of Hitchcock's formula to account for roll deformation; use of constant coefficients of friction; and lack of corrections for elastic entry and exit. My students and I developed a mathematical model that accounts for all of these parameters. In a publication that appeared in December 1981 [27] I also compared roll pressures, separating forces, and roll torques to Al-Salehi's results. In all instances better agreement was shown than that given by Orowan's equations.

In light of these observations it is disappointing to see that Li and Kobayashi use a rigid-plastic model that ignores elastic entry and exit corrections. Their assumption of rigid rolls further limits the usefulness of the analysis. The finite element method would have allowed easy use of Al-Salehi's data on the variation of the coefficient of friction. It would have been interesting to see the results thus obtained. The important point is however that Li and Kobayashi reproduced the multi-peak pressure distribution – no doubt, their inclusion of nonhomogeneous compression caused it. In my model, using a one-dimensional equation of equilibrium – hence, planes remained planes – but including noncircular roll deformation and variable friction coefficients, the double pressure peaks were also reproduced. The question then is which parameters cause multiple pressure peaks. At this stage I have no answers.

In my paper [27] I assumed certain parameters, such that the Orowan analysis did not converge. When the same parameters were used and roll deformation and friction were properly accounted for, convergence, albeit very slow, was evident. As the Li and Kobayashi model uses rigid rolls the aforementioned problems would not be noticed. Including in their model a description of roll flattening would allow a test of convergence. Would the technique then take much too long?

Additional References

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26 Orowan, E., "The Calculation of Roll Pressure in Hot and Cold Flat Rolling," *Proc. I. Mech. E.*, Vol. 150, 1943, pp. 140–167.

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Authors' Closure

The authors wish to thank Professor Lenard for his comments on our paper. This paper is a first attempt to analyze the flat rolling process by the finite element method, and should not be expected to be either perfect or complete. We used a rigid-plastic model, because it offers the possibility of examining the effect of process parameters on the mechanics involved in the process with good computational efficiency. It is true that we could have used a variable coefficient of friction in the analysis easily. We did not do that, however, because the variation of the coefficient of friction along the arc of contact has not been well established and the available experimental data are only for a few cases. With regard to the question of convergence with roll flattening, we do not know whether the technique will give converged solution if roll deformation is included in the model.

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