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RELATIONSHIP BETWEEN STREAM WATER TEMPERATURE AND AMBIENT AIR TEMPERATURE

A Simple Autoregressive Model for Mean Daily Stream Water Temperature Fluctuations

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The water temperature of streams and rivers is required for various practical purposes and is frequently obtained by calculating the heat budget. This method is tedious and yields rather inaccurate values of the water temperature. This paper presents an alternative approach using cheap and simple means to measure air temperatures, which are believed to be a major factor controlling the water temperature. It is demonstrated that a useful separation can be made between the seasonal cyclic variations and the daily stochastic fluctuations of these temperatures.

The water temperature is a very important criterion in water quality since most of the quality parameters (physical, chemical and biochemical) are temperature-dependent.

The water temperature of a natural stream is determined by two kinds of factors: the meteorological and the geophysical ones. The principal meteorological factors are the air temperature, the intensity and duration of solar radiation, the wind velocity at the surface, and the pressure of water vapor in the air. The main geophysical factors are the depth of the river, the discharge, the degree of mixing, the dimension of the free surface, and the temperature of the superficial and underground inflows. The purpose of this study is to determine, after subtraction of the seasonal climatological cyclic component of the region, the precision with which one can predict the daily stream temperature fluctuations as a function of the daily meteorological records; the latter are characterized by one factor integrating best the different causes: the air temperature.

Climatic Cycles of Temperature

It has long been recognized that air temperatures measured at the same time each day are distributed about an annual sine curve of good interannual stability. The fluctuations around such a curve represent the effect of the stochastic events such as the precipitation, the passage of air masses of different characteristics and the change in cloud cover. More recently, it was shown by Ward (1963) that the daily temperatures of a well-mixed stream also follow a cyclic pattern. In this paper, we develop a relationship between the stochastic components of the water and air temperatures.

For both air and water temperatures, the representative sine curve can be written:

$$Ts = a + b \sin c(n+n_0) \tag{1}$$

with $$c = \frac{2\pi}{365} = 0.0172 \text{ radian.day}^{-1}$$

and n being the number of the day in the year. The three other coefficients a, b and n_0 can be determined by a least square fit of a sufficient number of data points to the curve.

This fitting requires the solution of a set of three normal equations derived from equation (1) and is described by Ward (1963). Edinger et al. (1968) have shown that the water temperature follows an "equilibrium" air temperature with a time-lag and a damping of amplitude.

The climatic data normally available provide daily maximum and minimum air temperatures and therefore we have to define the equilibrium temperature in terms of these two temperatures. Edinger used a simple mean; however it is not obvious that the mean is the best measure of the temperature we seek. One might, for example, expect that, in view of the diurnal fluctuations of atmospheric stability, nocturnal temperatures are less important because of the reduction in turbulent transfer of heat. In this study the temperature best representing the equilibrium temperature was found in a different way.

After determination of the sine curve representing the seasonal cyclic component of the water temperature variations, the maximum temperature of this curve is found. At this point there is no net heat transfer to the water and thus the equilibrium temperature must be the same as the water temperature.

This enables an optimum combination of the maximum and minimum in temperatures to be defined by the following equation:

$$Ta_{eq} = \alpha\, Ta_{min} + (1-\alpha)\, Ta_{max} = Tw_{max} \tag{2}$$

where

Ta_{eq} = equilibrium air temperature

Ta_{min} = maximum of the seasonal cyclic curve of minimum air temperature

Ta_{max} = maximum of the seasonal cyclic curve of maximum air temperature

Tw_{max} = maximum of seasonal cyclic curve of water temperature.

The value of α is evaluated by a trial and error procedure.

Meteorological Fluctuations of Temperature

We call meteorogical fluctuations the daily differences between actual temperature and the cyclical component, both for water temperature and the "representative" temperature of the air as calculated before. The standard calculation of the autocorrelations of the two series shows that, for a period of less than two weeks, no residual periodicity remains. If one considers the passage of atmospheric perturbations as a disruption of the thermal equilibrium corresponding to the "steady state" climatic sine curves calculated before, it is interesting to investigate the way these perturbations are followed by the water temperatures and to specify the part due to water thermal inertia and the part due to the air-water exchanges.

In this analysis a second order Markov process is assumed in order to describe better the persistence of meteorologic conditions. After calculating the autocorrelation coefficients for various data available, it was shown that the coefficient for two days still explained a non-negligible part of the variance of the phenomena.

If R_1 and R_2 are the auto-correlation coefficients for one and two days, the coefficients of the second-order Markov process as calculated by Quimpo (1967) are given by:

$$A_1 = \frac{R_1\,(1-R_2)}{1 - R_1^2} \qquad A_2 = \frac{1 - R_1^2}{R_2 - R_1^2} \tag{3}$$

The fluctuations for the water temperature can then be calculated as follows:

$$\begin{aligned} Tw\,(n) - Tsw\,(n) = {} & A_1\,[Tw\,(n-1) - Tsw\,(n-1)] \\ & + A_2\,[Tw\,(n-2) - Tsw\,(n-2)] \\ & + K\,[Ta\,(n) - Tsa\,(n)] \end{aligned} \tag{4}$$

where n = number of the day in the year
Tw = actual water temperature
Tsw = seasonal trend water temperature
Ta = actual air temperature (equilibrium air temperature)
Tsa = seasonal trend air temperature
$A_1 A_2$ = second-order Markov process coefficients
K = exchange coefficient to be optimized

Once the seasonal sine curves are known, one only needs Tw (1) and Tw (2) to calculate any subsequent temperature. Those two temperatures themselves may be taken from the sine curve since dependence on them rapidly diminishes during subsequent days.

Recently Kothandaraman (1971) has developed a similar model based solely on the air temperature fluctuations of the day and the two previous days. The latter approach includes the effect of air temperatures on preceding days as driving terms, whereas in the method presented here they are effective only via the thermal inertia of the water. This difference is important in that it permits a distinction to be made between the coefficients A_1 and A_2 in equation (1), which will vary from one stream to another, but will not depend on meteorological conditions, and the coefficient K which should be independent of the stream and should depend only on the meteorological conditions.

Experimental Test of the Recurrence Equation

The equation has been tested on the Rivière du Nord, at Saint-Jérôme, forty-eight (48) km north of Montreal, P.Q., Canada. The intercepted area is 1167 km^2 and the flow is not disturbed by engineering works. The river is shallow and well mixed. The water temperature measurements were recorded by the Department of Natural Resources of the Province of Quebec with an accuracy of 1 F° (= 0.55 C°). The mean daily values were computed as the averages of the 24 hourly readings. The maximum and minimum daily temperatures used were taken at the meteorological station of Sainte-Agathe-des-Monts. This station is by the river in the center of the basin.

The records from other stations in the basin have been examined with no significant difference in results.

In order to avoid the effect of phase-change due to the winter conditions (jamming, snowmelt on some parts of the basin), the study used the data of 170 consecutive days from June 11, 1969.

The fitting of the sine curve coefficients was executed on an APL/360 computer and the results are given in Table 1.

Table 1.
Computed coefficients of the cyclic component of the temperature.

Adjusted temperature	a °C	No. of days	b °C	Annual maximum °C
Water temperature	8.61	44.94	12.16	20.77
Minimum air temperature	0.05	45.00	12.67	12.72
Maximum air temperature	7.33	45.17	16.50	23.83
"Representative" air temperature ($\alpha = 0.2755$)	5.32	45.12	15.45	20.77

On this table, it appears that the cyclic components of the temperatures are in phase within one day and that the maximum appears 45 days after the 11th of June. The value obtained for α is 0.2755 and thus the weight given to the maximum air temperature is about three times that given to the minimum air temperature.

The two first auto-correlation coefficients and Markov coefficients of the fluctuations series have been computed and are shown in Table 2.

This table shows the importance of the inertia terms for the water series. The coefficient K, standing for the driving term of the air-water heat exchange, has been adjusted on a sample of the data by minimizing the discrepancies between prediction and measurement. This adjustment gives $K = 0.062$ and was verified on the remaining data.

The recurrence equation was run for the whole set of data and the results are shown in Figures 1 and 2.

The mean of the absolute values of the measured fluctuations was 1.24 C°.

Table 2.
Auto-correlation coefficients and Markov coefficients of the stochastic components of the temperature.

Fluctuation series	R_1	R_2	A_1	A_2
Water	0.916	0.736	1.150	–0.647
Air	0.619	0.290	0.712	–0.150

Fig. 1.

Measured and computed values of water temperature (seasonal variations and stochastic fluctuations combined).

With the use of the autoregressive equation, this value was reduced to 0.59 C°. It can be seen that the remaining scatter is of the order of the error of measurement (0.55 C°) of the water temperature. (The error in the measurement of the air temperature can be neglected because of the relative magnitudes of K and R_1, R_2.) Thus whilst the air temperature is not the sole factor controlling the water temperature, it is the most important one for this prediction model.

This kind of model could be useful for certain applications. For example, it could be used to demonstrate thermal pollution caused by industry. This requires that a suitable period of calibration should exist before the industry is established. Then subsequent measurements would reveal the magnitude of the problem by the temperature fluctuations unexplained by the model.

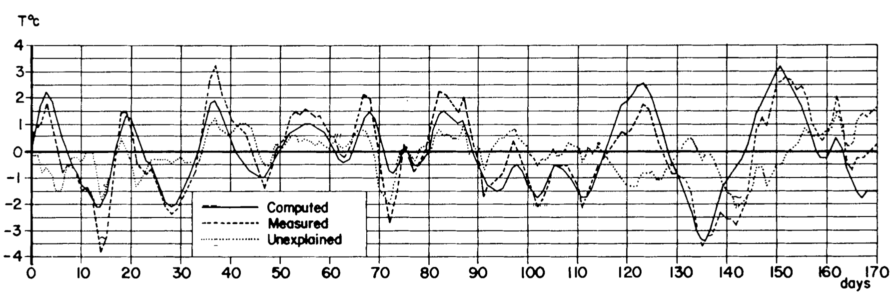

Fig. 2.

Measured and computed water temperatures (stochastic fluctuations alone).

Further Developments

Tests of the autoregressive scheme are currently being conducted on different rivers of the province where data are available. Improvements will also be sought using cyclic values of the air-water transfer coefficient K.

Another step will be to investigate the shape of the hourly fluctuations.

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