Equivalent hydraulic resistance to simulate pipes subject to diffuse outflows

M. Ferrante, E. Todini, C. Massari, B. Brunone and S. Meniconi

ABSTRACT

In water distribution network simulation models, pipes subject to diffuse outflow, either due to connections or to distributed demand or to leaks along their length, are generally converted into pipe elements only subject to lumped demand at their ending nodes. This approximation, which disregards the flow variation along the pipes, generates a loss of axial momentum, which is not correctly taken into account in the present generation of water distribution network models. In this paper a correction to the lumped demand approximation is provided and this equivalence is analyzed within the framework of the Global Gradient Algorithm. This is obtained through a correction of the pipe hydraulic resistance; this approach has proven to be more effective than the use of an asymmetrical lumped demand of the total distributed outflow at the pipe ending nodes. In order to assess the effect of the introduced correction, an application to a simple water distribution system is finally provided.

Key words | diffuse outflow, hydraulic resistance, water distribution systems

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INTRODUCTION

Engineers face the study of diffused flow supply along pipes in many fields of application. Simulation of manifolds supplying canal locks, gas burners, devices to control circulation around aircraft, sprinkling irrigation systems and ocean outfalls all require appropriate modeling of a pipe element subject to diffused discharge along its length. When dealing with water distribution networks (WDN), hydraulic engineers face similar phenomena in pipes with several spills either due to actual demand or due to leakages, as in damaged pipes.

The commonly used WDN models require that the flow exchange between the pipe network and the external environment takes place at nodes, i.e. the ends of each pipe element. As a consequence several nodes should be introduced along the pipes to simulate the diffused demand, which inevitably leads to unacceptable model complexity. When simulating a leaking pipe such additional nodes cannot be introduced since the leaks' location and number are unknown. This paper deals with the possibility of correctly representing flow and head losses in numerical models of WDN using an 'equivalent' simpler pipe element exchanging flows at its ending nodes, as an alternative to the introduction of an extremely large number of additional nodes, with particular concern about the effects of the diffused outflow in the model equations of each pipe element. The modified equations, that take into account the effect of the diffused outflow, are formulated so as to preserve the equivalence between a pipe with a diffused outflow along its length and the corresponding pipe with a lumped total demand at its ending nodes. This equivalent pipe element is then introduced in the framework of the Global Gradient Algorithm (GGA) (Todini & Pilati 1988) following the same approach taken by Giustolisi & Todini (2009), namely by introducing a correction to the pipe hydraulic resistance. The equivalence based on this correction has proven to be more effective than the equivalence based on an asymmetric lumping of the flow at the pipe ends, providing more stability and reliability to the numerical model. An application to the simple system used in Giustolisi & Todini (2009) is finally provided to clearly show the effects of the introduced corrections.

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THE MODEL EQUATIONS

The analysis of a pipe subject to a diffused outflow requires a proper formulation both in terms of the continuity and the momentum equations.

Under steady-state conditions, for an infinitesimal length, dx, of a pipe subject to a uniform outflow of an incompressible liquid along its length, the continuity equation can be written as

$$\frac{dQ}{dx} = -q \tag{1}$$

where the variation with the distance x along the pipe of a flow Q equals the outflow per unit length, q. If q is uniformly distributed along the generic kth pipe element of length L_k such as in the considered demand-driven case, it is $q_k = P_k/L_k$, where P_k is the total outflow along the pipe (Figure 1). If the pipe does not distribute flow along the path, $q_k = 0$ and Equation (1) confirms that a single constant flow, Q_k, can be assigned to the whole length of the generic kth pipe, from the starting node i where $x = 0$, to the ending node j ($x = L_k$).

In terms of the momentum equation, when a lateral uniform outflow occurs an additional term has to be introduced

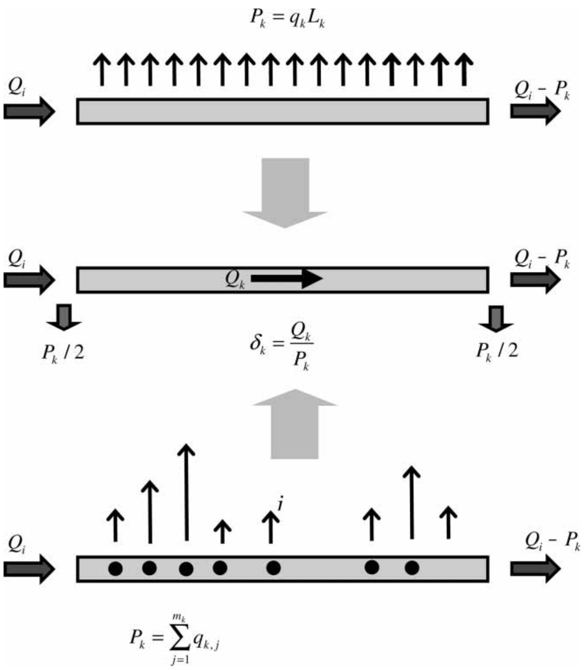

Figure 1 | Lumped and distributed approaches: the uniformly distributing pipe, the equivalent pipe and the pipe with connections.

into the classical momentum equation (McNown 1954; Bajura 1971) to give

$$\frac{dH}{dx} + J + G = 0 \tag{2}$$

Equation (2) demonstrates that, when a pipe with diffused outflow is considered, the variation along x of the total head, H, cannot be completely explained by the total head slope J evaluated by means of the Darcy–Weisbach formula:

$$J = \frac{f(\text{Re, Ke})}{2gDA^2} Q|Q| = K(\text{Re, Ke}, D)Q|Q| \tag{3}$$

In Equation (3) D is the pipe diameter, f is the pipe friction factor which depends on pipe equivalent roughness, Ke, and Reynolds Number, Re, g is the gravitational acceleration and $K(\text{Re, Ke}, D)$ is the pipe unitary hydraulic resistance. In many practical applications, the monomial expression

$$J = K_\infty(\text{Ke}, D)Q|Q|^{n-1} \tag{4}$$

is used instead of Equation (3), with $n = 2$ (i.e. Chezy–Manning or Gauckler–Strickler) mainly in Europe and $n = 1.852$ (Hazen–Williams) as a popular choice in the US (Walski *et al.* 2001). Although Equation (4) is not as general as (3), the main advantage of using such an expression descends from computing K_∞(Ke, D) only once during the simulation. For a fully turbulent flow, the Darcy–Weisbach formula can be expressed by Equation (3) with $n = 2$, $f = f_\infty$ and $K_\infty = f_\infty/2gDA^2$.

The G term in Equation (2) has to be considered for a pipe subject to diffused outflow since this outflow causes a decrease of the discharge in the pipe and a loss of axial momentum. In fact, when $G = 0$ Equation (2) becomes the usual equation of motion, derived under the hypothesis of a constant discharge of real fluid along the streamtube. G takes into account the loss of axial momentum due to the lateral outflow and assuming, without a lack of generality, that the Boussinesq coefficient β is equal to 1, it is

$$G = \frac{C_\beta - 1}{gA^2} Qq \tag{5}$$

where A is the cross-sectional area of the pipe and C_β is a coefficient, experimentally determined, that varies between 0 and 1. For usual applications, several papers (e.g. McNown 1954; Bajura 1971) provide different values of C_β depending on the outflow conditions (e.g. hole in the pipe, T junction, etc.). It is worth noting that Jaumouillé *et al.* (2007) and Jaumouillé (2009), for constant background leakages, suggest a dependence of the distributed discharge effect in Equation (2) on the Coriolis and Boussinesq coefficients. The definition of a relationship between C_β and these coefficients requires a deeper analysis and it is outside the aim of this paper. In the following, values for C_β ranging from 0.7 to 1.0 are used but a more extensive experimental activity could provide better estimates for WDN simulations.

In principle, since $C_\beta \leq 1$ and $G \geq 0$, the sum of the two terms $J + G$ can be positive or negative. In the limit when G prevails over J, the derivative of H with respect to x is positive and hence the total head increases along the flow direction. This result seems to contradict the expected decrease of H with x in steady-state conditions that is associated with an energy dissipation. The apparent paradox can be explained considering that the Bernoulli equation cannot be rigorously used for the diffused outflow case since its extension to a streamtube implies constant discharge. When the more general momentum equation is used, as in the paper by Bajura (1971), the effect of the momentum of the diffused outflow is taken into account, giving rise to the term G. Although the prevalence of G on J is quite unusual in practical applications, as will be shown in the following, the possible head increase along the flow also enhances the relevance of a deeper exploration of the implications of a correct and complete momentum equation for diffused flow in simulating WDNs.

THE EQUIVALENCE DEFINITION

If a uniformly distributed pipe demand is assumed, the differential Equation (2) can be integrated on the pipe element, considering that, at node i, the origin of the coordinate system, it is $H = H_i$ and $Q = Q_i$ while at the other pipe end, j, it is $H = H_j$ and $Q = Q_j$, Q_i (Q_j), being the water flow entering or exiting at the ending node i (j). By means of some algebraic manipulations (Giustolisi & Todini 2009) the integration of Equation (2) yields

$$\begin{aligned}(H_i - H_j)_{distr.} &= \int_0^{L_k} \left(K_{k,\infty} Q_x |Q_x|^{n-1} + \frac{C_{\beta,k} - 1}{gA_k^2 L_k} Q_x P_k \right) dx \\ &= K_{k,\infty} L_k \left(\frac{|\delta|^{n+1} - |\delta - 1|^{n+1}}{n+1} P_k^n \right. \\ &\quad \left. + \frac{(C_{\beta,k} - 1)}{gA_k^2 K_{k,\infty} L_k} \frac{\delta^2 - (\delta - 1)^2}{2} P_k^2 \right) \\ &= R_{k,\infty} \left(\frac{|\delta|^{n+1} - |\delta - 1|^{n+1}}{n+1} P_k^n + \gamma_k \frac{\delta^2 - (\delta - 1)^2}{2} P_k^2 \right) \end{aligned} \quad (6)$$

In Equation (6) the discharge dimensionless parameter δ is defined as

$$\delta = \frac{Q_i}{P_k} = \frac{Q_j}{P_k} + 1 \quad (7)$$

where P_k is the total outflow along the pipe and $R_{k,\infty}$ is the pipe hydraulic resistance, with $R_{k,\infty} = K_{k,\infty} L_k$.

The dimensionless parameter

$$\gamma_k = \frac{(C_{\beta,k} - 1)}{gA_k^2 R_{k,\infty}} \quad (8)$$

can be used to evaluate the relevance of the axial momentum loss with respect to the other terms in the momentum equation (Ferrante *et al.* 2009): when $\gamma_k = 0$ $G_k = 0$ also and this effect can be neglected. It is worth noting that γ_k is dimensional when $n \neq 2$. When the Darcy–Weisbach formula is used for fully turbulent flows, it is

$$\gamma_k = \frac{(C_{\beta,k} - 1)}{gA_k^2 \left(\frac{f_{k,\infty} L_k}{2gA_k^2 D_k} \right)} = \frac{2(C_{\beta,k} - 1) D_k}{f_{k,\infty} L_k} \quad (9)$$

and it can be seen that low values of the ratio L_k/D_k and $f_{k,\infty}$ enhance the axial momentum loss effects. Values of γ_k from -0.2 to 0 are considered in the following for practical applications, since $\gamma_k = -0.2$ corresponds to the concomitant values for $C_{\beta,k} = 0.7$, $L_k/D_k = 200$, $f_{k,\infty} = 0.015$ and $n = 2$.

When a constant flow pipe is considered with the outflow $P_k/2$ lumped at each end node, the integration of Equation (2) with $G = 0$ simply yields

$$(H_i - H_j)_{lumped} = K_{k,\infty}\left(Q_i - \frac{P_k}{2}\right)\left|Q_i - \frac{P_k}{2}\right|^{n-1} L_k = K_{k,\infty}\left(\delta - \frac{1}{2}\right)\left|\delta - \frac{1}{2}\right|^{n-1} P_k^n L_k \quad (10)$$

Equation (10) can be simplified as

$$(H_i - H_j)_{lumped} = R_{k,\infty}\delta_k|\delta_k|^{n-1}P_k^n \quad (11)$$

with

$$\delta_k = \frac{Q_k}{P_k} = \delta - \frac{1}{2} \quad (12)$$

Using this quantity Equation (6) becomes

$$(H_i - H_j)_{distr.} = R_{k,\infty}\left(\frac{\left|\delta_k + \frac{1}{2}\right|^{n+1} - \left|\delta_k - \frac{1}{2}\right|^{n+1}}{n+1} P_k^n + \gamma_k \delta_k P_k^2\right) \quad (13)$$

The equivalence between a pipe subject to diffuse outflow and a pipe with the same total outflow amount equally divided at the ending nodes can be achieved by introducing an equivalence factor ε_k, on the basis of the previously defined equations and using the same methodological approach introduced by Giustolisi & Todini (2009). Using this term the equivalent pipe can be defined similarly to Equation (11) as

$$(H_i - H_j)_{model} = (1 + \varepsilon_{k,n})R_{k,\infty}\delta_k|\delta_k|^{n-1}P_k^n \quad (14)$$

and the term $\varepsilon_{k,n}$ can be evaluated by equating the right-hand sides of Equations (13) and (14):

$$\varepsilon_{k,n} = \frac{\left|\delta_k + \frac{1}{2}\right|^{n+1} - \left|\delta_k - \frac{1}{2}\right|^{n+1}}{(n+1)\delta_k|\delta_k|^{n-1}} + \gamma_k \frac{P_k^{2-n}}{|\delta_k|^{n-1}} - 1 \quad (15)$$

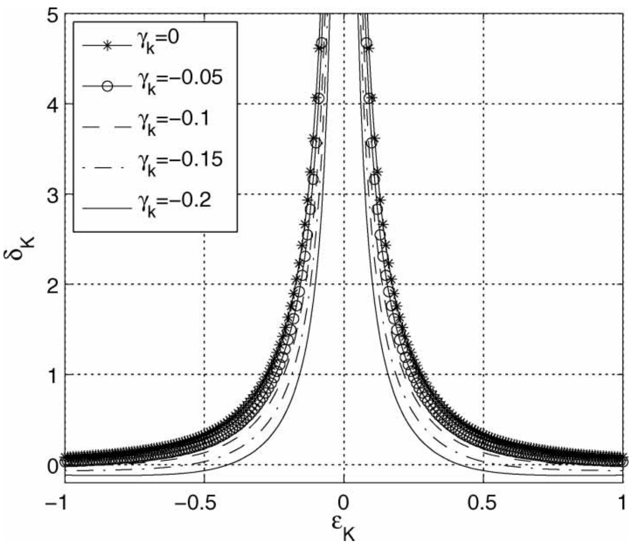

Figure 2 | Variation of ε_k with δ_k and γ_k.

The dependence of the equivalence factor on the parameters δ_k and γ_k is also shown in Figure 2 for $n = 2$. The effect of $G \neq 0$ can be expressed as a reduction in the values of ε_k that can also be negative.

When the equivalent pipe element is considered in the model, the pipe head difference can then be expressed as the sum of the classical head loss plus a head loss correction, E_k^ε, which depends on $\varepsilon_{k,n}$:

$$\begin{aligned}(H_i - H_j)_{model} &= R_{k,\infty}\delta_k|\delta_k|^{n-1}P_k^n + E_k^\varepsilon \\ E_k^\varepsilon &= R_{k,\infty} z_{k,n} P_k^n\end{aligned} \quad (16)$$

with the dimensionless parameter $z_{k,n}$ defined as

$$z_{k,n} = \varepsilon_{k,n}\delta_k|\delta_k|^{n-1} = \frac{\left|\delta_k + \frac{1}{2}\right|^{n+1} - \left|\delta_k - \frac{1}{2}\right|^{n+1}}{n+1} + \gamma_k \delta_k P_k^{2-n} - \delta_k|\delta_k|^{n-1} \quad (17)$$

$z_{k,n}$ can be regarded as the head loss correction E_k^ε for a pipe with $P_k = 1\ \mathrm{m^3\,s^{-1}}$ and $R_{k,\infty} = 1\ \mathrm{s^n\,m^{1-3n}}$ (Giustolisi & Todini 2009; Berardi et al. 2010). Figure 3 shows that a value of γ_k as low as −0.05 introduces remarkable variation of $z_{k,n}$ with δ_k (the curves shown refer to $n = 2$).

If m_k connections are located along the kth pipe, each delivering the demand q_{kj} (Figure 1), instead of considering a uniform distributed demand (Giustolisi et al. 2010),

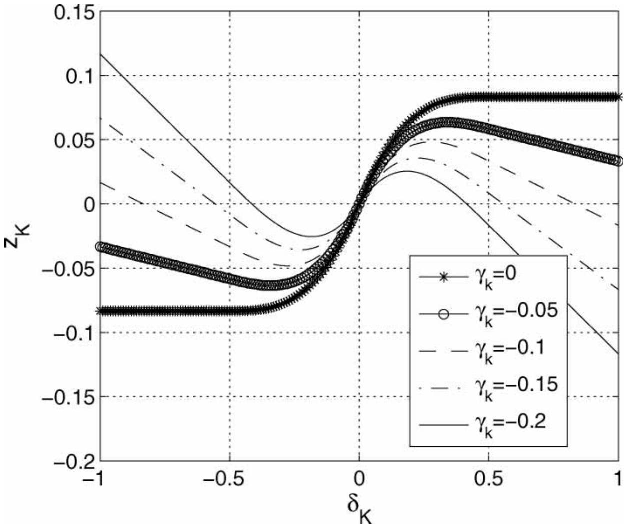

Figure 3 | Variation of z_k with δ_k and γ_k.

Equation (6) becomes

$$(H_i - H_j)_{conn.} = \sum_{i=0}^{m_k} K_{k,i+1}\left(Q_{k,1} - \sum_{j=1}^{i} q_{k,j}\right)\left|Q_{k,1} - \sum_{j=1}^{i} q_{k,j}\right|^{n-1} \times L_{k,i+1} + \sum_{i=1}^{m_k} \frac{(C_{\beta,k,i}-1)}{gA_{k,i}^2}\left(Q_{k,1} - \sum_{j=1}^{i} q_{k,j}\right) q_{k,i} \tag{18}$$

where the i index denotes the ith trunk between the ith and $(i+1)$th connection along the kth pipe. Assuming that

$$\kappa_{k,i} = \frac{K_{k,i}}{K_{k,\infty}}, \lambda_{k,i} = \frac{L_{k,i}}{L_k}, \pi_{k,i} = \frac{\sum_{j=1}^{i} q_{k,j}}{P_k}, \gamma_{k,i} = \frac{(C_{\beta,k,i}-1)}{gA_k^2 R_{k,\infty}}, \delta_k = \frac{Q_k}{P_k}, \frac{Q_{k,1}}{P_k} = \delta_k + \frac{1}{2}, c_{k,i} = \frac{q_{k,i}}{P_k} \tag{19}$$

Equation (18) can be written as

$$(H_i - H_j)_{conn.} = L_k K_{k,\infty}\left\{\sum_{i=0}^{m_k} \kappa_{k,i+1}\left(\delta_k + \frac{1}{2} - \pi_{k,i}\right)\left|\delta_k + \frac{1}{2} - \pi_{k,i}\right|^{n-1} \lambda_{k,i+1} P_k^n + \sum_{i=1}^{m_k} \gamma_{k,i}\left(\delta_k + \frac{1}{2} - \pi_{k,i}\right) c_{k,i} P_k^2\right\} \tag{20}$$

Following the same approach, Equations (15) and (17) become, respectively,

$$\varepsilon_k = \left\{\frac{\sum_{i=0}^{m_k} \kappa_{k,i+1}\left(\delta_k + \frac{1}{2} - \pi_{k,i}\right)\left|\delta_k + \frac{1}{2} - \pi_{k,i}\right|^{n-1} \lambda_{k,i+1}}{\delta_k|\delta_k|^{n-1}} + \frac{\sum_{i=1}^{m_k} \gamma_{k,i}\left(\delta_k + \frac{1}{2} - \pi_{k,i}\right) c_{k,i} P_k^{2-n}}{\delta_k|\delta_k|^{n-1}}\right\} - 1 \tag{21}$$

and

$$z_k = \left\{\sum_{i=0}^{m_k} \kappa_{k,i+1}\left(\delta_k + \frac{1}{2} - \pi_{k,i}\right)\left|\delta_k + \frac{1}{2} - \pi_{k,i}\right|^{n-1} \lambda_{k,i+1} + \sum_{i=1}^{m_k} \gamma_{k,i}\left(\delta_k + \frac{1}{2} - \pi_{k,i}\right) c_{k,i} P_k^{2-n}\right\} - \delta_k|\delta_k|^{n-1} \tag{22}$$

IMPLEMENTATION IN THE GGA

Global Gradient Algorithm

The system of the continuity and momentum equations for a WDN can be expressed in terms of matrices as in Todini & Pilati (1988):

$$\begin{aligned} \mathbf{A}_{11}\mathbf{Q} + \mathbf{A}_{12}\mathbf{H} &= -\mathbf{A}_{10}\mathbf{H}_0 \\ \mathbf{A}_{21}\mathbf{Q} &= \mathbf{d} \end{aligned} \tag{23}$$

where $\mathbf{Q}$, $\mathbf{H}$ and $\mathbf{H_0}$ are column vectors containing, respectively, the unknown discharges in the n_p pipes, the unknown heads in the n_n inner nodes and the known head values at the n_0 outer nodes; $\mathbf{A}_{21} = \mathbf{A}_{12}^T$ is the incidence matrix of the inner nodes and $\mathbf{A}_{10}$ is the incidence matrix of the inner nodes; $\mathbf{d}$ is the column vector of pipe demands lumped at nodes. The classical GGA provides at the generic iteration *iter* an approximated solution of the nonlinear system of Equations (23) by evaluating

$$\mathbf{B}^{iter} = \left(\mathbf{D}_{11}^{iter}\right)^{-1}\mathbf{A}_{11}^{iter}$$

$$\mathbf{F}^{iter} = \mathbf{A}_{21}\left(\mathbf{Q}^{iter} - \mathbf{B}^{iter}\mathbf{Q}^{iter}\right) - \mathbf{d} - \mathbf{A}_{21}\left(\mathbf{D}_{11}^{iter}\right)^{-1}(\mathbf{A}_{10}\mathbf{H}_0)$$

$$\mathbf{H}^{iter+1} = \left(\mathbf{A}_{21}\left(\mathbf{D}_{11}^{iter}\right)^{-1}\mathbf{A}_{12}\right)^{-1}\mathbf{F}^{iter}$$
$$\mathbf{Q}^{iter+1} = \left(\mathbf{Q}^{iter} - \mathbf{B}^{iter}\mathbf{Q}^{iter}\right) - \left(\mathbf{D}_{11}^{iter}\right)^{-1}\left(\mathbf{A}_{10}\mathbf{H}_0 + \mathbf{A}_{21}\mathbf{H}^{iter+1}\right) \tag{24}$$

where $\mathbf{D_{11}}$ contains the derivatives of the head losses with respect to the kth pipe discharge. Instead of considering the classical GGA, the inclusion of the head loss correction term requires the wider framework of the so-called *Enhanced GGA* (Berardi *et al.* 2010) where the matrices defined in (24) can be rewritten as

$$\begin{aligned}
D_{11}^{enh}(k,k) &= nR_{k,\infty}|\delta_K|^{n-1}P_k^{n-1} + R_{k,\infty}P_k^n\frac{dz_k}{d\delta_k}\frac{d\delta_k}{dQ_k} \\
&= D_{11}^{orig}(k,k) + R_{k,\infty}zD_{k,n}P_k^{n-1} \\
A_{11}^{enh}(k,k) &= R_{k,\infty}\left(|\delta_k|^{n-1} + zA_{k,n}\right)P_k^{n-1} \\
&= A_{11}^{orig}(k,k) + R_{k,\infty}zA_{k,n}P_k^{n-1} \\
B^{enh}(k,k) &= \frac{A_{pp}^{enh}(k,k)}{D_{pp}^{enh}(k,k)} \\
&= zB_{k,n} = \frac{|\delta_k|^{n-1} + zA_{k,n}}{n|\delta_k|^{n-1} + zD_{k,n}}
\end{aligned} \tag{25}$$

with the superscripts *orig* and *enh* denoting the original and the enhanced GGA and

$$\begin{aligned}
zD_{k,n} &= \frac{dz_{k,n}}{d\delta_k} = \left(\delta_k + \frac{1}{2}\right)\left|\delta_k + \frac{1}{2}\right|^{n-1} - \left(\delta_k - \frac{1}{2}\right) \\
&\quad\times\left|\delta_k - \frac{1}{2}\right|^{n-1} + 2\gamma_k P_k^{2-n} - n|\delta_k|^{n-1} \\
zA_{k,n} &= \frac{z_{k,n}}{\delta_k} = \frac{\left|\delta_k + \frac{1}{2}\right|^{n+1} - \left|\delta_k - \frac{1}{2}\right|^{n+1}}{(n+1)\delta_k} \\
&\quad + \gamma_k P_k^{2-n} - |\delta_k|^{n-1}
\end{aligned} \tag{26}$$

The variation with δ_k and γ_k of the dimensionless parameters defined by Equation (26) is shown in Figures 4 and 5, where it is clear that a low value of γ_k can also appreciably change the values of the parameters.

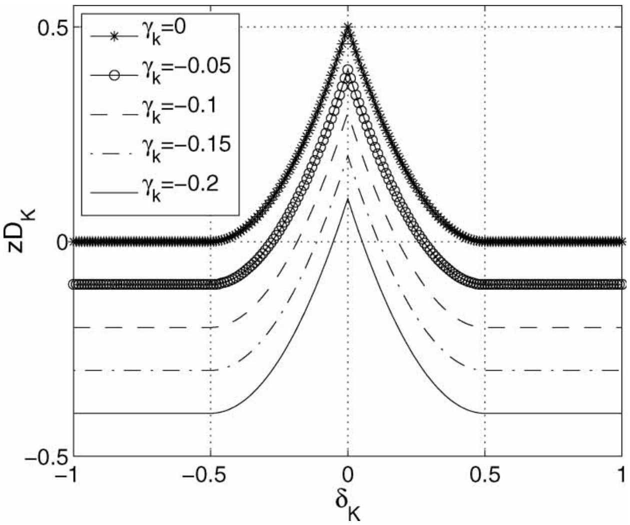

Figure 4 | Variation of zD_k with δ_k and γ_k.

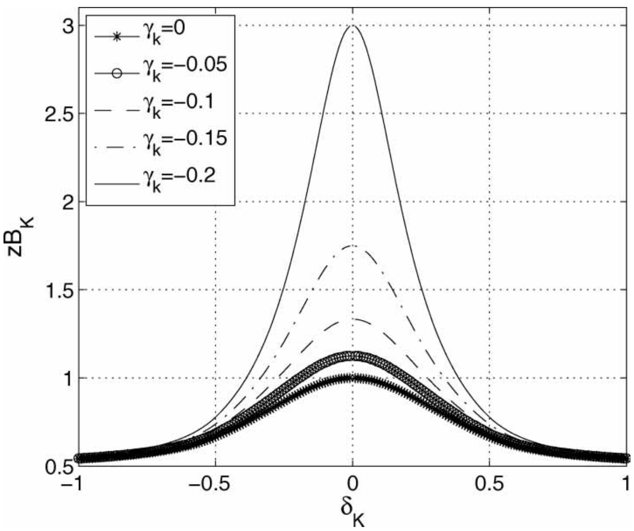

Figure 5 | Variation of zB_k with δ_k and γ_k.

When m_k connections are considered instead of a distributed demand, it is

$$\begin{aligned}
zD_k = \frac{dz_k}{d\delta_k} &= \sum_{i=0}^{m_k}\left\{\left(\frac{d\kappa_{k,i+1}}{dQ_{k,i+1}}\left(\delta_k + \frac{1}{2} - \pi_{k,i}\right)\right.\right. \\
&\quad \left.\left. + n\kappa_{k,i+1}\right)\left|\delta_k + \frac{1}{2} - \pi_{k,i}\right|^{n-1}\lambda_{k,i+1}\right\} \\
&\quad + \sum_{i=1}^{m_k}\gamma_{k,i}c_{k,i}P_k^{2-n} - n|\delta_k|^{n-1}
\end{aligned} \tag{27}$$

and zA_k is evaluated by means of Equation (27).

CASE STUDY

To analyze the effects of the loss of axial momentum, a case study is used with the same simple system considered in Giustolisi & Todini (2008, 2009). This case is particularly useful to understand under which conditions C_β has a significant effect. Figure 6 shows a scheme for the case study, where the two reservoirs T1 ($H_{T1} = 60$ m) and T2 ($H_{T2} = 40$ m) are linked by a series of three pipes with the same diameter ($D_1 = D_2 = D_3 = 0.2$ m) and length ($L_1 = L_2 = L_3 = 300$ m). The first and the third pipe do not distribute flow along their length while pipe 2 has a uniform distributed total outflow P_2. An extended period simulation is considered, with the system changing its operating conditions in time slowly enough so that the steady-state equations always apply. A one-hour time step is used for the 24 h total duration of the simulation. The total distributed flow varies between the minimum value of $P_{2,\mathrm{Min}} = 5\ \mathrm{l\,s^{-1}}$ during the night up to the maximum of $P_{2,\mathrm{Max}} = 20\ \mathrm{l\,s^{-1}}$ in the peak hour, as shown in Figure 7. In order to explore the effect of the γ_k values in different operating conditions, three different cases are considered, with $f_{2,\mathrm{I}} = 0.08$, $f_{2,\mathrm{II}} = 0.04$ and $f_{2,\mathrm{III}} = 0.02$, and the corresponding values of γ_2 given by Equation (8), $\gamma_{2,\mathrm{I}} = -0.005$, $\gamma_{2,\mathrm{II}} = -0.01$ and $\gamma_{2,\mathrm{III}} = -0.02$, respectively, with $C_\beta = 0.7$. The different cases with $\gamma_2 \neq 0$ are compared with the case $\gamma_2 = 0$.

Differences in hydraulic head ΔH_{AB} between A and B for the three considered cases are shown in Figure 8; a comparison with the results of a classical GGA is reported as well.

Figure 9 shows the variation in time of ε_2; the effects of $\gamma_2 \neq 0$ are also appreciable in Figure 10 where relative differences $\Delta\varepsilon_2$ reach a maximum value of about 250% with respect to the case of $\gamma_2 = 0$.

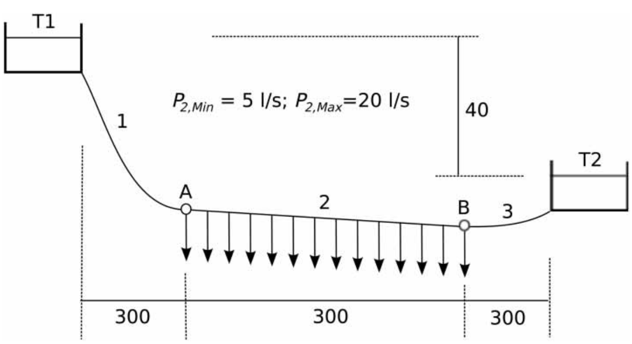

Figure 6 | The system considered in the case study.

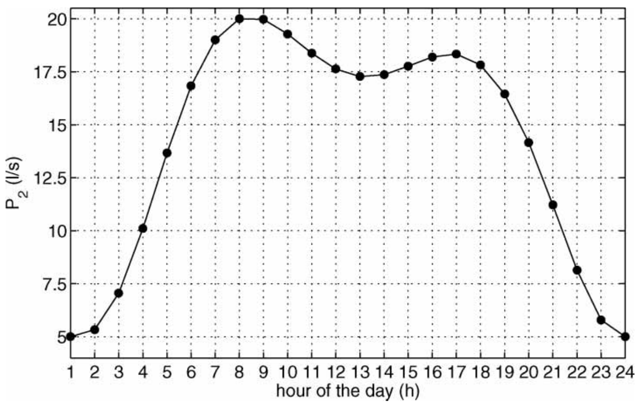

Figure 7 | Variation of P_2 in time for the considered extended-period simulations.

RESULTS AND CONCLUSIONS

The effect on the WDN simulation of the loss of axial momentum in distributing pipes is analyzed, using the GGA approach, as an extension of the work by Giustolisi & Todini (2009). It is demonstrated in this paper that a modified momentum equation must be considered when dealing with pipes subject to diffuse outflow, where a term G balances the effect of the usual friction term J on the derivative of the total head. The modified equations are implemented within the enhanced GGA proposed by Giustolisi & Todini (2009) with the introduction of an additional parameter γ_k (Ferrante *et al.* 2009). Finally, a case study is used to explore the effect of this modified algorithm in practical applications.

As shown in Figure 2, the equivalence between the two momentum equations, the one used in the enhanced GGA and that modified to take into account the loss of axial momentum, seems to introduce a remarkable effect of γ_k on the correction parameter ε_k. The decrease in the values of ε_k with a decreasing γ_k can be explained by considering that this term balances the head losses and hence the amount of the correction. This behavior reflects the variation with γ_k of the parameters z_k and zD_k of the enhanced GGA shown in Figures 3 and 4, since both parameters strongly depend on ε_k: the effect of γ_k on these

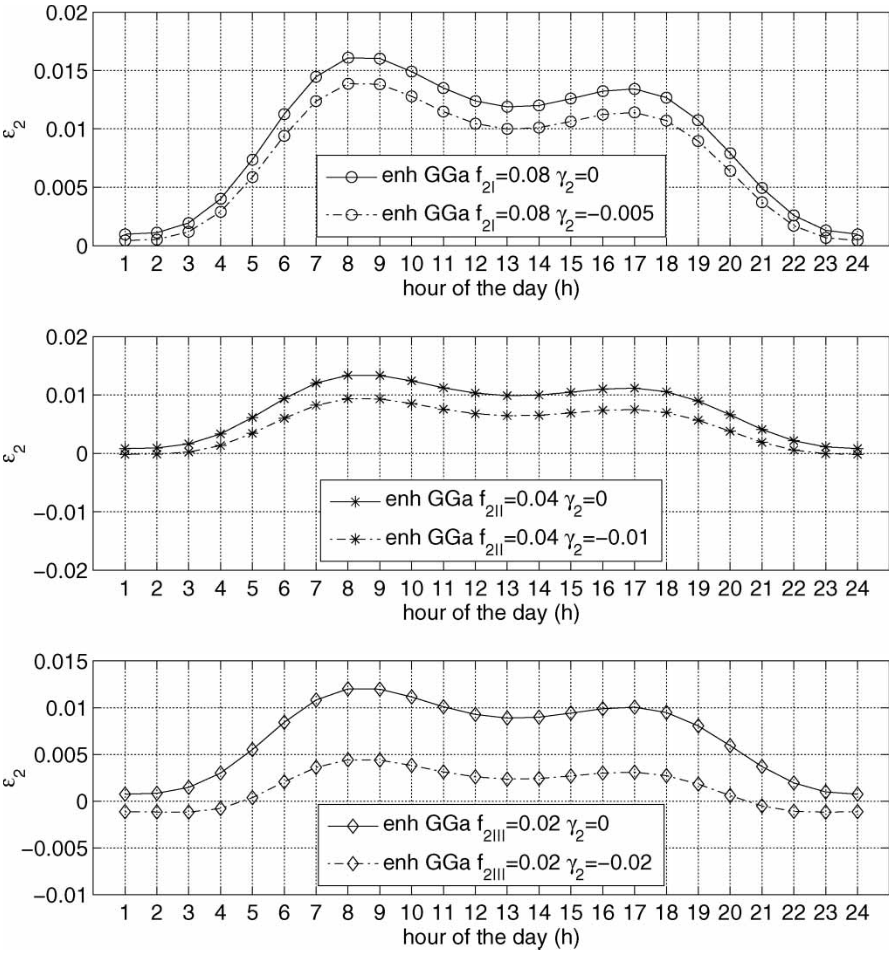

Figure 8 | Variation of ΔH_{AB} during the day for the three considered cases. Results for $\gamma_2 = 0$ are also shown, both for the classical and the enhanced GGA.

parameters seems to be relevant, especially when the limit values are considered for $\delta_k = \pm 1$.

The above-mentioned results are confirmed by the practical application of the case study, where the chosen values of γ_k are very low. Nevertheless, differences of the modified algorithm if compared with the enhanced GGA are still relevant and are of the same order of magnitude as the differences introduced by the enhanced algorithm when compared to the classical GGA, which uses the classical scheme where a lumped demand (or leakage loss) is applied to the ending nodes (e.g. Figure 8). The comparison between enhanced GGA with and without the γ_k term is even more evident in Figures 9 and 10. In Figure 10 the values of $\Delta\varepsilon_k$, i.e. the relative error of ε_k with respect to the $\gamma_k = 0$ case, are as high as 250%.

Two concluding remarks can be drawn. On the one hand, a modified equation needs to be used when considering pipe elements in WDN equivalent to uniformly outflow diffusing pipes. The effects of the modified equations seem to be negligible when low values of γ_k are considered, i.e. long and rough pipes, but still some practical applications could involve these low values and low values could also introduce appreciable effects in the correction term ε_k as shown in this paper.

On the other hand, although uniformly outflow diffusing pipes have been experimentally investigated in the past, the available data on C_β in the literature seem to be inadequate to properly set the value of this parameter in a practical application for WDN. Furthermore, although the proposed case study gives a general idea of what can happen in operational systems, an extensive application of the modified equations to the simulation models of actual WDN will certainly give more

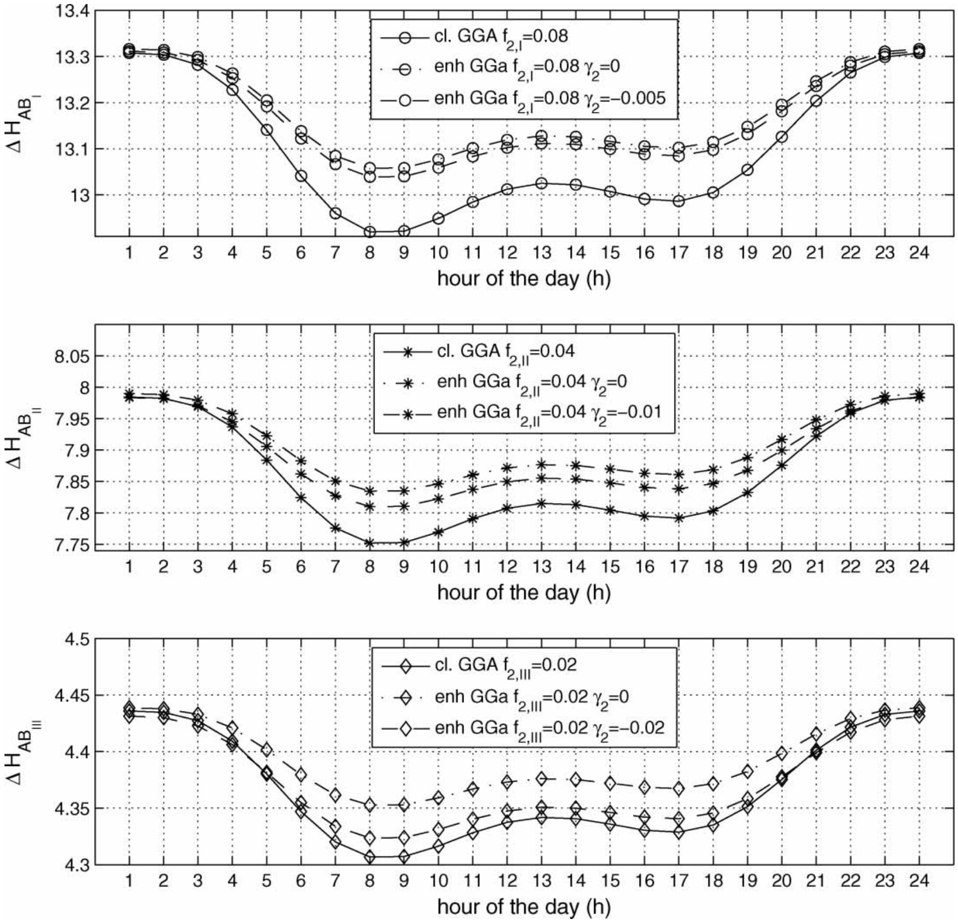

Figure 9 | Variation of ε_2 during the day for the three considered cases, with comparison to the case with $\gamma_2 = 0$.

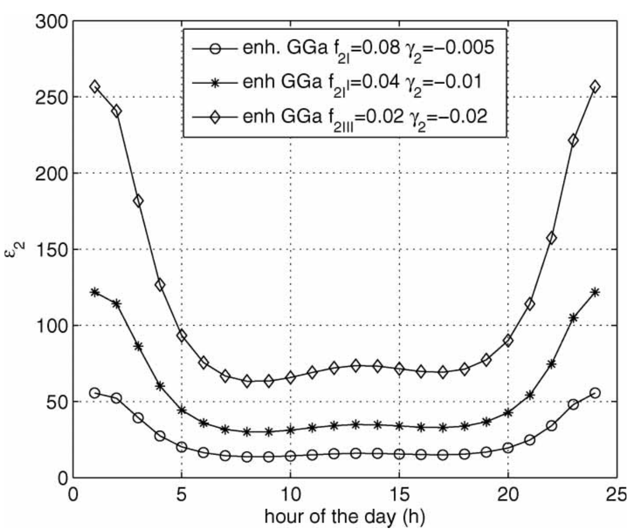

Figure 10 | Relative difference of ε_2 during the day for the three considered cases, as a percentage of the case with $\gamma_2 = 0$.

insights into the practical effects of the proposed corrections.

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