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An Indirect Method for Determination of the Parameters of Flood Plain Aquifers

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An indirect method of determining flood plain aquifer parameters from estimated autocorrelation of groundwater table data is presented. For this purpose a stream-aquifer system is approximated by a cascade of linear reservoirs. The response of groundwater piezometric head to an arbitrarily varying flood pulse in the river is given by the convolution equation. The autocorrelation function of groundwater piezometric head is expressed as a function of the parameters of the aquifer and the autocorrelation function of the water stage in the river. Observations performed in Gaupne in western Norway are used to determine the parameters of a flood plain aquifer.

Introduction

The parameters of an aquifer; hydraulic conductivity and coefficient of storage, should be known in order to predict the response of the aquifer to applied stresses, e.g. pumping from wells or changing water levels in a near stream. In a number of works the linear differential equation describing flow of water in an aquifer in terms of water table height for various boundary conditions have been solved. The hydraulic conductivity and coefficient of storage are identified by comparing observed aquifer response to the response given by the solution of the differential equation.

In case of a river having pervious banks there will be hydraulic contact between the water in the stream and the groundwater in adjacent flood plain aquifers.

Fluctuations in the water level of the stream lead to corresponding fluctuations in the groundwater table which become dampened and delayed as the distance from the river increases. The speed of propagation and the rate of dampening of the fluctuations depend on the characteristics of the aquifer.

Venetis (1968, 1970) and Hall and Moench (1972) consider the case of a flood plain aquifer influenced by varying water levels in a stream. Assuming that the groundwater table height in the river bank is equal to the water level in the river they solve the linearized differential equation describing flow of water in the aquifer as a convolution equation.

In this paper the river and the aquifer are approximated by a cascade of linear reservoirs. The response of the aquifer to an arbitrarily varying flood pulse in the stream is given by the convolution equation. An expression for the autocorrelation function of groundwater table height which depends on the parameters of the aquifer and the autocorrelation function of the water level in the river is derived. If observations are performed in a river and an adjacent flood plain aquifer the transmissivity and coefficient of storage can be determined by comparing empirical and theoretical values of the autocorrelation function of groundwater piezometric height.

A Linear Model for Flood Plain Aquifers

We consider a cross-section of a flood plain as shown in Fig. 1. The one directional, linear differential equation describing horizontal flow of water in this case for a homogeneous, isotropic aquifer is

$$\frac{\partial^2 h(x,t)}{\partial x^2} \equiv \frac{S}{KH}\frac{\partial h(x,t)}{\partial t} \tag{1}$$

where

S – the coefficient of storage.
K – the hydraulic conductivity.
H – the thickness of the aquifer.

Eq. (1) is valid for confined flow and is a good approximation for unconfined flow if changes in the groundwater level $h(x,t)$ are small compared with the saturated thickness H.

Boundary condition is given at $x = 0 : h(0,t) = h_r(t)$.

The general solution to Eq. (1) is then written

$$h(x,t) \equiv \int_0^t U(x,t-\tau)\ h_r(\tau)\,d\tau \tag{2}$$

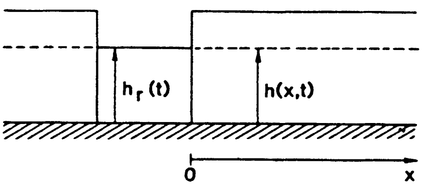

Fig. 1. Simplified cross-section of a flood plain.

where $U(x,t\text{-}\tau)$ is the impulse response function. It can be expressed as follows for the given boundary conditions (Venetis 1970)

$$U(x,t) = (t^*)^{-3/2} \exp(-\frac{1}{t^*}) \frac{Sx^2}{4HK} \tag{3}$$

where t^* is a dimensionless time equal to $t^* = 4HKt/SX^2$

If x is fixed, the convolution integral Eq. (2) also expresses the general solution to a linear ordinar differential equation. In its simplest form such a lumped system is described by the equation

$$T_a(x) \frac{dh(x,t)}{dt} + h(x,t) \equiv h_r(t) \tag{4}$$

where

- $T_a\,(x)$ is the recession coefficient of the linear system. It depends on the parameters of the aquifer and the fixed distance x from the river.
- the input to the system is the water stage in the river: $h_r(t)$
- the response or output from the system is the piezometric heigth at distance x from the river: $h(x,t)$.
- $h_r(t)$ and $h(x,t)$ are measured with reference to an arbitrary datum level.

We thus assume in this case that the part of the aquifer lying between the river and the point under consideration at a fixed distance x from the river is a linear reservoir characterized by a linear relationship between storage and output.

We postulate as before that the system is at rest at time $t = 0$. The boundary condition is given at $x = 0$: $h(0, t) = h_r(t)$. The solution to Eq. (4) is given by the convolution Eq. (2) where

$$U_a(x,t) = \begin{cases} 0 & \text{for} \quad t < 0 \\ \frac{1}{T_a(x)} e^{-t/T_a(x)} & \text{for} \quad t >= 0 \end{cases} \tag{5}$$

The impulse response function $U_a(x,t)$ is the reponse of the linear reservoir to an input at $x = 0$ given by the Dirac delta function.

The autocorrelation of groundwater level for the general case Eq. (1) has been derived by Gottschalk (1977). Here the simpler Eq. (4) will be used. The results for the general case will however be used so that the time scale, as defined from the autocorrelation function, is preserved by the simplified model. This will be further developed below.

Autocorrelation Function of Groundwater Table Height

In a stream-aquifer system the groundwater piezometric height depends on the variations in the water level of the river and on the characteristics of the aquifer. The same is true for the autocorrelation function of groundwater piezometric height.

The autocovariance function $C_h(x,\tau)$ of $h(x, t)$ is calculated as

$$C_h(x,\tau) = E\{[h(x,t)-\mu_h(x)]\,[h(x,t+\tau)-\mu_h(x)]\} \tag{6}$$

where $\mu_h(x) = E\{h(x, t)\}$ is the mean of the process $h(x,t)$.

The autocorrelation function $R_h(x, \tau)$ of $h(x, t)$ is defined as

$$R_h(x,\tau) \equiv \frac{C_h(x,\tau)}{C_h(x,0)} \tag{7}$$

where $C_h(x, 0) = \sigma^2_h(x)$ is the variance of the process $h(x, t)$.

We approximate the river and the aquifer by a cascade of linear reservoirs as shown in Fig. 2.

The river represents the whole drainage basin above the considered cross-sction of the river plain. Input $i(t)$ to this reservoir is the climatic input to this part of the drainage basin. The output $h_r(t) = h(0, t)$ is the water level in the river. Input and output to the linear reservoir representing the aquifer are $h_r(t)$ and $h(x, t)$ as before. We assume stationarity.

The impulse response function $U(x, t)$ of the composite system can be found by letting the input $i(t)$ be a Dirac delta function. Then the output from the first reservoir is the impulse response function to the river

$$U_r(t) \equiv \frac{1}{T_r} e^{-t/T_r} \tag{8}$$

where T_r is the recession coefficient of the river. The input to the aquifer is $U_r(t)$ and the impulse response function of the composite system is given by Eq. (2).

If $T_r \neq T_a(x)$ then

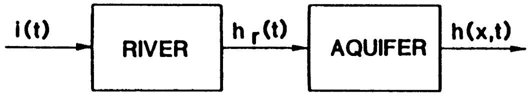

Fig. 2. Composite linear system for river and aquifer.

$$U(x,t) = \frac{e^{-t/T_r} - e^{-t/T_a(x)}}{T_r - T_a(x)} \qquad (9)$$

If $T_r = T_a(x)$ then

$$U(x,t) = \frac{t}{T_a(x)^2} e^{-t/T_a(x)} \qquad (10)$$

We assume that the climatic input $i(t)$ to the composite system is a time independent process defined by

$$\mu_i = E\{i(t)\} = 0 \quad \text{and} \quad C_i(\tau) = E\{i(t)\, i(t+\tau)\} = \sigma_i^2 \delta(\tau)$$

where $\delta(\tau)$ is the Dirac delta function.

The autocovariance function of the output from the composite system is

$$C_h(x,\tau) \equiv \sigma_i^2 \int_0^\infty U(x,v)\, U(x,v+\tau)\, dv \qquad (11)$$

When $T_r \neq T_a(x)$ we use Eq. (9) for $U(x, v)$ and get

$$C_h(x,\tau) = \frac{\sigma_i^2}{(T_r - T_a(x))^2} \left[\frac{T_r}{2} e^{-\tau/T_r} + \frac{T_a(x)}{2} e^{-\tau/T_a(x)} - \frac{T_r T_a(x)}{T_r + T_a(x)} \left(e^{-\tau/T_r} + e^{-\tau/T_a(x)} \right) \right] \qquad (12)$$

The autocorrelation function is

$$R_h(x,\tau) = \frac{C_h(x,\tau)}{C_h(x,0)} = \frac{T_r e^{-\tau/T_r} - T_a(x) e^{-\tau/T_a(x)}}{T_r - T_a(x)} \qquad (13)$$

When $T_r = T_a(x)$ we use Eq. (10) for $U(x, v)$ and get

$$C_h(x,\tau) = \frac{\sigma_i^2}{4T_a(x)} e^{-\tau/T_a(x)} \left(1 + \frac{\tau}{T_a(x)}\right) \qquad (14)$$

$$R_h(x,\tau) = e^{-\tau/T_a(x)} \left(1 + \frac{\tau}{T_a(x)}\right) \qquad (15)$$

The Recession Coefficients of the River and the Aquifer

When $i(t)$ is a time independent process Eq. (11) gives the autocovariance function of the output from the composite system in Fig. 2. It is also valid for the output from the first reservoir, the river

$$C_r(\tau) \equiv \sigma_i^2 \int_0^\infty U_r(v)\, U_r(v+\tau)\, dv$$

Inserting $U_r(t) = T_r^{-1} \exp(-t/T_r)$ yields

$$C_r(\tau) \equiv \frac{\sigma_i^2}{2T_r} e^{-\tau/T_r} \qquad (16)$$

The autocorrelation function is

$$R_r(\tau) = \frac{C_r(\tau)}{C_r(0)} = e^{-\tau/T_r} \qquad (17)$$

The time scale is a simple measure of the memory in a process. It is defined as the integral of the autocorrelation function over time $\tau>0$. If the input to a linear reservoir is a time independent process the time scale of the output is equal to the recession coefficient of the reservoir. This is seen by calculating the integral of $R_r(\tau)$ for $\tau>0$.

$$\int_0^\infty R_r(\tau)\, d\tau = \int_0^\infty e^{-\tau/T_r}\, d\tau = T_r \qquad (18)$$

T_r can be calculated by using the method of least squares to fit the function given by Eq. (18) to empirical values of the autocorrelation function of observed water level in the river. In Fig. 3 the empirical autocorrelation function is shown together with a fitted theoretical function in accordance with Eq. (17). T_r was determined to 14.5 days.

It was shown above that the convolution integral Eq. (2) is the solution to both the equation of groundwater flow Eq. (1) and the equation describing a simple linear system Eq. (4). In the former case the impulse response function depends on the parameters of the aquifer as well as the distance from the river. Gottschalk (1977) uses the expression for the impulse response function Eq. (3) to obtain the time scale of groundwater table height for a time independent input at $x = 0$:

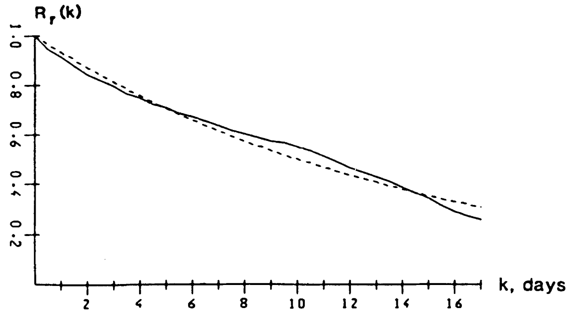

Fig. 3. Empirical (—) and fitted (---) autocorrelation functions based on 12 hour river stage observations. The theoretical function Eq. (17) has been fitted by the method of least squares.

$$\int_0^\infty R_h(x,\tau)\,d\tau = \frac{\Pi S x^2}{2KH} \qquad (19)$$

This expression will be used as an approximation for the recession coefficient $T_a(x)$, i.e. $T_a(x) = \Pi S x^2/2KH$ which was shown to be equal to the time scale (Eq. (18)). The correlation structure related to to the impulse response function Eq. (3) is of course more complex than the simple exponential decay (Eq. (5)). The gain in using the more complex expression is very small with respect to the practical results and the mathematical and numerical treatment is very much simplified if we use the exponential correlation function instead.

The Study Area

The study area is situated in Gaupne in western Norway. It lies in the upper part of the delta deposited by the river Jostedøla at its outlet in the Gaupnefjord. The location is shown in Fig. 4.

The drainage basin of Jostedøla has an area of 861 km. It is characterized by a glacially sculptured topography into Caledonian gneisses of granitic composition. Quaternary ice front deltas and glaciofluvial and marine deposits are the main sources of the sediments comprising the delta and the river plain at Gaupne (Vorren 1973). Together with recent glacial erosion they are still important sources for the sediment load in Jostedøla.

The area lies in a transition zone between maritime and continental climate with precipitation maximum during the autumn and minimum during the spring. Roughly 27% of the drainage basin is covered with glaciers. Annual hydrographs correspond closely to the mass balance of the glaciers and a little less than 90% of annual runoff is discharged during the summer months.

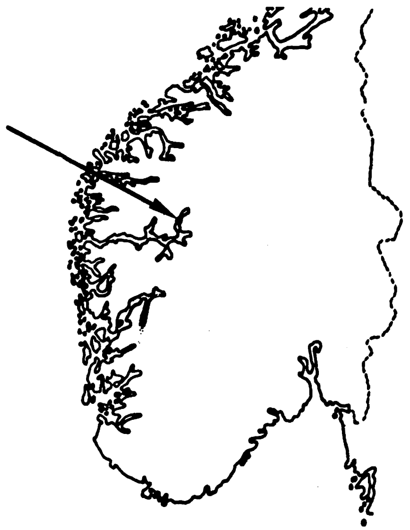

Fig. 4. Location of the study area.

Results

Data from eleven groundwater observation wells on the flood plain and two water gauges in the river have been used in the investigations. Fig. 5 shows location of the observation points. L1-L5 are groundwater stations Nos. 310010-310014 of Norwegian Water Resources and Energy Administration (NVE). All the wells are drilled down to a depth of 5-5.5 m below the ground surface.

Simple soundings, sediment samples, seismic refraction surveys, vertical electrical soundings and pumping tests gave the following results: The aquifer is phreatic, consists mainly of sand and gravel and has a saturated thickness of about 30 m at the river and about 23 m at observation point L5. Its transmissivity is 0.01 m^2/s and the coefficient of storage is 0.2. The pumping well is situated between stations R1 and R2. Sediment samples give values of the hydraulic conductivity between 10^{-4} m/s and 3 10^{-3} m/s.

Water level observations in the wells and the river were performed during the period from April to October 1984. Since there is no systematic change in the mean or the variance and no strictly periodic variations the observed time series are assumed to be stationary.

Empirical values of the autocorrelation function of observed groundwater level in the wells are compared with theoretical values given by Eq. (13). When the values are equal the recession coefficient $T_a(x)$ of the part of the aquifer between the river and each observation well can be determined. Fig. 6 shows empirical and theoretical values of the autocorrelation function for observations made in well L3.

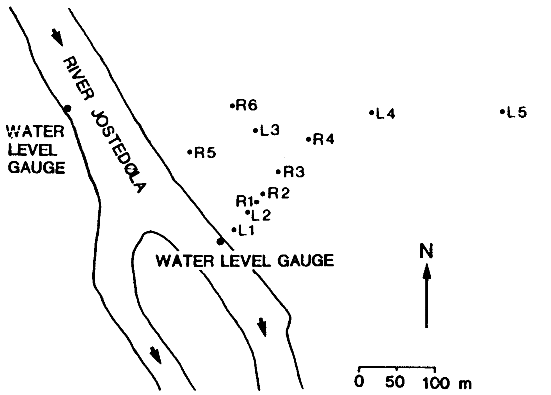

Fig. 5. Schematic map of the investigated flood plain.

When $T_a(x)$ is known Eq. (19) can be used to determine the diffusivity of the aquifer which is equal to the ratio between transmissivity and coefficient of storage. If we choose $S = 0.2$ in accordance with the results of the pumping test the data from the different observation wells can be used to determine the transmissivity. The results are presented in Table 1.

The transmissivity increases with the distance from the river to the observation well that has been applied. Although the sediment samples show no such tencency this may be due to an increase in grain sizes in this direction. There is, however, another reason for this increase: Variations in the water level of the river will quickly affect the groundwater level in its vicinity. As the fluctuations propagate into the aquifer they become dampened. This means that other processes influencing the aquifer, e.g. infiltration after precipitation or throughflow from the valley sides become more important. Since they have a more random variation than the water level of the river the empirical values of the autocorrelation function of groundwater piezometric height become smaller than what would have been the case if the river alone had influenced the aquifer. Eq. (13) and Eq. (15) show that

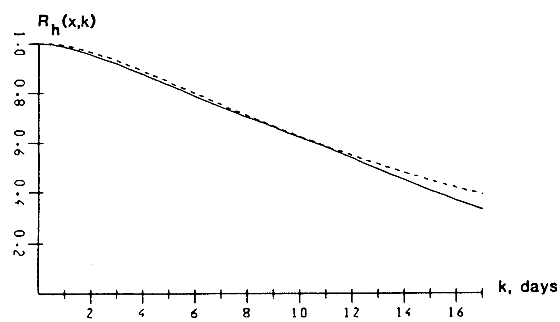

Fig. 6. Empirical (—) and theoretical (---) autocorrelation functions in well L 4. Data has been sampled every 12th hour from April 23, 1984 until October 10, 1984.

Table 1 – Calculated transmissivities from empirical data.

Well	Distance from river x (m)	Recession coefficient $T_a(x)$ (days)	Transmissivity KH (m^2/s)	Storage coefficient S
L1	19	0.06	0.022	0.2
R5	39	0.15	0.035	0.2
L2	48	0.2	0.042	0.2
R1	65	0.3	0.051	0.2
R2	75	0.4	0.051	0.2
R3	113	0.8	0.058	0.2
R6	115	0.9	0.053	0.2
L3	125	1.0	0.057	0.2
R4	170	1.6	0.066	0.2
L4	255	3.0	0.079	0.2
L5	385	6.0	0.090	0.2

$T_a(x)$ must decrease in order to obtain smaller values of the theoretical autocorrelation functions. When $T_a(x)$ decreases the transmissivity increases if the coefficient of storage is constant.

The autocorrelation functions obtained theoretically and from the empirical data show good agreement except for well L5. This is due to the fact that the effects of fluctuations of the water level in the river diminish with the growth of the distance from the river.

If we assume that the diffusivity is constant throughout the aquifer Eq. (19) can be simplified

$$T_a(x) \equiv ax^2 \tag{20}$$

where a is constant throughout the aquifer. The method of least squares has been used to fit the function given by Eq. (19) to the values of x and $T_a(x)$ presented in Table 1. We obtain a = 4.2 10^{-5} days/m^2 and a correlation coefficient of 0.99.

Fig. 7 shows values of x, $T_a(x)$ and the function given by Eq. (20). The transmissivity is equal to the product of hydraulic conductivity K and the saturated thickness H. If we let H vary between 30 m at the river and 23 m at L5, K varies between 7 10^{-4} m/s and 3.9 10^{-3} m/s, which is in agreement with the analysis of the sediment samples.

The pumping test gives smaller values of the transmissivity than the analysis of water level variations in the river and the observation wells. This may be caused by a local decease in grain sizes around the pumping well. The effect of the river on the groundwater level can be traced over a large area and will not be affected much by a local variation in the hydraulic parameters. The pumping test has a small radius of influence and is sensitive to variations in the characteristics of the aquifer.

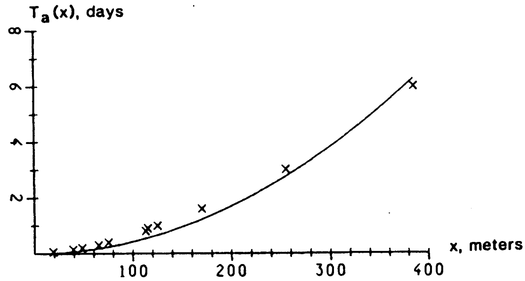

Fig. 7. Fitted functional relationship between aquifer recession coefficients $T_a(x)$ and distance x from the river.

During a pumping test it is assumed that the water flows horizontally. The pumping well is only partially penetrating the aquifer and vertical flow components will exist. The bedded character of fluvial deposits imparts a strong anisotropy to the system, giving horizontal hydraulic conductivity of much larger values than the vertical (Freeze and Cherry 1979). This fact will also reduce the values of transmissivity obtained during the pumping test.

The coefficient of storage is chosen equal to 0.2. This is in agreement with the results of the pumping test and with values usually obtained in phreatic aquifers consisting of sand and gravel.

Observed aquifer response to a varying flood pulse in the river has been simulated by a discrete version of the convolution equation Eq. (2). Since the system is not initially at rest the difference between the mean of the water level in the river and the mean of the water level in the well during the observation period must be subtracted from the values given by Eq. (2). The agreement between observed and predicted groundwater level is good for all observation wells. Fig. 8 and Fig. 9 show examples of the results.

The investigated aquifer has good hydraulic contact with the river and high values of hydraulic conductivity. This is confirmed by the observations which show that the aquifer responds quickly to variations in the water stage of the river.

Conclusions

An expression has been found which relates the autocorrelation function of groundwater piezometric height in a river plain aquifer to the parameters of the aquifer and the autocorrelation function of the water stage in the river. By comparing empirical and theoretical values of the autocorrelation function of groundwater piezometric height in a stream-aquifer system the ratio between transmissivity and coefficient of storage can be determined. The results of other methods, e.g. pump-

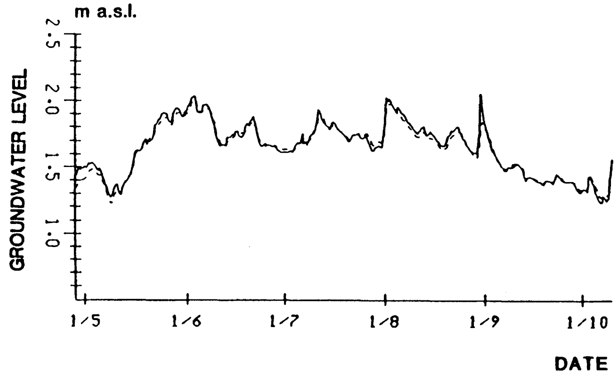

Fig. 8. Observed (—) and calculated (---) groundwater table fluctuations in observation well L 3 (see Fig. 4 for location).

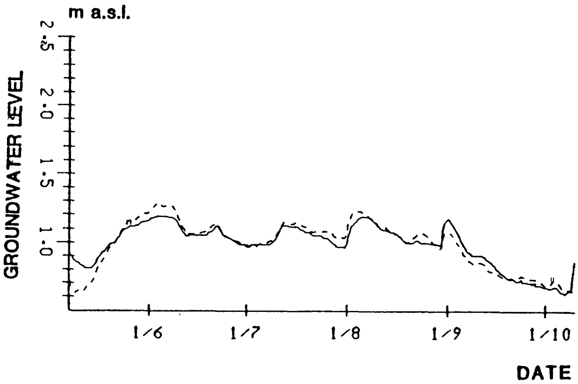

Fig. 9. Observed (—) and calculated (---) groundwater table fluctuations in observation well L 4 (see Fig. 4 for location).

ing test or sediment samples must be known in order to determine the exact value of each parameter.

The convolution equation permits great generality in describing the response of the aquifer to varying water stage in a nearby stream since it permits flood pulses of arbitrary shapes.

Determination of the diffusivity of an aquifer by comparing empirical and theoretical values of groundwater piezometric height may be difficult because the oscillations often are rapid. Comparing the smooth autocorrelation functions is more easy.

The mathematical model that has been applied calculates the response of an aquifer to fluctuations in the water stage of a nearby stream. It does not consider the effect of other factors influencing the aquifer, e.g. infiltration after precipitation, evapotranspiration, throughflow from the valley sides. This gives errors when the groundwater piezometeric height is calculated. The errors increase with the distance from the river bank since the fluctuations having their origin there become dampened as they penetrate into the aquifer.

The methods obtained have been applied to data from a phreatic aquifer. We could also have used data from a confined aquifer. The theory is more correct for this case since one does not have to linearize the continuity equation which describes flow of the water in the aquifer.

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