QCD Effects to Parity-Violating $\Delta S = \Delta C = 0$ Nonleptonic Interactions in the Kobayashi-Maskawa Model

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We calculate the QCD corrections for parity-violating $\Delta S = \Delta C = 0$ nonleptonic weak interactions in the Kobayashi-Maskawa model. The effective Hamiltonians are evaluated for its isospin $\Delta I = 0, 1$ and 2 parts.

§ 1. Introduction

The recent experimental results on parity-violating nuclear forces show some discrepancies between experimental values and theoretical estimations. The experimental values of the asymmetry $A_L$ of proton-proton (-nucleus) scattering for polarized beams and the asymmetry $A_{\gamma}$ of $\gamma$ rays emitted in the polarized thermal neutron-deuteron capture have been found to be several times as large as the simple estimates by the standard theories. In order to make clear whether or not the inconsistency exists, it is necessary to carry out more detailed theoretical estimations.

The unified gauge theory which combines the quantum chromodynamics (QCD) and Weinberg-Salam's $SU(2) \times U(1)$ gauge model seems most promising in the fundamental interactions. As to the weak interactions, the QCD corrections to the nonleptonic decays have given a possible explanation of the $\Delta I = 1/2$ enhancement. Moreover in the parity-violating nuclear interactions, these QCD corrections have improved the results. All of these calculations have assumed the four quark model, while the evidence of $Y$ and $\tau$ particles seems to have made the six quark model plausible. The QCD corrections with the six quark model have been calculated to the CP-violation of the nonleptonic decays and to the charmed mesons decays with the favourable effects. The effects on the parity-violating nuclear interactions have not been studied yet.

In this paper we calculate the QCD corrections to all of the isospin parts $\Delta I = 0, 1$ and 2 of the parity-violating nuclear interactions, taking the six quark model of Kobayashi-Maskawa. In calculating the QCD corrections, we divide the virtual momentum into four regions in the same way as Ref. 10).
In the next section we present the Hamiltonians and operator bases in the six quark model. The effective Hamiltonians to the parity-violating nuclear interactions are calculated in § 3. The results and remarks are given in the final section.

§ 2. Operator bases in six quark model

To start with, we write down the basic weak interaction Hamiltonian in the six quark model,

\[ H_w = \frac{G_F}{2\sqrt{2}} (\{J^w, J^w\} + \{J^w, J^{w\alpha}\}), \]

\[ J^w = \bar{u} \gamma^\mu (1 - \gamma_5) d' + \bar{c} \gamma^\mu (1 - \gamma_5) s' + \bar{\ell} \gamma^\mu (1 - \gamma_5) b', \]

\[ = (\bar{u}d')_L + (\bar{c}s')_L + (\bar{\ell}b')_L, \]

\[ \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \]

\[ J^w = J^w - 2 \sin^2 \theta_w J^{w\alpha}, \]

\[ J^w = \frac{1}{2}(\bar{u}u + \bar{c}c + \bar{\ell}\ell - \bar{d}d - \bar{s}s - \bar{b}b)_L, \]

\[ J^{w\alpha} = \frac{2}{3}(\bar{u}u + \bar{c}c + \bar{\ell}\ell)_V - \frac{1}{3}(\bar{d}d + \bar{s}s + \bar{b}b)_V, \]

where \( G_F \) is the Fermi coupling constant, \( c_i = \cos \theta_i, s_i = \sin \theta_i \) (\( i = 1, 2, 3 \)), \( \delta \) is the CP-violation phase and \( (\bar{q}q)_V \equiv \bar{q}\gamma_\mu q, \) etc. From Eq. (1) the parity violating nuclear interaction Hamiltonians are obtained as follows:

\[ H^{w\alpha}_V = H^{w\alpha}_V + H^{w\alpha}_E, \]

\[ H^{w\alpha}_V = -\frac{G_F}{2\sqrt{2}} \left[ c_1^2 \{(\bar{u}d), (\bar{u}u)\} + s_1^2 c_3^2 \{(\bar{s}u), (\bar{s}s)\} \right. \]

\[ + s_1^2 s_3^2 \{(\bar{u}b), (\bar{u}u)\} + s_1^2 c_2^2 \{(\bar{d}c), (\bar{c}d)\} + s_1^2 s_2^2 \{(\bar{d}t), (\bar{t}d)\} \left. \right|_{\alpha = \alpha V}, \]

\[ H^{w\alpha}_E = -\frac{G_F}{4\sqrt{2}} \left[ (1 - 2x)\{(\bar{u}u - \bar{d}d), (\bar{u}u - \bar{d}d)\} \right. \]

\[ + (1 - 2x)\{(\bar{u}u - \bar{d}d), (\bar{c}c + \bar{\ell}\ell - \bar{s}s - \bar{b}b)\} \]

\[ - \frac{2}{3} x\{(\bar{u}u + \bar{d}d), (\bar{u}u + \bar{d}d)\} - \frac{2}{3} x\{(\bar{u}u + \bar{d}d), (\bar{c}c + \bar{\ell}\ell - \bar{s}s - \bar{b}b)\} \]
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\[ + \left( 1 - \frac{8}{3} x \right) \left( (c\bar{c} + i\tau), (\bar{u}u - \bar{d}d) \right) \]

\[ - \left( 1 - \frac{4}{3} x \right) \left( (s\bar{s} + \bar{b}b), (\bar{u}u - \bar{d}d) \right) \] \(V_A\),

(2)

where \( \{ (\bar{u}d), (\bar{d}u) \}_V \cdot \lambda = (\bar{u}\gamma_\nu d)(d\gamma_\nu^* u) + (d\gamma_\nu^* u)(\bar{u}\gamma_\nu d) \), etc. and \( x = \sin^2 \theta_w \).

Here we have shown only the terms relevant in our calculation.

For calculating the QCD correction to Eq. (2), we prepare the following operator bases which are classified by the isospin property:

\[ \Delta I = 2 \]

\[ O_1 = \{ (\bar{u}d), (\bar{d}u) \} + \{ (\bar{d}u), (\bar{u}d) \} - \{ (\bar{u}u - \bar{d}d), (\bar{u}u - \bar{d}d) \}, \]  

(3)

\[ \Delta I = 1 \]

\[ O_2 = \{ (\bar{u}d), (\bar{d}u) \} - \{ (\bar{d}u), (\bar{u}d) \}, \]

\[ O_3 = \{ (\bar{u}u - \bar{d}d), (\bar{u}u + \bar{d}d) \}, \]

\[ O_4 = \{ (\bar{u}u - \bar{d}d), (\bar{s}s + \bar{b}b) \}, \]

\[ O_5 = \{ (\bar{u}u - \bar{d}d), (\bar{c}\bar{c} + i\tau) \}, \]

\[ O_6 = \{ (\bar{u}s), (\bar{s}u) \} - \{ (\bar{s}d), (\bar{d}s) \}, \]

\[ O_7 = \{ (\bar{u}c), (\bar{c}u) \} - \{ (\bar{c}d), (\bar{d}c) \}, \]

\[ O_8 = \{ (\bar{u}b), (\bar{b}u) \} - \{ (\bar{b}d), (\bar{d}b) \}, \]

\[ O_9 = \{ (\bar{u}t), (\bar{t}u) \} - \{ (\bar{t}d), (\bar{d}t) \}. \]

(4)

\[ \Delta I = 0 \]

\[ O_{10} = 2\{ (\bar{u}d), (\bar{d}u) \} + 2\{ (\bar{d}u), (\bar{u}d) \} + \{ (\bar{u}u - \bar{d}d), (\bar{u}u - \bar{d}d) \}, \]

\[ O_{11} = \{ (\bar{u}u + \bar{d}d), (\bar{u}u + \bar{d}d) \}, \]

\[ O_{12} = \{ (\bar{u}u + \bar{d}d), (\bar{s}s) \}, \]

\[ O_{13} = \{ (\bar{u}u + \bar{d}d), (\bar{c}\bar{c}) \}, \]

\[ O_{14} = \{ (\bar{u}u + \bar{d}d), (\bar{b}b) \}, \]

\[ O_{15} = \{ (\bar{u}u + \bar{d}d), (\bar{\tau}) \}, \]

\[ O_{16} = \{ (\bar{u}s), (\bar{s}u) \} + \{ (\bar{s}d), (\bar{d}s) \}, \]

\[ O_{17} = \{ (\bar{u}c), (\bar{c}u) \} + \{ (\bar{c}d), (\bar{d}c) \}, \]

\[ O_{18} = \{ (\bar{u}b), (\bar{b}u) \} + \{ (\bar{b}d), (\bar{d}b) \}, \]

\[ O_{19} = \{ (\bar{u}t), (\bar{t}u) \} + \{ (\bar{t}d), (\bar{d}t) \}, \]

\[ O_{20} = \{ (\bar{s}s + \bar{c}\bar{c} + \bar{b}b + i\tau), (\bar{s}s) \}, \]

\[ O_{21} = \{ (\bar{s}s + \bar{c}\bar{c} + \bar{b}b + i\tau), (\bar{c}\bar{c}) \}, \]

\[ O_{22} = \{ (\bar{s}s + \bar{c}\bar{c} + \bar{b}b + i\tau), (\bar{b}b) \}. \]
In terms of these operators, we rewrite Eq. (2) as

\[ H_{PV}^Q = - \frac{G_F}{2\sqrt{2}} \left[ \frac{c_1^2}{6} (O_+ + O_{10})_{V+A} + \frac{s_1^2 c_2^2}{2} (O_6 + O_{16})_{V+A} \right. \\
+ \frac{s_1^2 s_2^2}{2} (O_9 + O_{18})_{V,A+AV} + \frac{s_1^2 c_3^2}{2} (O_{17} - O_{17})_{V,A+AV} \\
+ \left. \frac{s_1^2 s_3^2}{2} (O_9 - O_9)_{V,A+AV} \right]. \]

\[ H_{PV}^Q = - \frac{G_F}{4\sqrt{2}} \left[ \frac{1}{3} \left( 2O_1 - O_{10} \right)_{V+A} + (1 - 2x)(O_6 - O_6)_{V,A} \right. \\
- \frac{2}{3} x \left[ O_{24} + (O_{13} - O_{12})_{V,A} + (O_{15} - O_{14})_{V,A} \right] \\
+ \left. \left( 1 - \frac{8}{3} x \right) O_{54} - \left( 1 - \frac{4}{3} x \right) O_{44} \right]. \] (6)

**§ 3. QCD corrections to the four-body quark operators**

Now we evaluate the QCD corrections to the parity violating weak interactions of Eq. (6). Following Ref. 10), we divide the whole range of integration over the virtual momenta \( Q \) into the following four intervals:

1. **Region I:** \( M_w^2 > Q^2 > m_t^2 \),
2. **Region II:** \( m_t^2 > Q^2 > m_b^2 \),
3. **Region III:** \( m_b^2 > Q^2 > m_e^2 \),
4. **Region IV:** \( m_e^2 > Q^2 > \mu^2 \),

where \( M_w, m_t, m_b \) and \( m_e \) are the weak boson mass, \( t-, b-, c- \) quark mass respectively and \( \mu \) is the infrared cutoff.

In order to derive the effective parity-violating nuclear interaction Hamiltonian, we expand the interaction Hamiltonian as follows:\(^{12}\)

\[ H_{PV}^Q = - \frac{G_F}{2\sqrt{2}} \sum_a A_a O_a . \] (8)

By solving the renormalization group equations, the corrected interaction Hamiltonian is obtained as

\[ H_{\text{eff}}^{PV} = - \frac{G_F}{2\sqrt{2} \alpha_1 M_w^2} \sum_{a,b,c,d} A_a \left[ \frac{\alpha_1(m_e^2)}{\alpha_1(M_w^2)} \right]_{ab} \left[ \frac{\alpha_2(m_b^2)}{\alpha_2(m_t^2)} \right]_{bc} \left[ \frac{\alpha_3(m_c^2)}{\alpha_3(m_e^2)} \right]_{cd} \]
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$$
\times \left[ \left( \frac{\alpha_s(m_i^2)}{\alpha_s(m_j^2)} \right)^\nu \right]_{\nu_\lambda} \left( \frac{\alpha_s(m_j^2)}{\alpha_s(m_i^2)} \right)^{\nu_\lambda} \tilde{O}_\nu ,
$$

where $\alpha_s(m_j^2)$ is the strong coupling constant at the $j$-particle mass in the region $i$ and $D_i$ is a matrix which is proportional to the anomalous dimension of the four quark operators in the region $i$. Here $D_i$'s are determined by calculating the diagrams of Figs. 1(a) and (b) which give the lowest order QCD corrections to the local four-body quark operators. Calculations are made by eliminating the $(t, t)$, $(t, b)$ and $(t, b, c)$ quarks from the operator bases in regions II, III and IV respectively. The details are the following:

a) $\Delta I = 2$ operators

We begin with the $\Delta I = 2$ part. By using the dimensional regularization technique, the corrected operators in region $i$ evaluated from the diagram of Fig. 1 are

$$
\left\{ \frac{1}{16 \pi^2} g^2 \Gamma(\epsilon) (-2 C_2(R) + M_{\mu = 2}) \right\}
\begin{pmatrix}
O_{1VAV}\ \\
O_{1VAV+AV}
\end{pmatrix},
$$

$$
M_{\mu = 2} = \begin{pmatrix}
0 & 6 \\
\frac{3}{2} & -2
\end{pmatrix},
$$

where $g$ is the quark-gluon running coupling constant, \((\bar{q}_1 q_2, (\bar{q}_3 q_4))^{1\lambda}\ = \sum\{(\bar{q}_1 t a q_2)_{\lambda}, (\bar{q}_3 t a q_4)_{\lambda}\}, Tr(t a t b) = \delta_{a b}, C_2(R) = (N^2 - 1)/2N \text{ and } \epsilon = (4 - D)/2$. From Eq. (10) and the renormalization equation technique, the renormalized operators $O'$ become

$$
\begin{pmatrix}
O'_{1VAV}\ \\
O'_{1VAV+AV}
\end{pmatrix} = \left[ \left( \frac{\alpha_s(m_i^2)}{\alpha_s(m_j^2)} \right)^{\nu_\lambda} \left( \frac{\alpha_s(m_j^2)}{\alpha_s(m_i^2)} \right)^{\nu_\lambda} \right]_{\nu_\lambda} \begin{pmatrix}
O_{1VAV}\ \\
O_{1VAV+AV}
\end{pmatrix}
$$

*) The $\Delta I = 2$ operators have the same $M_{\mu = 2}$ in all regions.
where $D_i = -(1/\beta_{0_i}) M_i^{\mu_2} \beta_0' = (32 - 2f)/3$ and the relation between flavour number $f$ and region number $i$ is given by $f = 7 - i$. In proceeding the calculation, we diagonalize the matrix $D_i$ by a transformation matrix $V_i$ as

\[ V_i D_i V_i^{-1} = X_i, \quad X_i = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}, \quad V_i = \begin{pmatrix} \frac{2}{3} & 1 \\ -1 & 1 \end{pmatrix} \]

and Eq. (11) becomes

\[
\begin{align*}
(O_{1,VA+AV}^{i}) \times V_i^{-1} \left( \begin{array}{c} a_3(m_3^2) \\ a_3(m_2^2) \end{array} \right) x_i 
+ V_i^{-1} \left( \begin{array}{c} a_2(m_3^2) \\ a_2(m_2^2) \end{array} \right) \right) 
\times V_i^{-1} \left( \begin{array}{c} a_3(m_3^2) \\ a_3(m_2^2) \end{array} \right) \right) 
\end{align*}
\]

Then we obtain

\[
O_{1,VA+AV} = (\frac{2}{3} a - \frac{1}{3} b) O_{1,VA+AV} + (a - b) O_{1,VA+AV}.
\]

Using the Fierz identity, Eq. (14) becomes

\[
O_{1,VA+AV} = a O_{1,VA+AV}.
\]

If the following values are assumed,

\[
\begin{align*}
M_w &= 80 \text{ GeV}, \quad m_t = 25 \text{ GeV}, \quad m_b = 4.5 \text{ GeV}, \\
m_c &= 1.5 \text{ GeV}, \quad \mu = 0.7 \text{ GeV}, \quad a(\mu^2) = 1,
\end{align*}
\]

one obtains

\[
O_{1,VA+AV} = 0.6127 O_{1,VA+AV}.
\]

b) $\Delta I = 1$ operators

We evaluate the QCD correction to the $\Delta I = 1$ part; the operators from $O_2$ to $O_9$. The QCD corrected operators for $O_j$ ($j = 2, \cdots, 9$) are evaluated in the same way as those for $O_i$. 
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\[
\begin{pmatrix}
O_{JVA} \\
O_{JAV} \\
O_{JVA}^2 \\
O_{JAV}^2
\end{pmatrix}
+ \frac{g^2}{16\pi^2} \Gamma(e) \begin{pmatrix}
-2C_2(R) \\
O_{JAV} \\
O_{JVA} \\
O_{JAV}^2
\end{pmatrix}
+ N(j, i)
\begin{pmatrix}
O_{JVA} \\
O_{JAV} \\
O_{JVA}^2 \\
O_{JAV}^2
\end{pmatrix}
\}
\]

\[
R_{AV} \equiv (O_0 + O_4 + O_5)_{AV},
\]

where \( N(j, i) \) are the 4 × 5 matrices shown in Appendix 1. We can rewrite Eq. (18) as follows:

\[
\begin{pmatrix}
O_{JVA} \\
O_{JAV} \\
O_{JVA}^2 \\
O_{JAV}^2
\end{pmatrix}
\begin{pmatrix}
1 + \frac{g^2}{16\pi^2} \Gamma(e)(-2C_2(R) + M^{n-1})
\end{pmatrix}
\begin{pmatrix}
O_2 \\
O_3 \\
\vdots \\
O_6
\end{pmatrix}
\]

where \( M^{n-1} \) is the 32 × 32 matrix obtained from \( N(j, i) \). Therefore the renormalized operators \( O_j' \) are expressed in the same way as the \( ΔI = 2 \) operators by

\[
O_j' = \begin{pmatrix}
O_{JVA} \\
O_{JAV} \\
O_{JVA}^2 \\
O_{JAV}^2
\end{pmatrix}
\begin{pmatrix}
O_2 \\
O_3 \\
\vdots \\
O_6
\end{pmatrix}
\]

\[
D_j = -\frac{1}{\beta^I_0} M^{n-1}
\]

By diagonalizing the matrix \( M^{n-1} \) and using Eq. (16), the results are

\[
\begin{pmatrix}
O_2' \\
O_3' \\
O_4' \\
O_5'
\end{pmatrix}
= T(ΔI = 1)
\begin{pmatrix}
O_2 \\
O_3 \\
O_4 \\
O_5
\end{pmatrix}
\]

where the numerical values of the matrix \( T(ΔI = 1) \) are shown in Table I. Note that only the four operators \( O_2, O_3, O_4, \) and \( O_5 \) remain in our calculation.

c) \( ΔI = 0 \) operators

Finally we give the QCD corrections for the \( ΔI = 0 \) part: The operators from
Table I. The value of the $T(M=1)$.

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<td>0.010</td>
<td>0.110</td>
<td>-0.300</td>
<td>-0.656</td>
<td>0.230</td>
<td>-2.096</td>
<td>3.886</td>
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<td>0.045</td>
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<td>-0.371</td>
<td>0.045</td>
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<td>-0.371</td>
<td>1.390</td>
<td>-0.094</td>
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<td>-0.033</td>
<td>0.117</td>
<td>0.240</td>
<td>-0.695</td>
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<td>0.117</td>
<td>-0.012</td>
<td>0.003</td>
<td>-0.033</td>
<td>0.117</td>
<td>-0.695</td>
<td>0.240</td>
<td>-2.207</td>
</tr>
</tbody>
</table>
QCD Effects to Parity-Violating $\Delta S=\Delta C=0$ Nonleptonic Interactions

$O_{i0}$ to $O_{i3}$ in the ways analogous to the case $\Delta I=1$ part (see Appendix 2). After the diagonalization of the matrix $M^{\mu=0}$ the final results are

$$
\begin{bmatrix}
O_{i0} \\
O_{i1} \\
O_{i2} \\
O_{i6}
\end{bmatrix} = T(\Delta I=0) 
\begin{bmatrix}
O_{i0} \\
O_{i1} \\
O_{i2} \\
O_{i6}
\end{bmatrix} .
$$

(22)

The numerical results of $T(\Delta I=0)$ are given in Table II.

§ 4. Results and remarks

From the results of §§ 2 and 3, we obtain the effective Hamiltonian for the parity-violating nuclear interactions as follows:

$$H_{\text{eff}}^{\Delta I=2} = H_{\text{eff}}^{\Delta I=2} + H_{\text{eff}}^{\Delta I=1} + H_{\text{eff}}^{\Delta I=0},$$

$$H_{\text{eff}}^{\Delta I=2} = -\frac{G_F}{12\sqrt{2}} \left[ 0.613(c_1^2 - 1 + 2x) O_{1V+1V} ight],$$

$$H_{\text{eff}}^{\Delta I=1} = -\frac{G_F}{4\sqrt{2}} \left[ (0.089 s_1^2 c_3^2 + 0.028 - 0.030x) O_{3V+1V} ight. + (-0.021 s_1^2 c_3^2 - 0.066 - 0.906x) O_{3V+1V} + (0.245 s_1^2 c_3^2 + 0.072 + 1.837x) O_{5V+1V} + (-0.742 s_1^2 c_3^2 - 0.289 + 0.078x) O_{5V+1V} + (0.089 s_1^2 c_3^2 - 1.269 + 2.563x) O_{4V+1V} + (-0.021 s_1^2 c_3^2 - 1.302 + 1.687x) O_{4V+1V} + (0.245 s_1^2 c_3^2 + 2.123 - 2.265x) O_{6V+1V} + (-0.742 s_1^2 c_3^2 + 1.782 - 4.024x) O_{6V+1V} + s_1^2 c_3^2 \{1.296 (O_{6V+1V} + O_{6V+1V}) - 2.051 (O_{6V+1V} + O_{6V+1V}) \}],$$

$$H_{\text{eff}}^{\Delta I=0} = -\frac{G_F}{12\sqrt{2}} \left[ (c_1^2 + \frac{1}{2} - x) \{1.296 (O_{10V+1V} + O_{10V+1V}) - 2.051 (O_{10V+1V} + O_{10V+1V}) + (0.510 c_1^2 + 0.242 s_1^2 c_3^2 + 0.255 - 0.529x) O_{10V+1V} + (-0.122 c_1^2 - 0.059 s_1^2 c_3^2 - 0.061 + 0.125x) O_{10V+1V} + (1.406 c_1^2 + 0.767 s_1^2 c_3^2 + 0.703 - 1.453x) O_{10V+1V} + (-4.176 c_1^2 - 1.950 s_1^2 c_3^2 - 2.062 + 4.465x) O_{10V+1V} \}. \right]$$
| Table II  The value of the $T(Ml=0)$. |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 1.390 -0.094 1.079 -3.130 | 0.255 -0.061 0.703 -2.086 | 0.257 -0.073 0.841 -2.120 | 0.0 0.0 0.0 0.0 |
| -0.094 1.390 -3.130 1.079 | 0.255 -0.061 0.703 -2.086 | 0.257 -0.073 0.841 -2.120 | 0.0 0.0 0.0 0.0 |
| 0.240 -0.695 4.187 -2.207 | -0.072 0.015 0.190 0.662 | -0.072 0.018 0.229 0.670 | 0.0 0.0 0.0 0.0 |
| -0.695 0.240 -2.207 4.187 | -0.072 0.015 0.190 0.662 | -0.072 0.018 0.229 0.670 | 0.0 0.0 0.0 0.0 |
| 0.0 0.0 0.0 0.0 | 1.437 -0.107 1.215 -3.407 | 0.047 -0.016 0.162 -0.284 | 0.0 0.0 0.0 0.0 |
| 0.0 0.0 0.0 0.0 | -0.048 1.377 -2.993 0.801 | 0.047 -0.016 0.162 -0.284 | 0.0 0.0 0.0 0.0 |
| 0.0 0.0 0.0 0.0 | 0.307 -0.712 4.374 -2.752 | 0.068 -0.020 0.224 -0.554 | 0.0 0.0 0.0 0.0 |
| 0.0 0.0 0.0 0.0 | -0.628 0.223 -2.020 3.642 | 0.068 -0.020 0.224 -0.554 | 0.0 0.0 0.0 0.0 |
| 0.0 0.0 0.0 0.0 | 0.009 0.002 -0.023 0.009 | 1.384 -0.112 1.256 -3.079 | 0.0 0.0 0.0 0.0 |
| 0.0 0.0 0.0 0.0 | -0.009 0.002 -0.023 0.009 | -0.100 1.373 -2.944 1.129 | 0.0 0.0 0.0 0.0 |
| 0.0 0.0 0.0 0.0 | 0.022 -0.005 0.060 -0.179 | 0.251 -0.655 3.864 -2.261 | 0.0 0.0 0.0 0.0 |
| 0.0 0.0 0.0 0.0 | 0.022 -0.005 0.060 -0.179 | -0.684 0.280 -2.530 4.133 | 0.0 0.0 0.0 0.0 |
| 0.0 0.0 0.0 0.0 | 0.040 -0.010 0.112 -0.325 | 0.020 0.075 -0.504 -0.991 | 1.590 -0.094 1.079 -5.130 |
| 0.0 0.0 0.0 0.0 | 0.040 -0.010 0.112 -0.325 | 0.020 0.075 -0.504 -0.991 | -0.094 1.390 -3.130 1.079 |
| 0.0 0.0 0.0 0.0 | -0.011 0.002 -0.030 0.104 | -0.008 -0.021 0.190 0.040 | 0.240 -0.695 4.187 -2.207 |
| 0.0 0.0 0.0 0.0 | -0.011 0.002 -0.030 0.104 | -0.008 -0.021 0.190 0.040 | -0.695 0.240 -2.207 4.187 |
QCD Effects to Parity-Violating $\Delta S = \Delta C = 0$ Nonleptonic Interactions

\[(0.514c_1^2 + 0.121s_1^2c_3^2 + 0.257 + 2.254x)O_{12VA} + (-0.147c_1^2 + 0.451s_1^2c_3^2 - 0.073 - 0.076x)O_{12AV} + (1.683c_1^2 - 3.505s_1^2c_3^2 + 0.841 + 0.846x)O_{12VA} + (-4.240c_1^2 - 0.544s_1^2c_3^2 - 2.120 - 1.918x)O_{12AV} + s_1^2c_3^2(3.889(O_{16VA} + O_{16AV}) - 6.153(O_{16VA} + O_{16AV}))\]

Now we discuss the dependence of the effective Hamiltonian on the infrared cutoff parameter $\mu$. In the leading logarithmic approximation, the quark-gluon running coupling constant is

\[a(Q^2) = \frac{12\pi}{33 - 2f} \frac{1}{\ln Q^2/\Lambda^2}\]

with

\[\Lambda^2 = \mu^2 \exp\left(-\frac{12\pi}{33 - 2f} \frac{1}{a(\mu^2)}\right).

The value of $\Lambda^2$ suggested by the analyses of experiments and theories is roughly within the range $0.1 \sim 0.01$ GeV$^2$. We have taken here $\Lambda^2 \sim 0.08$ GeV$^2$ ($\mu = 0.7$ GeV in Eq. (16)). We should remark that the calculation is fairly insensitive to the value of $\Lambda^2$. For example, if we change $\Lambda^2$ to $\sim 0.01$ GeV$^2$ ($\mu = 0.3$ GeV), the factor 0.613 in the effective Hamiltonian of $\Delta I = 2$ part (Eq. (23a)) changes to 0.596, and the magnitudes of other numerical factors in Eqs. (23b, c) increase only within 8%. The effective Hamiltonian (Eq. (23)) will be used to estimate the parity violating $NN\pi, NN\rho$ and $NNe\gamma$-coupling constants in our subsequent paper.

Finally we should like to compare our result with the calculation by the four quark model. The magnitudes of the correction factors in the effective Hamiltonian in the four quark model are about 5% smaller than those in the six quark model. Although the four quark model seems to give roughly the same result as that in the six quark model at least as to the parity-violating nuclear interactions, the six quark model seems to be plausible from the evidence of $J$ and $\tau$ particles.

Acknowledgements

The authors would like to thank Professor M. Konuma and the participants of the study meeting on 'Parity Violating Nuclear Forces' held at Research Institute for Fundamental Physics, Kyoto University, on February 14~16, 1980.
for comments and discussions. They also wish to thank Professor M. Yonezawa and Dr. T. Hayashi for their encouragement and discussions.

Appendix 1

The matrices $N(j, i)$ in Eq. (18) are

$$
N(1, k) = \begin{pmatrix}
0 & 0 & 0 & 6 & 0 \\
0 & 0 & 6 & 0 & 0 \\
0 & \frac{4}{3} & -\frac{9}{2} & \frac{5}{2} & 0 \\
\frac{4}{3} & 0 & \frac{5}{2} & -\frac{9}{2} & 0
\end{pmatrix} \equiv \begin{pmatrix}
0 \\
0 \\
N_1 \\
0
\end{pmatrix},
$$

$$
N(2, k) = \begin{pmatrix}
\frac{4}{3} \\
\frac{4}{3} \\
\frac{4}{9} \\
\frac{4}{9}
\end{pmatrix} N_1,
$$

$$
N(3, 1) = N(3, 2) = N(4, 1) = \begin{pmatrix}
0 \\
0 \\
N_1, \frac{2}{3} \\
N_1, \frac{2}{3}
\end{pmatrix},
$$

$$
N(3, 3) = N(3, 4) = N(4, 2) = \begin{pmatrix}
0 \\
0 \\
N_1, \frac{1}{3} \\
N_1, \frac{1}{3}
\end{pmatrix},
$$

$$
N(5, k) = N(6, k') = N(7, k'') = N(8, 1) = \begin{pmatrix}
\frac{2}{3} \\
\frac{2}{3} \\
N_1, \frac{1}{9} \\
N_1, \frac{1}{9}
\end{pmatrix}.
$$
where \((k=1, 2, 3, 4), (k'=1, 2, 3)\) and \((k''=1, 2)\). All the other matrices become zero in our calculation.

Appendix 2

As in the case \(\Delta I=1\) (Eq. (18)), we define the matrix \(N(j, i)\) as follows:

\[
\begin{bmatrix}
O_{jVA} \\
O_{jAV} \\
O_{jVA}' \\
O_{jAV}'
\end{bmatrix} + \frac{g^2}{16\pi^2} \Gamma'(\varepsilon) \begin{bmatrix}
-2C_2(R) \\
O_{jVA} \\
O_{jAV} \\
O_{jVA}'
\end{bmatrix} + N(j, i) = \begin{bmatrix}
O_{jVA} \\
O_{jAV} \\
O_{jVA}' \\
O_{jAV}'
\end{bmatrix}.
\]

For region I, the matrix \(N(j, i)\) and \(P_{jVA}'\), \(R_{jVA}'\) and \(S_{jVA}'\) are shown as

\[
N(10, 1) = \begin{bmatrix}
0 & 0 & 0 & 6 & 4 & 0 & 0 \\
0 & 0 & 6 & 0 & 4 & 0 & 0 \\
0 & 4 & -9 & 2 & 5 & 2 & 0 & 0 \\
4 & 0 & 5 & -9 & 2 & 2 & 0 & 0 \\
3 & 0 & 2 & 2 & 2 & 3 & 3 \\
4 & 0 & 0 & 3 \\
4 & 0 & 0 & 0
\end{bmatrix}, \quad
N_0, \quad
\begin{bmatrix}
4 & 0 & 0 \\
4 & 0 & 0
\end{bmatrix}, \quad
P_{jVA}' = (O_{11} + O_{12} + O_{13} + O_{14} + O_{15})_{jVA}, \quad
R_{jVA}' = (O_{16} + O_{17})_{jVA},
\]

\[
N(11, 1) = \begin{bmatrix}
4/3 & 0 & 0 \\
4/3 & 0 & 0 \\
10/9 & 0 & 0 \\
10/9 & 0 & 0
\end{bmatrix}, \quad
N_0, \quad
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}, \quad
S_{jVA}' = (O_{26} + O_{27})_{jVA},
\]

\[
N(k, 1) = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
1/3 & 2/3 & 0 \\
1/3 & 2/3 & 0
\end{bmatrix}, \quad
N_0, \quad
\begin{bmatrix}
1/3 & 2/3 \\
1/3 & 2/3
\end{bmatrix}, \quad
R_{jVA}' = (O_{16} + O_{17})_{jVA}.
\]
\[ N(k', 1) = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & \frac{4}{3} & 0 \\ -1 & -\frac{2}{9} & 0 \end{pmatrix}, \quad R_{k'}^{V, A} = (O_{k' - 8}) + O_{k' + 3}) V, A, \]

\[ N(k'', 1) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad R_{k''}^{V, A} = (O_{k'' - 8}) + O_{k''} V, A, \]

\[ S_{k''}^{V, A} = (O_{12} + O_{13} + O_{14} + O_{15} + O_{20} + O_{21} + O_{22} + O_{23} - O_{(k'' - 8)} - O_{k''}) V, A, \]

where the number of flavor \( f = 6 \), \((k = 12, 13, 14, 15, k' = 16, 17, 18, 19, k'' = 20, 21, 22, 23)\). Here the elements \( R_{k'}^{V, A} \) \((k = 10, 11)\) and \( S_{k''}^{V, A} \) \((k = 10, 11, \cdots, 19)\) equal zero. For regions II, III and IV, \( f \) takes the values of 5, 4 and 3, respectively and the quarks \( \{t, b\} \) and \( \{t, b, c\} \) are dropped from the operators \( O_{10} \sim O_{23} \), respectively. The matrix \( M'_{\mu - 6} \) is defined in the same way as Eq. (19).

References


A. Salam, in Elementary Particle Theory, edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367; and later extensions.


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