Projectile Fragmentation in Hadron-Nucleus Collisions and the Quark Model

Fujio Takagi

Department of Physics, Tohoku University, Sendai 980

(Received December 1, 1980)

It is shown that recent experimental results of Brenner et al. on the \( A \) (target mass number)-dependence of inclusive cross sections for \( h + A \rightarrow c + \text{anything} \) (\( h \) = a hadron, \( A \) = a nucleus, \( c \) = a hadron) in the projectile fragmentation region are in good agreement with the prediction from the quark model proposed by Dar and the present author. In particular, the observed deviation at small \( A \) from the power law behavior \( A^z \) is correctly predicted by the model.

§ 1. Introduction

It is now widely recognized that the study of multiple production off nuclear targets offers important and unique information on the structure of hadrons, the interactions of constituents of hadrons and the mechanism or the space-time structure of hadronization at high energies. Among various attempts to explain multiple production in hadron-nucleus collisions in terms of the quark or parton language, the additive quark model (AQM) combined with the recombination picture for the fragmentation of projectile hadrons has been found to be very successful in describing the \( A \) (= target mass number)-dependence of the projectile fragmentation.\(^{3-5}\) Predictions from the model of Ref. 4) that was proposed by Dar and the present author were compared with experimental data on \( \gamma_e + A \rightarrow \text{charged hadron} + \text{anything} \), \( p + A \rightarrow (\pi^\pm, K^\pm \text{or } K^0_s) + \text{anything} \) and \( p + A \rightarrow (n \text{ or } A) + \text{anything} \). It was shown that the model can explain well the observed \( A \)-dependence including its characteristic dependence on the type of the process.\(^7\)

Brenner et al. have recently reported new experimental results on the \( A \)-dependence of the inclusive reactions \( h + A \rightarrow c + \text{anything} \) in the projectile fragmentation region, where \( h = \pi^\pm \) and \( p \), \( c = \pi^\pm, K^\pm \text{and } p \), and \( A = C, \text{Al, Cu, Ag and Pb} \).\(^6\) Since this experiment contains new results on the reactions that were not studied in Ref. 4), it is very interesting to compare the new data with our model, which is the purpose of this paper. As a result, a good agreement between the two data and the model is obtained.

The assumptions of and the predictions from the model are summarized in § 2. The predictions are compared with the data in § 3. Conclusions are given in § 4.

Crucial parameters in our model are the total inelastic cross sections for \( h + A \)
and $q$-$A$ collisions, where $h$ is a hadron and $q$ is a constituent quark. One may use experimental values of $\sigma_{hA}$ if available. However, there is no data on $\sigma_{qA}$ because free quark beams are not available. Since the relative magnitude between $\sigma_{hA}$ and $\sigma_{qA}$ has a rather important effect on quantitative predictions, one needs a theoretically consistent set of $\sigma_{hA}$ and $\sigma_{qA}$, actually $\sigma_{\bar{q}A}$, $\sigma_{qA}$ and $\sigma_{qA}$ in the present analysis. Such a set of cross sections is calculated in the Appendix.

§ 2. Summary of the model

2.1. Assumptions

The model is based on three fundamental assumptions, each relating to the structure of hadrons, the initial interactions between constituents of both projectile and target hadrons (or nuclei) and the mechanism of hadronization of leading quarks:

(i) A hadron $h$ consists of a definite number ($=C_h$) of spatially separated constituent quarks, of which flavour quantum numbers are the same as those of the corresponding valence quarks.$^{7,10,11}$ For example, $C_h=3$ for $h=a$ (non-exotic) baryon, $C_h=2$ for $h=a$ (non-exotic) meson. So-called sea quarks, sea antiquarks and gluons form a virtual cloud of a constituent quark. A constituent quark will simply be called a quark hereafter in this paper.

(ii) In any of hadron-hadron, hadron-nucleus and nucleus-nucleus collisions, some quarks from the projectile interact inelastically with quarks from the target thus losing a considerable fraction of their initial momenta, while the remaining quarks in both the projectile and the target pass through as the leading (or the spectator) quarks retaining their initial momenta.

(iii) Those leading quarks hadronize eventually via fragmentation and recombination mechanisms. The hadrons produced in the projectile fragmentation region are supposed to be dominated by the products of the hadronization (fragmentation and recombination) of the leading quarks from the projectile.

The cross sections for the initial state interactions are calculated by using a usual geometrical-optical formula. For simplicity and as the lowest order approximation, the quark additivity is used for the quark-quark, quark-hadron or hadron-hadron cross sections.$^{11}$

2.2. Predictions

$^{11}$There is an important difference between our opinion and that of Dar, the coauthor of Ref. 4) and his collaborators$^{11,9,10}$ on this point. They identify the relevant hadronic constituents with the valence quarks themselves that are probed in deep inelastic lepton-hadron scattering at, say, $-\left(Q^2\right)\approx 2-4 \text{ GeV}^2$. Since the processes under consideration are presumably dominated by soft interactions, we think that the valence quarks probed at such large $Q^2$ are not directly relevant to the processes. We agree with Amisovich et al.$^{11}$ and Nikolichev et al.$^{9}$ on this point.
Consider ratios of the single particle inclusive cross sections defined as

\[ R_{pA}(x, p_T) = \frac{E \frac{d^3 \sigma(hA \rightarrow cX)}{dp_T^3}}{E \frac{d^3 \sigma(hp \rightarrow cX)}{dp_T^3}}. \]  

(2.1)

where \( x \) is the Feynman scaling variable defined in the \( h \)-nucleon c.m. system and \( p_T \) is the transverse momentum. Predictions from our model for the relevant processes are:\(^{(\text{A})}\)

\[ R_{pA}(x, p_T) = 2(\sigma_{pA} - \sigma_{hA})/\sigma_{pA} \]  

(2.2)

for any hadron \( c \) with large \( x \) and small \( p_T \) provided that the spectrum of hadron \( c \) is dominated there by the fragmentation or recombination of leading quarks,

\[ R_{pA}^{c}(x, p_T) = \frac{3}{\sigma_{pp}} \left( \sigma_{pA} - \sigma_{hA} + \frac{2(\sigma_{pA} - \sigma_{hA})}{\gamma_{p}^{c}(x, p_T)} \right) \]  

(2.3)

for \( c \) a (non-exotic) meson with large \( x \) and small \( p_T \) provided again that production of hadron \( c \) is dominated by the leading quark hadronization, and

\[ R_{pA}^{c}(x \approx 1, p_T) = 3(\sigma_{pA} - \sigma_{hA})/\sigma_{pp} \]  

(2.4)

for \( c \) a baryon that contains \((u, u)\) or \((u, d)\) as valence quarks, where \( \sigma_{pp} \) and \( \sigma_{pA} \) are \( p-p \) (\( \approx \) nucleon-nucleon) and \( p-p \) (\( \approx \) nucleon-nucleon) inelastic cross sections, respectively. In Eq. (2.3), \( \gamma_{p}^{c}(x, p_T) \) is the ratio of a combination of spectra of a non-exotic meson \( c \) produced from leading diquark systems to that from leading single quark systems,

\[ \gamma_{p}^{c}(x) = 2n_{qA,c}^{p}(x) + n_{qU,c}^{p}(x) \]  

(2.5)

\[ n_{qA,c}^{p}(x) + 2n_{qU,c}^{p}(x) \]

where the variable \( p_T \) is suppressed or to be understood as has already been integrated. Here, for example, \( n_{qA,c}^{p}(x) \) is the \( x \)-distribution (per leading system) of \( c \) that are produced from leading \((u, d)\)-systems of which parents are incident protons. Note that, for example, \( n_{qA,c}^{p}(x) \) is not a usual fragmentation function itself of leading single \( d \)-quarks but a convolution of the \( d \)-quark distribution \( d_{p}(x) \) in a proton and the \( d \)-quark hadronization function \( D_{d,c}^{p}(z) \):

\[ n_{qA,c}^{p}(x) = \int_{x}^{1} \frac{dy}{y} d_{p}(y) D_{d,c}^{p}\left(\frac{x}{y}\right). \]  

(2.6)

Here, the effects from both fragmentation and recombination (and absence of accompanying leading quarks) should be taken into account in \( D_{d,c}^{p}(z) \). That is why we call it the hadronization function rather than the fragmentation function.

In the previous paper,\(^{(\text{I})}\) we assumed implicitly the independent quark hadronization into mesons;

\[ n_{qA,c}^{p}(x) = n_{qA,c}^{p}(x) + n_{qU,c}^{p}(x), \]  

(2.7a)
\[ n_{\text{mb}^\text{c}}(x) = 2n_{\text{m}^\text{c}}(x), \nonumber \] (2.7b)

which implies that \( \gamma_{e^e}(x) = 2 \). In this case, (2.3) turns out to be
\[ R_{e^e}(x, p_T) = 4(\sigma_{pA} - \sigma_{AA})/(2\sigma_{pp}). \] (2.8)

Since the diquark system may have a higher probability of producing a baryon via recombination than the single quark system, it is quite probable that \( \gamma_{e^e}(x) \) is considerably smaller than 2. For example, Anisovich et al. suggest that \( \gamma_{e^e}(x \approx 1/3) = 5/4 \) on the basis of quark combinatorics.\(^3\) The result of an analysis by Bialas and Bialas even indicates that \( \gamma_{e^e}(x) < 1 \) for \( 0.2 \leq x \leq 0.4 \).\(^4\) Hence, we do not necessarily adhere to the assumption of independent quark hadronization. However, it will be shown in the next section that the prediction (2.8) is actually in good agreement with the data.

§ 3. Comparison with the data

3.1. How to compare with the data

Brenner et al. fitted the measured cross sections to the form
\[ E_c^3 d^3 \sigma(hA \rightarrow cX)/dp^3 = E_c^3 d^3 \sigma(hp \rightarrow cX)/dp^3 \eta_{e^e}(x, p_T) \exp\{a_{e^e}(x, p_T) \ln A\} \] (3.1)

for \( 12 \leq A \leq 207 \) and presented the values of \( a_{e^e}(x, p_T) \), \( E_c^3 d^3 \sigma(hp \rightarrow cX)/dp^3 \times \eta_{e^e}(x, p_T) \) (\(-\sigma_0(x, p_T)\) in their notation) and \( E_c^3 d^3 \sigma(hp \rightarrow cX)/dp^3 \) \((-\sigma_n(x, p_T)\) in their notation).\(^6\) It should be noted here that \( \eta_{e^e}(x, p_T) \) must be equal to unity if the power law \( (A^a) \) fit applied to \( A \geq 12 \) is valid down to \( A = 1 \) while the data show a considerable deviation from unity.\(^6\) From (2.1) and (3.1), one has
\[ R_{e^e}(x, p_T) = \eta_{e^e}(x, p_T) \exp\{a_{e^e}(x, p_T) \ln A\}. \] (3.2)

Now one can calculate \( \eta_{e^e}(x, p_T) \) and \( a_{e^e}(x, p_T) \) by using predicted values of \( R_{e^e}(x, p_T) \). For example, from (2.2) and (3.2), one obtains
\[ a_{e^e}(x, p_T) = \left( \frac{\sigma_{pA} - \sigma_{AA}}{\sigma_{pA} - \sigma_{AA}} \right) \ln \frac{A_2}{A_1}, \] (3.3)

and
\[ \eta_{e^e}(x, p_T) = 2(\sigma_{pA} - \sigma_{AA})/(\sigma_{pp} \exp\{a_{e^e}(x, p_T) \ln A_i\}) \] (3.4)

for a pair of nuclei \( A = A_1 \) and \( A_2, i = 1 \) or 2. Since the predicted values of \( R_{e^e} \) do not obey a rigorous power law in general, the values of \( a_{e^e}(x, p_T) \) and \( \eta_{e^e}(x, p_T) \) depend on the choice of \( A_1 \) and \( A_2 \).

As an alternative to the above method, we also fitted our prediction for \( R_{e^e}(x, p_T) \) to the power law form (3.2) by minimizing the normalized \( \chi^2 \) defined by
\[ \chi^2 = \sum_{i=1}^{S} \left(1 - \frac{a_{n}^c \exp(a_{n}^c \ln A)}{K_{n}^c} \right)^2, \]  \hspace{1cm} (3.5)

where \( A_1 = 12, A_2 = 27, A_3 = 64, A_4 = 118 \) and \( A_5 = 207 \).

3.2. \( \pi^+ + A \rightarrow c + \) anything; \( c = \) any hadron

As is given by (2.2), our model predicts that \( R_{n}^c(x, \rho_T) \) and hence \( a_n^c(x, \rho_T) \) and \( \eta_n^c(x, \rho_T) \) are independent of \( x \) and \( \rho_T \) as long as \( x \) is not too small and \( \rho_T \) is not too large. Numerically the least \( \chi^2 \) fit to the calculated values of \( R_{n}^c \) for \( 12 \leq A \leq 207 \) yields

\[ a_n^c(x, \rho_T) = 0.635; \quad \eta_n^c(x, \rho_T) = 1.72, \]  \hspace{1cm} (3.6)

which are in good agreement with the data as is shown in Figs. 1 and 2. The upper (lower) dashed line in Fig. 1 and the lower (upper) dashed line in Fig. 2 represent the theoretical results when one uses (3.3), (3.4) with \( A_1 = 12, A_2 = 27 \) \((A_1 = 118, A_2 = 207)\). The experimental values of \( a_n^c \) and \( \eta_n^c \) averaged over all the available data, i.e., \( c = \pi^+, K^+ \) and \( \rho, 0.3 \leq x \leq 0.88, \rho_T = 0.3 \) and 0.5 GeV/c, are

\[ \langle a_n^c(x, \rho_T) \rangle_{\exp} = 0.632 \pm 0.010; \quad \langle \eta_n^c(x, \rho_T) \rangle_{\exp} = 1.74 \pm 0.07, \]  \hspace{1cm} (3.7)

which are in good agreement with (3.6).

It should be noted here that (i) the observed \( A \)-dependence of the reaction \( \pi^+ + A \rightarrow c + \) anything including the deviation from \( A_a \) behavior at small \( A \) is

---

**Fig. 1.** Comparison between the calculated and the experimental values of \( a_n^c(x, \rho_T) \). Data are taken from Ref. 6). See text for the meaning of dashed lines.

**Fig. 2.** Comparison between the calculated and the experimental values of \( \eta_n^c(x, \rho_T) \). Data are taken from Ref. 6).
correctly predicted by the model, and (ii) though the errors are fairly large, there is experimentally no appreciable difference between $a^+_\pi$ for different $c$ and also between $\eta^+_\pi$ for different $c$ in agreement with the prediction.

3.3. $p + A \rightarrow \pi^+ + \text{anything}$

This process is more complicated than $\pi^+ + A \rightarrow \pi^+ + \text{anything}$ because both leading diquark systems and leading single quark systems from incident protons can produce fast pions. The appearance of the parameter $\gamma^\pi_+(x, p_T)$ is due to this circumstance. Though there is no convincing reason, we first assume the independent quark hadronization into pions, i.e., $\gamma^\pi_+(x, p_T) = 2$. The prediction is then given by (2.8). The least $\chi^2$ fit for $12 \leq A \leq 207$ yields

$$a^\pi_+(x, p_T) = 0.594; \quad \eta^\pi_+(x, p_T) = 1.73.$$  \hspace{1cm} (3.8)

These predictions are compared with the data in Figs. 3 and 4. The dashed lines correspond again to the choices $(A_1, A_2) = (12, 27)$ and $(118, 207)$. The experimental values averaged over $\pi^\pm$, $0.3 \leq x \leq 0.8$, $p_T = 0.3$ and $0.5$ GeV/c are

$$\langle a^\pi_+(x, p_T)\rangle_{\exp} = 0.555 \pm 0.020; \quad \langle \eta^\pi_+(x, p_T)\rangle_{\exp} = 1.96 \pm 0.17.$$  \hspace{1cm} (3.9)

The agreement between the theory and the experiment is good.

In order to check the sensitivity of the result on the choice of $\gamma^\pi_+(x, p_T)$, we have also calculated $a^\pi_+$ and $\eta^\pi_+$ by assuming that $\gamma^\pi_+(x, p_T) = 1$, which means a strong suppression of pion production from diquark systems compared to single quark systems. The result of the least $\chi^2$ fit is

![Graphs showing the comparison between calculated and experimental values of $a^\pi_+(x, p_T)$ and $\eta^\pi_+(x, p_T)$.](image)

Fig. 3. Comparison between the calculated and the experimental values of $a^\pi_+(x, p_T)$. Data are taken from Ref. 6.

Fig. 4. Comparison between the calculated and the experimental values of $\eta^\pi_+(x, p_T)$. Data are taken from Ref. 6.
\[ a_\rho^p(x, p_T) = 0.635; \quad \eta_\rho^p(x, p_T) = 1.72. \]

Now the agreement between the theory and the experiment is not as good as in the first case.

3.4. \( p + A \rightarrow p + \text{anything} \)

In the previous paper,\(^4\) we did not apply our model to this type of reactions where all the valence quarks of the detected particles can be common to those of the projectile hadrons. The reason was that the cross section of such a reaction might be dominated by the process where quarks are not “wounded” (only gluons interact or be exchanged). However, it is tempting to identify such a process with the diffractive dissociation process. Then, our prediction (2.4) can be compared with the data on \( p + A \rightarrow p + \text{anything} \) at appropriate \( x \) and \( p_T \) where the cross section is dominated by the nondiffractive component. Since the region very near to \( x = 1 \) is certainly dominated by the diffractive component, we compare the prediction (2.4) for \( x \approx 1 \) with the data at \( x = 0.8 \sim 0.88 \). The least \( \chi^2 \) fit yields

\[ a_\rho^p(x \approx 1, p_T) = 0.537; \quad \eta_\rho^p(x \approx 1, p_T) = 1.79. \] \( (3 \cdot 10) \)

These results are compared with the data in Figs. 5 and 6. The experimental values of \( a_\rho^p \) and \( \eta_\rho^p \) averaged over \( x = 0.8, 0.88 \) and \( p_T = 0.3 \) and 0.5 GeV/c are

Fig. 5. Prediction for \( a_\rho^p(x \approx 1) \) is compared with data on \( a_\rho^p(x) \).\(^6\) Solid line is the result from the least \( \chi^2 \) fit to the model prediction, while dashed lines correspond to the choice \((A_1, A_2) = (12, 27)\) and \((118, 207)\). Data on \( a_\rho^p(x, p_T) \) from Ref. 12 are also plotted for comparison.

Fig. 6. Prediction for \( \eta_\rho^p(x \approx 1) \) is compared with data on \( \eta_\rho^p(x) \).\(^6\) Meaning of solid and dashed lines is the same as in Fig. 5.
$\langle a_\rho(x, p_T) \rangle_{\text{exp}} = 0.48 \pm 0.01$; $\langle \eta_\rho(x, p_T) \rangle_{\text{exp}} = 1.58 \pm 0.07. \quad (3.11)$

There seems to be a slight discrepancy between the experimental and the theoretical values of $a_\rho$. However, the results of the least $\chi^2$ fit (and also those of the alternative method using a pair of nuclei $A_1$ and $A_2$) are rather sensitive to the choice of the cross sections and a typical uncertainty in $a_\rho$ due to this is $\pm 0.04$. (Of course the same uncertainty applies to the better results of §§ 3.2 and 3.3.) Therefore, we do not consider that the slight discrepancy is physically significant.

Independent evidence for the prediction (2.4) comes from comparing data on $p + A \rightarrow p + \text{anything}^0$ with those on $p + A \rightarrow A + \text{anything}$,12,13 which are also plotted in Fig. 5. Equation (2.4) predicts that the $A$ dependence of these reactions is the same at large $x$ provided that the non-diffractive component is dominant. The prediction is certainly realized experimentally as is seen from Fig. 5. In fact, it is remarkable that an approximate equality $a_\rho^p(x, p_T) \approx a_\rho^A(x, p_T)$ holds experimentally for a wide range of $x$.

§ 4. Conclusion

The results of the preceding section are summarized with some remarks as follows:

(i) The most definite prediction (2.2) is in good agreement with the data on $\pi^+ + A \rightarrow \pi^+ + \text{anything}$ for $c = \pi^\pm, K^\pm$ and $p$,6 including the deviation from the $A^\omega$ behavior at small $A$.

(ii) The agreement between the prediction (2.3) and the data on $p + A \rightarrow c + \text{anything}$ for $c = \pi^\pm$ is also good again including the deviation from the $A^\omega$ behavior at small $A$ when one assumes that $\gamma_{\pi^\pm}(x) = 2$. There is no indication for the suppression of fast pion production from the leading diquark systems. However, we do not claim that this is evidence for the independent hadronization of leading quarks into mesons. Only a detailed study of hadronization of various leading quark systems into mesons, baryons and antibaryons can answer this problem.

(iii) The agreement between (2.4) and the data on $p + A \rightarrow p + \text{anything}$ is not as good as in the former two cases. However, there is an uncertainty due to different choices of the cross sections. Hence, one can at least say that the prediction is not inconsistent with the data. Furthermore, the data on $p + A \rightarrow p + \text{anything}$ and $p + A \rightarrow A + \text{anything}$ show the same degree of nuclear attenuation at large $x$ as is predicted by our model.

(iv) As an entire result, our model predicts correctly the following qualitative feature that is observed experimentally:6

$$a_\rho(x \simeq 1, p_T) < a_\rho^{\pi\pm}(x, p_T) < a_\rho^{\pi\pm}(x, p_T) \quad \text{for } c = \pi^\pm, K^\pm \text{ and } p. \quad (4.1)$$

To conclude, the main features of the new experimental data on the $A$-
dependence of the reactions $h + A \rightarrow c + \text{anything}$ in the projectile fragmentation region\textsuperscript{49} are well explained by the quark model\textsuperscript{49} based on the picture of spatially separated constituent quarks in a hadron.\textsuperscript{7,8} The present results add further evidence for the validity of the model.\textsuperscript{44} However, it is a little trouble how to choose a correct set of cross sections $\sigma_{hA}$ and $\sigma_{cA}$. We do not know experimentally $\sigma_{hA}$. Though $\sigma_{hA}$ are available, there are some discrepancies between the results from different experimental groups.\textsuperscript{47,48,49,50}

After completion of this paper, the author received a paper by A. Bia{\l}as and W. Czyz entitled "Projectile Fragmentation in Hadron-Nucleus Collisions at High Energies" (Jagellonian University and Institute of Nuclear Physics preprint, September 1980), where the quark model prediction is compared with the data of Ref. 6). However, their way of presentation is quite different from the present paper and their emphasis is laid on a comparison with multiple scattering models and on the contribution of diffractive interactions. Otherwise their conclusion is consistent with this paper.

Acknowledgement

The author would like to thank N. Masuda for bringing the data of Ref. 6) to his attention.

Appendix

The cross sections for inelastic $c-A$ scattering can be calculated from the well-known formula:

$$\sigma_{cA} = \int d^2 b \left[ 1 - (1 - \rho_{cA}(b))^{1/4} \right], \quad (A\cdot1)$$

where

$$\rho_{cA}(b) = 4A^{-1} \int d^2 b' \rho_A(b') f_{c\bar{p}}(b - b'), \quad (A\cdot2)$$

$\rho_A(b)$ being the nucleon density distribution of the nucleus $A$ normalized as

$$\int d^2 b \rho_A(b) = A, \quad (A\cdot3)$$

and $f_{c\bar{p}}(b)$ being the inelastic overlap function for $c$-proton scattering normalized as

$$4 \int d^2 b f_{c\bar{p}}(b) = \sigma_{c\bar{p}}, \quad (A\cdot4)$$

Here, we have used the conventional approximation $\sigma_{c\bar{p}} \approx \sigma_{ch}$ ($n=$neutron) that is
valid at high energies. The particle $c$ may be a hadron $h$ or a quark $q$.

For simplicity we assume that both $f_{cp}(b)$ and $\rho_{A}(b)$ are Gaussian,

\[ f_{cp}(b) = \sigma_{cp} \exp\left(-b^2 / \langle b^2 \rangle_{cp}\right) / (\pi \langle b^2 \rangle_{cp}), \]
\[ \rho_{A}(b) = A \exp\left(-b^2 / \langle b^2 \rangle_{A}\right) / (\pi \langle b^2 \rangle_{A}), \]

(A.5)

(A.6)

where $\langle b^2 \rangle_{A}$ is related to the equivalent radius $R_{A}$ of the nucleus $A$ as

\[ \langle b^2 \rangle_{A} = \frac{2}{5} R_{A}^2. \]

(A.7)

From (A.1), (A.2), (A.5) and (A.6), one obtains

\[ \sigma_{cA} = \pi B_{cA}^2 \int_{0}^{1} \frac{dt}{t} \left(1 - \left(1 - \frac{\sigma_{cp} t}{\pi B_{cA}^2}ight)^t\right), \]

(A.8)

where

\[ B_{cA}^2 = \langle b^2 \rangle_{cA} - \langle b^2 \rangle_{cp}. \]

(A.9)

As nondiffusive inelastic cross sections on the nucleon, we take

\[ \sigma_{pp} = 28.5 \text{ mb}, \quad \sigma_{np} = 19.0 \text{ mb}, \quad \sigma_{pp} = 9.5 \text{ mb} \]

(A.10)

based on the quark additivity approximation.\(^{11}\) For $\langle b^2 \rangle_{cp}$, we take

\[ \langle b^2 \rangle_{pp} = 9.6 \text{ mb}, \quad \langle b^2 \rangle_{np} = 6.4 \text{ mb}, \quad \langle b^2 \rangle_{np} = 3.2 \text{ mb}, \]

(A.11)

where the values of $\langle b^2 \rangle_{pp}$ is taken from the result of a $p$-$p$ data analysis,\(^{17}\) and then the values of $\langle b^2 \rangle_{np}$ and $\langle b^2 \rangle_{cp}$ are determined on the analogy of the quark additivity for $\sigma_{cp}$. (A.11) with (A.10) implies an approximation that $f_{cp}(0) = f_{np}(0) = 0.236$. Note that the unitarity upper bound for $f_{cp}(b)$ is 1/4 in our normalization.

Finally the $A$-dependence of $R_{A}$ is adjusted to reproduce the experimental values of $\sigma_{pA}$ and $\sigma_{nA}$\(^{14}\) with a reasonable accuracy. The result is

<table>
<thead>
<tr>
<th>$A$</th>
<th>$\sigma_{cA}$</th>
<th>$\sigma_{nA}$</th>
<th>$\sigma_{pA}$</th>
<th>$\sigma_{nA}/\sigma_{pA}$</th>
<th>$\sigma_{pA}/\sigma_{nA}$</th>
<th>$\sigma_{pA}/\sigma_{nA}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>75.5</td>
<td>136.3</td>
<td>188.1</td>
<td>1.38</td>
<td>152 ± 2</td>
<td>210 ± 2</td>
</tr>
<tr>
<td>12</td>
<td>98.2</td>
<td>174.3</td>
<td>237.0</td>
<td>1.36</td>
<td>181 ± 3</td>
<td>248 ± 2</td>
</tr>
<tr>
<td>27</td>
<td>203.8</td>
<td>341.0</td>
<td>443.9</td>
<td>1.30</td>
<td>342 ± 3</td>
<td>445 ± 4</td>
</tr>
<tr>
<td>64</td>
<td>429.8</td>
<td>670.0</td>
<td>833.6</td>
<td>1.24</td>
<td>647 ± 7</td>
<td>796 ± 8</td>
</tr>
<tr>
<td>118</td>
<td>712.9</td>
<td>1055.8</td>
<td>1276.9</td>
<td>1.21</td>
<td>1045 ± 25</td>
<td>1259 ± 15</td>
</tr>
<tr>
<td>207</td>
<td>1117.8</td>
<td>1583.2</td>
<td>1873.3</td>
<td>1.18</td>
<td>1578 ± 30</td>
<td>1812 ± 35</td>
</tr>
<tr>
<td>238</td>
<td>1241.1</td>
<td>1739.6</td>
<td>2049.0</td>
<td>1.18</td>
<td>1740 ± 20</td>
<td>2052 ± 20</td>
</tr>
</tbody>
</table>
\[ R_{A}^2 = r_{0}^2 A^{0.5} \text{ with } r_{0} = 1.9 \text{ fm for } A \geq 9. \]  \hspace{1cm} (A\cdot12)

The result for the cross sections are shown in Table I in comparison with the experimental values (average of 20 - 60 GeV data).\(^{(4)}\) Validity of the calculation is justified by the fact that, as is seen from Table I, the calculated ratios \( \sigma_{pA}/\sigma_{\pi A} \) agree very well with the experimental values once \( R_{A} \) is adjusted to fit \( \sigma_{pA} \).

References