The effects of dinucleon resonances in $\pi$-deuteron elastic process are discussed. We first construct the background amplitudes using the Glauber model and investigate them in detail. A dinucleon formation term is superimposed on the background part. We examine all the world data of $\pi$-deuteron elastic scattering around dinucleon energy regions and have found that, except around 600 MeV/c, they can be well explained by introducing the dinucleon resonances which are suggested in $pp$ scattering.

§ 1. Introduction

Though the possibility of dibaryon resonances was suggested a long time ago, there had been no strong evidence of their existence. They were outside the main interests of nuclear and subnuclear physics.

This situation has changed in the last few years owing to the highly precise experiment with polarized proton beams and targets at Argonne. Hoshizaki included these data in his partial wave analysis and found the resonance-like behaviors in $^1D_2$ and $^3F_3$ waves. Arndt has recently extended his phase-shift analysis and confirmed these results. The existence of these two dibaryon resonances in proton-proton ($p-p$) channel is now beyond doubt.

Study of dibaryon resonances will be an important area in nuclear and/or elementary particle physics. It is very difficult to produce resonance states of two protons using usual nuclear potential. If we want to interpret the dibaryon state as a new type of nucleus, we will have to introduce a new degree of freedom into the conventional nuclear theory where only nucleons are considered as...
constituents of nuclei. The small elasticities of $^1D_2$ and $^3F_2$ might suggest that these dibaryons are composed of two nucleons and pion(s), if we consider them in terms of the nuclear theory. From the subnuclear point of view, dibaryons are the typical example of exotics. If we regard dibaryons as bound states of six quarks, the small couplings to $pp$ states are not so unnatural and there should be many decay channels; the coupling to each channel is determined according to the configuration of six quarks in dibaryons. In order to draw a lot of information about hadron physics, we will have to search and study dibaryon resonances in many channels.

What is the largest trouble in searching for dibaryons in various channels outside proton-nucleon elastic scattering? Some candidates of dibaryons were reported so far. They failed, however, to be confirmed mainly due to the lack of reliable calculation about background term. In order to investigate the effects of dibaryon resonances on a reaction, we must calculate not only resonance formation term but also non-resonant part of its amplitude, i.e., background part. Without precise knowledge of the background amplitudes we cannot estimate quantitatively the contribution of dibaryon resonances. In the case of $pp$ elastic scattering, its partial waves are slowly changing with energy and a lot of knowledge has been accumulated at low energies. Extrapolation of the low energy results to higher energies can give us a reliable background at dibaryon energy region. In the other reactions, however, we have to calculate it using some models. In the case of deuteron break-up processes, such as $\gamma d \rightarrow pn$ and $K^- d \rightarrow \Lambda n$, inevitable triangle diagram and off-shell deuteron-$N$-$N$ vertex prevent us from calculating the reliable background. (See Fig. 1.)

For $\pi$-deuteron ($\pi d$) elastic scattering we can calculate its background term with less ambiguity than for the other reactions. Two different approaches are available for calculating the background part; the Glauber model and the Faddeev equation. Especially the Glauber amplitudes can be calculated without free parameters.

In our earlier papers,$^3$ using the Glauber model with resonance term, we pointed out that effects of dibaryon resonance formation in $\pi d$ direct channel can be seen in the data: A dip structure found in $\pi d$ elastic scattering around 400 MeV/c can naturally be understood as the effect of the dibaryon resonances.

In this paper, we wish to discuss the following two subjects. Firstly we will

---

**Fig. 1.** Schematic graph of the deuteron disintegrated reactions including a triangle diagram.
check the validity of our background calculation by comparing the calculated result of the Glauber model with those of the Faddeev approach. It will be shown that the Glauber model provides a sound background at dibaryon energy regions. Secondly we will investigate all the world data in $\pi d$ elastic scattering at these energy regions and study effects of dibaryon resonances on the data.

§ 2. $\pi d$ elastic scattering amplitudes without dibaryon resonances

Glauber's multiple scattering model and the Faddeev equation are two reasonable theoretical approaches to describe the $\pi d$ elastic scattering. Some features of these models are as follows. The Glauber model, based on the eikonal approximation, is reliable at high energies but is doubtful at large angles. The Faddeev approach is grounded on a mathematically correct theory of nonrelativistic three-particle system. It may be sound even at large angles. Unfortunately it requires huge and tedious numerical calculation. It is difficult to include all $\pi N$ partial waves and to employ realistic deuteron wave function; higher partial waves are neglected and the deuteron form factor contains a parameter in this model. In the Glauber model these problems are out of question: We can easily utilize real $\pi N$ phase shift amplitudes and deuteron form factor.

The Glauber amplitudes, which we will calculate in the following, are essentially the same as derived by Michael and Wilkin. For the detailed expression, see Ref. 5). In the present analysis we further incorporate the Fermi motion inside deuteron, although we find it has little effect on the results in the next section.

Once the $\pi N$ amplitudes and the deuteron wave function are given, the Glauber model gives almost unique results without any free parameter. We examined Hamberstone's hom Moravcsik's fit to Gartenhaus and Reid's hard core function for the deuteron and also examined SACLA Y and CMU-LBL $\pi N$ phase shift analysis. We find that the calculated results are not so sensitive to the choice of these wave functions and $\pi N$ amplitudes. In the following we will show the results using Reid's hard core wave function and SACLAY $\pi N$ phase shift analysis.

We show a comparison of our calculations using the Glauber model with the results of the Faddeev approach. In Fig. 2 we show both Glauber and Faddeev calculations of the angular distributions at 245 and 290 MeV/c. Figure 3 shows the predictions of the analyzing tensor $T_{vq}$ at 245 MeV/c. Here we follow the Madison convention for the analyzing tensor. The spherical polarization $t_{vq}$ is given through the relation:

\footnote{There is an objective argument against the common saying that the eikonal approximation is applicable only to forward scattering. See Ref. 6).}
Dinucleon Resonances and π-Deuperon Elastic Scattering Amplitudes

Fig. 2. The differential cross section for elastic $\pi d$ scattering at 245 MeV/c. The three curves correspond to the Glauber model and the Faddeev approach. The Faddeev calculation is taken from Ref. 10.

Fig. 3. The angular dependence of the analyzing tensor for elastic $\pi d$ scattering at 245 MeV/c. The curves are the predictions of the Glauber model (the dashed line) and the Faddeev approach (the dot-dashed line). The Faddeev calculation is taken from Ref. 23.

For details, see Ref. 12).

The results of the two approaches agree well with each other up to large angles. In the Faddeev approach, multiple scattering and off-shell terms are taken into consideration, while the Glauber amplitudes consist of only the single and double scattering terms. It is very interesting that such different approaches give such similar results. Though we will not enter into this problem here, from the above argument, we can safely use the Glauber model as a reliable background.
in our program.

Finally, we briefly comment on the structure of the Argand diagram constructed from the calculated Glauber amplitudes. See Figs. 4(a) and (b). Here, following Hoshizaki,\textsuperscript{13)} we parametrize $S$ matrix for the partial waves with natural parity as follows:

$$ S' = \begin{pmatrix} \langle L = J - 1 | S' | L = J - 1 \rangle & \langle L = J - 1 | S' | L = J + 1 \rangle \\ \langle L = J + 1 | S' | L = J - 1 \rangle & \langle L = J + 1 | S' | L = J + 1 \rangle \end{pmatrix} = \begin{pmatrix} \sqrt{\eta} \ e^{i\theta} & 0 \\ 0 & \sqrt{\eta} \ e^{i\theta} \end{pmatrix} \begin{pmatrix} 1 - \frac{(\eta_+ / \eta_-) p_f^2}{\sqrt{1 - (\eta_+ / \eta_-) p_f^2}} & \frac{i \rho_f e^{i\theta_f}}{\sqrt{1 - (\eta_+ / \eta_-) p_f^2}} \\ \frac{i \rho_f e^{i\theta_f}}{\sqrt{1 - (\eta_+ / \eta_-) p_f^2}} & \sqrt{1 - (\eta_+ / \eta_-) p_f^2} \end{pmatrix} \begin{pmatrix} \sqrt{\eta} \ e^{i\theta} \\ 0 \end{pmatrix}, $$

Fig. 4. (a) Argand plots $\eta \ e^{i\theta}$, calculated from the Glauber amplitudes. The numerical values on the curves stand for incident pion momentum.
(b) Argand plots $\eta \ e^{i\theta}$, calculated from the Glauber amplitudes.
(c) Argand plots $\eta \ e^{i\theta}$, calculated from the Glauber amplitudes with addition of the dibaryon resonances.
(d) Argand plots $\eta \ e^{i\theta}$, calculated from the Glauber amplitudes with addition of the dibaryon resonances.
Though we do not yet add the resonant term, the Argand plots show resonance-like behavior. This is due to the $\pi N$ resonances, especially $J(1232)$, contained in the elementary $\pi N$ amplitudes. This strong energy dependence of $\pi d$ channel may be reflected in the other channels through a channel coupling mechanism.

\section{Effects of dibaryon resonances}

In this section we introduce dibaryon resonances and examine their effects on the differential cross section and the polarization parameters of $\pi d$ elastic scattering. We superimpose a dibaryon resonance formation term on the background which is calculated on the basis of the Glauber model discussed in \S\ 2.

The resonance formation terms are taken to be of Breit-Wigner type, which are the same form as in the previous analysis\(^5\) except that, this time, we have taken account of the relative phase of each resonance amplitude with respect to the background term. (See Appendix A.) We parametrize the partial wave amplitudes of dibaryon resonance formation as follows:

$$T'_{LL} = \frac{2M g_l g_v \Gamma_{\text{tot}}}{M^2 - s - iM\Gamma_{\text{tot}}} e^{i\gamma}$$

for $\sqrt{s} < M + 2\Gamma_{\text{tot}}$,

$$\times \left\{ \frac{1}{\sqrt{s} - M - 2\Gamma_{\text{tot}}} \right\}$$

for $\sqrt{s} > M + 2\Gamma_{\text{tot}}$,

where $M$ and $g_l$'s are the mass and the coupling to $L$ wave $\pi d$ system, respectively. The eigenphase-shift\(^{15}\) is denoted by $\gamma$ and $\sqrt{s}$ stands for the total energy in the c.m. system. The total width contains the threshold factor in the same form as in Ref. 5).

We take four dibaryon resonances, $B_1(2,17; 2^+)$, $B_1(2,22; 3^+)$, $B_1(2,32; 2^+)$ and $B_3(2,43; 0^+ \text{ or } 4^+)$, whose existence was suggested in the $pp$ phase shift analysis by Hoshizaki\(^{3,16}\). Here, after Yokosawa\(^{17}\) we write dinucleon resonance as $B_l^v$(Mass; $J^p$).

The resonance parameters, $M$, $\Gamma_{\text{tot}}$, $g_l$'s and $\gamma$, are determined to minimize $\chi^2$ through the least square fitting program SALS by Nakagawa and Oyanagi. Initial values of $M$ and $\Gamma_{\text{tot}}$ are taken from the results of the phase shift analysis of $pp$ scattering. In calculating $\chi^2$, we employ all the world data of the differential cross section of $\pi d$ elastic scattering in the energy region $2.1 < \sqrt{s} < 2.5$ GeV. Data list is given in Table I. The parameter space is restricted to the hypersurface where the unitarity condition is satisfied. The obtained resonance parameters are listed in Table II.
Table 1. List of references of the data used in the analysis.

<table>
<thead>
<tr>
<th>Author</th>
<th>Reaction</th>
<th>$P_{lab}(\text{MeV}/c)$</th>
<th>C. M. Angle</th>
<th>LAB.</th>
<th>EXP</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>K. Gabathuler</td>
<td>$\pi^+d$</td>
<td>245, 290, 328, 368, 408</td>
<td>20-140</td>
<td>SIN</td>
<td>C</td>
<td>SIN PR-80-11(1980)</td>
</tr>
<tr>
<td>A. V. Kravstov et al.</td>
<td>$\pi^+d$</td>
<td>438</td>
<td>25-170</td>
<td></td>
<td></td>
<td>Nucl. Phys. A322(1979), 439</td>
</tr>
</tbody>
</table>

EXP B—Bubble chamber, C—Counter experiments.

Table 2. List of the $\chi^2$ and the dibaryon resonance parameters. The number of the data points is 240.

A) Glauber without dibaryons: $\chi^2=2248$.

B) Glauber + four dibaryons (2', 2, 3 and 4'): $\chi^2=692$.

<table>
<thead>
<tr>
<th>$J^\pi$</th>
<th>Mass(GeV/$c^2$)</th>
<th>$\Gamma_{tot}$(MeV)</th>
<th>$\Gamma^1$(MeV)</th>
<th>$\Gamma^3$(MeV)</th>
<th>$\gamma$(degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2'</td>
<td>2.14</td>
<td>56</td>
<td>0.5</td>
<td>14.6</td>
<td>101</td>
</tr>
<tr>
<td>3'</td>
<td>2.26</td>
<td>181</td>
<td>6.3</td>
<td>0.4</td>
<td>-24</td>
</tr>
<tr>
<td>2'</td>
<td>2.29</td>
<td>139</td>
<td>4.1</td>
<td>---</td>
<td>-73</td>
</tr>
<tr>
<td>4'</td>
<td>2.51</td>
<td>122</td>
<td>0.0</td>
<td>2.3</td>
<td>64</td>
</tr>
</tbody>
</table>

C) Glauber + four dibaryons (2', 2, 3' and 0'): $\chi^2=718$.

<table>
<thead>
<tr>
<th>$J^\pi$</th>
<th>Mass(GeV/$c^2$)</th>
<th>$\Gamma_{tot}$(MeV)</th>
<th>$\Gamma^1$(MeV)</th>
<th>$\Gamma^3$(MeV)</th>
<th>$\gamma$(degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2'</td>
<td>2.14</td>
<td>54</td>
<td>0.5</td>
<td>14.5</td>
<td>101</td>
</tr>
<tr>
<td>3'</td>
<td>2.26</td>
<td>171</td>
<td>6.2</td>
<td>0.4</td>
<td>-25</td>
</tr>
<tr>
<td>2'</td>
<td>2.30</td>
<td>150</td>
<td>4.3</td>
<td>---</td>
<td>-81</td>
</tr>
<tr>
<td>0'</td>
<td>2.53</td>
<td>66</td>
<td>6.0</td>
<td>---</td>
<td>-60</td>
</tr>
</tbody>
</table>

$\Gamma^1 = \Gamma(DB \rightarrow \pi d(L=J+1))$, $\Gamma^3 = \Gamma(DB \rightarrow \pi d(L=J-L))$, ($\Gamma^3 = \Gamma(DB \rightarrow \pi d(L=J))$ for 2').
Fig. 5. (a). The angular distributions of \( nd \) elastic scattering at \( P_{cm} = 245, 290 \) and 328 MeV/c. The dashed lines show the results of the Glauber model, while the results with the dibaryon resonances in Table II are shown by solid lines.
(b). Same as (a) at 343, 368 and 408 MeV/c.
(c). Same as (a) at 438, 441 and 539 MeV/c.
(d). Same as (a) at 552 and 637 MeV/c.
We report in Fig. 5 the results of our calculation for the unpolarized differential cross section at various energies. In Fig. 6 the differential cross section at 180° is shown as a function of the incident π momentum. The solid (dashed) lines represent the results of the Glauber model with (without) four dibaryon resonances in Table II.

A rather deep minimum structure around 100° has been found at 368, 408, 438, 441 and 538 MeV/c. These structures are due to interference between the background term and the dibaryon resonances, \( B_i^4(2.22; 3^-) \) and \( B_i^4(2.32; 2^-) \). The backward enhancement is caused by the pion which is a decay product from the dibaryon resonances. Because of the large spatial extension of deuteron form factor and higher partial waves of \( \pi N \) amplitudes, the background term shows a very forward-peaked angular distribution. The dip structures resulting from the interference are expected to be clearer at higher energies because the background cross section is rapidly decreasing with increasing energy due to \( \Delta(1232) \) resonance.

Around \( B_i^4(2.17; 2^+) \) energy region the main contribution to \( \pi N \) amplitudes comes from the \( P \) wave due to \( \Delta(1232) \) resonance and then the forward peak is not so steep. Incorporating \( B_i^4(2.17; 2^+) \) we can get a rather good fit over all angles. It is dangerous, however, to confirm \( B_i^4(2.17; 2^+) \) in \( \pi d \) channel only from these featureless differential cross sections at 245 and 290 MeV/c. Fortunately the first measurement of tensor polarization \( t_{2\theta} \) was recently performed at 245 MeV/c\(^{18} \). The measured value, \( t_{2\theta} = -0.23 \pm 0.15 \) at 180°, could...
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not be explained by any available theoretical calculations. This data is not included in the above calculation of \( \chi^2 \)-minimization. We calculate \( t_{20} \) using the resonance parameters listed in Table II and find it to be \(-0.20\). See Fig. 7.

As for \( B_1(2.43; 0^+ \text{ or } 4^+) \), the situation is far from satisfactory. The bump structure at 700 MeV/c in Fig. 6 can be well explained by introducing the di-baryon resonance as pointed out first by Schroeder et al.\(^1\) We cannot obtain, however, good fits of angular distributions at 552 and 637 MeV/c. It is difficult to determine the spin and the parity of the resonance from these data only. Richer level structure besides the di-baryon resonances discussed here could exist at this energy region.

§ 4. Concluding remarks

In this paper we have tried to establish two main points: Firstly, that the Glauber model can provide us with reliable background amplitudes for \( \pi d \) elastic scattering; and, secondly, that most of the world data except around 600 MeV/c can be naturally explained if we take di-baryon resonances into consideration.

We cannot explain the remarkable agreement of the differential cross section and all the polarization tensors \( t_{20} \) between the Glauber model and the Faddeev approach. In order to solve this problem, more theoretical labour will be required concerning the scattering of the particle off bound state especially at large angles. Moreover the ambiguity in the Faddeev calculation is expected to be solved.\(^1\)\(^2\)\(^\text{20}\)

In the previous section we have found that all the world data of elastic \( \pi d \) scattering at \( B_1(2.17; 2^+) \), \( B_1(2.22; 3^+) \) and \( B_1(2.32; 2^+) \) energy regions can be described surprisingly well by taking these resonances into account, although it is difficult to distinguish clearly the role of \( B_1(2.17; 2^+) \) and \( B_1(2.32; 2^+) \) from the present data. Hereafter any phenomenological analysis of elastic \( \pi d \) scattering at these energy regions would lose its physical significance if di-baryon effects are not taken into account.

In order to see the effects of the di-baryon resonances on the amplitudes, we show the Argand diagrams in Fig. 4. In case of \( B_1(2.17; 2^+) \), the di-baryon formation amplitude is added destructively to the background due to its large eigenphase \( \gamma \). See the energy dependence and the shape of \( P \) wave of \( 2^+ \). \( G \) wave of \( 3^- \) comes to show strong energy dependence if we incorporate the di-baryons but \( D \) wave of \( 3^- \) is little altered. This is because \( \Gamma(DB \rightarrow \pi d(L=J+1)) \gg \Gamma(DB \rightarrow \pi d(L=J-1)) \) for \( B_1(2.22; 3^-) \). This suggests that \( B_1(2.22; 3^-) \) has large internal orbital angular momentum.

One of the most interesting results obtained here is that by the introduction of the di-baryon resonances we can get a very nice value of the polarization parameter \( t_{20} \) at 245 MeV/c which is in disagreement with all the other theoretical predictions. Now polarized deuteron targets are available at KEK\(^2\)\(^\text{1}\) and
Rutherford and the polarization of recoil deuteron can be measured. Measurements of polarization will play a very important role in studying dibaryon resonances and may open a new epoch into this field. The predictions of the polarization parameters at 408 MeV/c are displayed in Fig. 8. The dashed curves are the results of the Glauber model; the solid curves represent the results including the dibaryon effects.

For lack of sufficient data, we cannot determine the spin of \( B_1^*(2.43; 0^+ \text{ or } 4^+) \). There is a possibility that another dibaryon conceals itself in this region. Experimental data with high statistics around 500~700 MeV/c are desired.

It is important to search for higher dibaryon resonances because more fruitful level structure is expected if they are composed of six quarks. Though the differential cross section at 180° above 1 GeV/c is somewhat larger than the calculation without dibaryons, the experimental data are too poor to give an answer to the problem. Analysis of \( \pi d \) elastic scattering at these energy regions is of interest though our model will have to be improved to include recoiled deuteron effect there.

More efforts to search for dibaryon resonances in various channels are also required. We wish to suggest two interesting channels: \( \gamma d \to \pi d \) and \( K^- d \to K^- d \) because channels without deuteron break-up process can be treated with less ambiguity. We can apply to these processes the same framework as for \( \pi d \) elastic scattering.

So far we do not assume anything about dynamics of dibaryons in our analysis. We only require them to have Breit-Wigner behavior. For lack of knowledge about the dynamics, we could not help taking the mass \( M \) and the coupling \( g_\ell \) as free parameters. It is our future work to study and interpret these values on the basis of dynamical models.

Fig. 8. The angular dependence of spherical analyzing tensor at 408 MeV/c. The meaning of the lines is the same as in Fig. 5.
Acknowledgements

The authors would like to express their appreciation to K. Gabathuler and J. P. Egger for sending them their data before publication. They wish to thank T. Kobayashi for his critical reading of the manuscript. One of them (A. N.) is indebted to S. Furuichi and M. Namiki for helpful discussions and other (H. S.) acknowledges receipt of a Fellowship from Soryushi Shogakukai. The computer calculation for this work has been financially supported in part by Institute for Nuclear Study. They are grateful to the assistance of the staff of the KEK computer center where the earlier stage of the calculation was done.

Appendix A

—Problem of Relative Phase—

We will briefly explain here how to introduce resonances in our calculation. We discuss the case when a dibaryon has natural parity. The unnatural parity case is simpler. The dibaryon with natural parity can decay into the πd system with $L = J + 1$ and $J - 1$. In this case, $j$th partial $S$ matrix elements are as follows:

$$\begin{align*}
\langle s'_{r}(L = j + 1)S'_{r}(\pi d(\ell = J + 1)) &< \pi d(\ell = J + 1)|S'_{r}(\pi d(\ell = J - 1)) \rangle \\
&\langle \pi d(\ell = J - 1)|S'_{r}(\pi d(\ell = J + 1)) \rangle \\
&\langle \pi d(\ell = J + 1)|S'_{r}(\pi d(\ell = J - 1)) \rangle \\
&\langle \pi d(\ell = J - 1)|S'_{r}(\pi d(\ell = J + 1)) \rangle
\end{align*}
$$

Sym.

$$\langle s'_{r}(L = j + 1)S'_{r}(\pi d(\ell = J + 1)) \rangle = \sum_{s'_{r}} \langle s_{r}' \rangle,
$$

From the time-reversal invariance and unitarity, this matrix can be diagonalized by a real orthogonal matrix $U$:

$$S' = U\tilde{S}'U^\dagger
$$

$$U = \begin{bmatrix}
\langle \pi d(\ell = J + 1)|1 \rangle & \langle \pi d(\ell = J + 1)|2 \rangle & \cdots \\
\langle \pi d(\ell = J - 1)|1 \rangle & \langle \pi d(\ell = J - 1)|2 \rangle & \cdots \\
\langle pp|1 \rangle & \langle pp|2 \rangle & \cdots \\
\end{bmatrix}
$$

$$\tilde{S}' = e^{2i\gamma},
$$

$$S' = U\tilde{S}'U^\dagger
$$

$$U = \begin{bmatrix}
\langle \pi d(\ell = J + 1)|1 \rangle & \langle \pi d(\ell = J + 1)|2 \rangle & \cdots \\
\langle \pi d(\ell = J - 1)|1 \rangle & \langle \pi d(\ell = J - 1)|2 \rangle & \cdots \\
\langle pp|1 \rangle & \langle pp|2 \rangle & \cdots \\
\end{bmatrix}
$$
where $\gamma_i$'s, the famous eigenphase-shifts, are real. The physical $S$ matrix elements are written in terms of the eigenphase-shifts as

$$
\langle \pi d(L+1) | S | \pi d(L+1) \rangle = \sum_m \langle \pi d(L+1) | m \rangle e^{2i\gamma_m} \langle m | \pi d(L+1) \rangle,
$$

$$
\langle \pi d(L+1) | S \rangle | \pi d(L-1) \rangle = \sum_m \langle \pi d(L+1) | m \rangle e^{2i\gamma_m} \langle m | \pi d(L-1) \rangle.
$$

Resonances in multichannel systems may have poles in the eigenchannel. When the resonance occurs in the first eigenchannel, we get the eigenphase-shift in the change

$$
\tilde{S}_1 = e^{2i\gamma_1} \Rightarrow e^{2i\gamma_1} e^{2i\alpha_0},
$$

where

$$
e^{2i\alpha_0} = 1 + i \frac{2M\Gamma}{M^2 - s - iM\Gamma}.
$$

A simple calculation gives us

$$
\langle \pi d(L+1) | S \rangle | \pi d(L+1) \rangle = 1 + i \left\{ \langle \pi d(L+1) | F | \pi d(L+1) \rangle + \frac{2M\langle \pi d(L+1) | 1 \rangle | 1 \rangle \langle 1 | \pi d(L+1) \rangle \Gamma}{M^2 - s - iM\Gamma} e^{2i\gamma} \right\},
$$

$$
\langle \pi d(L+1) | S \rangle | \pi d(L-1) \rangle = 1 + i \left\{ \langle \pi d(L+1) | F | \pi d(L-1) \rangle + \frac{2M\langle \pi d(L+1) | 1 \rangle | 1 \rangle \langle 1 | \pi d(L-1) \rangle \Gamma}{M^2 - s - iM\Gamma} e^{2i\gamma} \right\},
$$

where

$$
F = \sum_m | m \rangle e^{2i\gamma_m} - \frac{1}{i} \langle m |.
$$

The terms $\langle | F \rangle$ represent the background. Resonance formation amplitudes have the eigenphase of the nonresonant scattering in the eigenchannel, $e^{2i\gamma}$, which is independent of the initial or final state.
Factorization of the resonance formation amplitudes assumed in § 3 is justified in this scheme. In § 3, we wrote
\[ g_{L=J=\pm 1} = \langle \pi d | L = J \pm 1 \rangle | 1 \rangle. \]  
(A.6)

**Appendix B**

---Effect of Fermi Motion---

In this paper we treat \( \pi d \) elastic scattering amplitude based on the Glauber model in the incident pion momentum between 240 and 640 MeV/c, where there are many resonances in \( \pi N \) channel. Since the \( \pi d \) amplitude would depend largely on energy, it is necessary to introduce an effect of motion of nucleons in the deuteron.

In the deuteron rest frame, the momentum distribution function, \( G(p) \), of the nucleon is given by the deuteron wave function \( \psi(r) \), i.e.,
\[ G(p) = (2\pi)^{-3/2} \int \psi(r) e^{ipr} d^3r, \]
where we use Moravcsik’s fit to Gartenhaus’ wave function and neglect the \( D \) wave for simplicity. In the calculation of the Glauber amplitude, the \( \pi N \) amplitude \( f_{\pi N} \) is averaged according to the following prescription:
\[ f_{\pi N}^M(s, t) = \int f_{\pi N}(\tilde{s}, t) |G(p)|^2 d^3p, \]
where \( \tilde{s} \) is the square of the invariant mass of the incident pion and the moving nucleon.

![Fig.9. The \( \pi d \) total cross section. The solid (dashed) lines show the results of the Glauber model with (without) the Fermi motion.](https://academic.oup.com/ptp/article-abstract/65/1/266/1856021)
We calculate the total cross section using the Glauber amplitude with (without) the Fermi motion and show them in solid (dashed) lines in Fig. 9. The effect of the Fermi motion is seen to be at most five percent and less scattering angle dependence.

Appendix C

---Glauber Amplitude in the Helicity State---

The s-channel helicity amplitudes of the background term, $f_{\mu}^{BG}$, are given by the Glauber amplitudes in the laboratory frame, $F_{\mu n'}$, through the following transformation:

$$f_{\mu}^{BG} = \sum_{n, m} \langle \mu | L | m' \rangle F_{\mu n'}^{G} \langle m | R | \nu \rangle \sqrt{\frac{dQ_{\text{cm}}}{dQ_{\text{lab}}}},$$

where

$$\langle \mu | L | m \rangle = \sum_{m'} e^{-i\theta_{m'}} d_{nm'}(\xi) \delta_{nm'} \left(0, -z, \frac{\pi}{2}\right),$$

$$\langle m | R | \nu \rangle = \delta_{nm} \left(-\frac{\pi}{2}, \xi', -\phi\right),$$

and $\xi$ stands for the angle between the incident $\pi$ momentum and the recoiled deuteron in the laboratory system.

References

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