Discussion

K. L. Johnson[1]

It is now well established that, at the high pressures encountered in elastohydrodynamic lubrication, common lubricants exhibit non-Newtonian properties. As the authors state, various constitutive equations have been proposed to describe this behavior, but all incorporate the well-documented feature of shear thinning, i.e., a reduction in effective viscosity with increased shear stress. Naturally I am pleased that the authors have selected the constitutive equations which I favor – the Eyring model – which has the advantage that, in addition to its good fit with experiment it has a physical basis in the thermal activation theory of fluid viscosity. Incidentally, further strong support for the Eyring equation is provided in Evans and Johnson: "Rheological Properties of EHD Lubricants" to be published by Inst. of Mechanical Engineers, London, Proc. Pt. C, *200*, 1986, pp. 303–312.

Non-Newtonian properties of the lubricant have a large influence on traction, but a rather minor effect on film thickness, mainly manifest when sliding accompanies rolling. It may be expedient, therefore, to use the non-Newtonian equation for the analysis of traction, whilst retaining a Newtonian analysis of film thickness. However this practical procedure is basically illogical and requires justification by results. The authors are performing a valuable service by investigating the influence of a firmly based non-Newtonian constitutive equation upon the film shape and thickness and the pressure distribution.

For this purpose it is disappointing that they have not performed comparative calculations assuming Newtonian behavior. This ought to be possible without difficulty in their computations by choosing an appropriately high value for the fluid parameter τ_0 (which we refer to as the *Eyring stress*, since it represents the shear stress above which the fluid becomes significantly nonlinear). The addition of the results of comparable Newtonian computations to the diagrams of film thickness, film shape, and pressure distribution would have greatly added to the educational value of the paper. Would it be possible for the authors to provide a sample, at least, of this information?

B. A. Gecim[2] and W. O. Winer[3]

The authors have made a valuable contribution in analyzing the non-Newtonian effects on film thickness and traction.

First, we should mention that the sinh expression used by the authors and $\tanh^{-1}$ expression used by the discussers (Gecim and Winer, 1980) behave similarly except at high shear rates where the $\tanh^{-1}$ expression approaches an experimentally observed "limiting shear stress" whereas the sinh expression continues to increase without boundary. Furthermore, the parameters τ_{L0} and m in equation (3) are measurable material parameters.

As to the non-Newtonian effects on film thickness, it is reassuring to see that the discussers' earlier results show good agreement with the present results. More specifically, the curves in Fig. 2 corresponding to 60°C indicate that at $g_1 = 1100$ the reduction in film thickness from low slide/roll ratios to a slide/roll ratio of two is approximately 20 percent. With the authors specific load of 10^6 N/m and the rolling speed of 5 m/s (at $g_1 = 1100$) one can find a similar reduction in the discussers' Fig. 13 even though the specific load used by the discussers was different. The discussers show, however, in their Fig. 15 that this reduction is not strongly influenced by the specific load.

Also, it is interesting to note that with an average inlet zone pressure of approximately 0.05 P_0 (as shown in Figs. 5 and 7), using $\tau_L = \tau_0 = 2.9$ MPa yields a value of $m \simeq 0.05$ (where $\tau_{L0} = 0.69$ MPa) in equation (3). The discussers used $m = 0.05$, which was based on experimental data, in their earlier work.

In Fig. 2, as g_1 decreases (or the rolling speed increases with everything else being constant) along a fixed slide/roll ratio curve or, as the slide/roll ratio increases at a fixed g_1, the non-Newtonian behavior becomes more pronounced. In either case shear rate increases and the shear stresses approach their limiting value. It would be interesting to see if more pronounced effects occur at higher rolling speeds.

It can be seen in Fig. 2 that at the same g_1 and slide/roll ratio, the non-Newtonian effects are less noticeable at 90°C than at 60°C. This is due to lower shear stresses resulting from a lower nominal viscosity even though the shear rates are comparable. This indicates the need for adding thermal effects to a non-Newtonian model in order to make the model more realistic, and implies that there is an offsetting behavior between the thermal effects and the non-Newtonian effects.

It is important to realize that the above discussion applies to operating conditions only. However, the non-Newtonian effects are obviously related to the lubricant material parameters characterizing nonlinear rheology. Different fluids can have quite different limiting-shear-stress parameters, τ_{L0} and m. The discussers presented Fig. 16 (as shown here) in their study which outlines the effect of different τ_{L0} and m values on film thickness under constant operating conditions. As τ_{L0} and/or m decreases, the results deviate from the results of Newtonian model and vice-versa. To give an example of low limiting-shear-stress, Jakobsen and Winer, 1975 experimentally determined that dimethylsiloxane had a limiting-shear-stress, $\tau_L = 4$ MPa, at a pressure of 550 MPa, hence the slope m was on the order of 0.001.

The special case of low limiting-shear-stress parameters was analyzed by Gecim and Winer, 1981. It would be interesting to

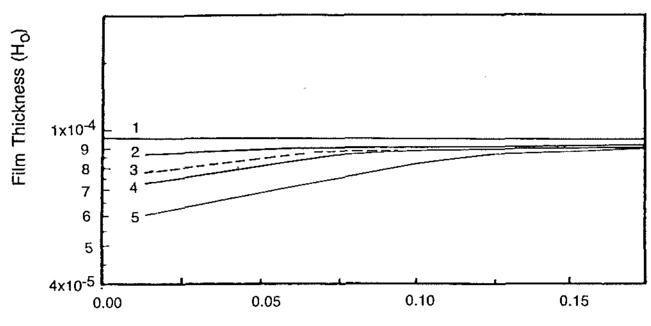

Fig. 16 Dimensionless film thickness versus slope of the limiting shear stress-pressure relation

$w/L = 876$ kN/m

$u_{11} + u_{21} = 5$ m/s

$\mu_0 = 69$ m Pas

$\tau_{L_0} = 0.69$ MPa except for the case 3

1) Grubin's film thickness prediction
2) Model with $\xi = 0$
3) Model with $\xi = 2$ and $\tau_{L_0} = 6.9$ MPa
4) Model with $\xi = 1$
5) Model with $\xi = 2$

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see the results obtained with a low representative stress, τ_0, from the present authors model.

Additional References

Jakobsen, J., and Winer, W. O., 1975, "High Shear Stress Behavior of Some Representative Lubricants," ASME JOURNAL OF LUBRICATION TECHNOLOGY, Vol. 97, p. 479.

Gecim, B., and Winer, W. O., 1981, "A Film Thickness Analysis for Line Contacts Under Pure Rolling Conditions with a Non-Newtonian Rheological Model," ASME JOURNAL OF LUBRICATION TECHNOLOGY, Vol. 103, pp. 305–316.

L. Houpert[4] and B. Hamrock[5]

The discussers would like to point out that an almost identical study has been performed by Houpert and Hamrock [1]. A new Reynolds-Eyring equation has also been derived, and solved in the case of smooth surfaces and surfaces having geometrical defects such as a scratch or a bump.

Identical problems were also found in the convergence in pure rolling condition (convergence was nevertheless obtained for ξ values as small as 0.1).

The authors should point out that their results and conclusions have been obtained for a given value of τ_0 and cannot be generalized. Especially the large reductions in film thickness as shown on Fig. 2 are due to the small value of τ_0 chosen.

τ_0 is obtained by curve-fitting experimental traction curves and is pressure dependent. The values obtained by curve fitting correspond to the average (and large) pressure found in the EHD contact. Also it is important to stress that very little is known about the τ_0 values in the low pressure range corresponding to the pressure in the inlet zone of the EHD contact where the film thickness is built up. Furthermore, since τ_0 is obtained by curve-fitting, its value is also very dependent on the viscosity model as described by Houpert (1985). If the Roelands viscosity model is used instead of the Barus one (as used presently by the authors), much larger values of τ_0 are defined leading to smaller effects of τ_0 on the film thickness. Using the Roelands viscosity model and a corresponding value for τ_0 equal to 1 10^7 Pa, the discussers found that τ_0 had no effect on the central film thickness (for a corresponding g_1 value equal to 142) but decreases the amplitude of the pressure spike as the sliding speed increases. Additional runs have been recently performed with τ_0 equal to 3×10^6 Pa, showing again no reduction of the central film thickness.

In Fig. 2, the authors use the parameter g_1 to describe the reduction of central film thickness due to the nonlinear viscous behavior of the lubricant. The discussers feel that a more appropriate parameter to use for this purpose is $L = E'H_0/(8\ \tau_0)$, where H_0 is the central film thickness calculated with the classical linear visccous model. L represents the ratio between the maximum shear stress at the surface and τ_0 in pure rolling conditions. The parameter L is based on the fact that the maximum shear stress on the surface can be estimated in pure rolling conditions, at abscissa $x = -a$, where the film thickness is H_0 and the pressure gradient dP/dX is approximately equal to P_0/a.

Expressing P_0 and a as a function of the dimensionless load, one can calculate the maximum stress equal to $L\tau_0$. Nonlinear viscous effects will start for value of L larger than 1 (or for $\tau_{max}/\tau_0 > 1$). For L larger than 1, a reduction due to τ_0 of the central film in pure rolling condition can therefore be expected, which is the case for large values of H_0, or for low values of τ_0. These effects cannot be described by the parameter g_1, since g_1 is independent of τ_0, and not proportional to H_0.

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An even stronger effect of τ_0 can be expected when sliding occurs. When sliding speed is present, the maximum shear stress is obtained by adding the shear stress components due to rolling and sliding, as shown in Houpert and Hamrock (1985). The shear stress ratio L' ($L' = \tau_{max}/\tau_0$) is then:

$$L' = L + sh^{-1}\left[\frac{8\xi L^2 U}{H_0^2 sh(L)}\right]$$

where U is the classical dimensionless speed parameter calculated theoretically with the viscosity corresponding to the pressure at abscissa $-a$. This pressure is usually not known, but small, so that one can as a first estimation take for the viscosity, 2 or 3 times the ambient pressure viscosity. A reduction of the central film thickness is then expected when L' is larger than 1, or when the sliding speed is large and τ_0 is small.

It would now be very instructive to see a curve showing the film thickness reduction as a function of the parameter L' (by varying for example τ_0) and curve fit this curve in order to predict later the film thickness reduction as a function of τ_0 and the sliding speed, or mores precisely, as a function of L'.

Concerning Figs. 3 and 4, the discussers believe that the sudden increase of the ratio h_{min}/h_0 could be a mesh effect. For large value of g_1 (or load for example), the pressure spike become very narrow and a finer mesh is required to detect its real amplitude. Using a coarse mesh will then lead to a smaller calculated spike amplitude and a larger minimum film thickness. For large value of ξ, the sudden increase of the ratio h_{min}/h_0 does not appear because the spike amplitude is reduced by the sliding speed.

Additional References

1 Houpert L., and Hamrock, B., "Elastohydrodynamic Lubrication Calculations Used as a Tool to Study Scuffing," *Proc. of the 12th Leeds Lyon Symposium*, paper VI (iv), 1985, pp. 146–159.

2 Houpert, L., "New Results of Traction Force Calculations in Elastohydrodynamic Contacts," ASME JOURNAL OF TRIBOLOGY, Vol. 107, 1985, pp. 241–248.

Authors' Closure

The authors wish to thank Professors Johnson, Winer and Hamrock and Dr. Houpert for their comments. Their observations and suggestions for additional data serve to help us increase our understanding of the effects of the Eyring equation in EHD lubrication. Because of the interrelationship of the various comments made by the discussers, generally no attempt will be made to address these comments on an individual basis. We hope that the comments below will answer most of the questions posed.

One of the questions posed was the effects of τ_0 on the results presented in the paper. Numerical solutions were obtained for values of the Eyring stress, τ_0, ranging from 2.9×10^6 Pa used in the paper to a value of 110×10^6 Pa. We considered the case of $g_1 = 1320$ (which corresponds to $U_0 = 3.5$ m/s), $T = 60°$C ($g_2 = 24.92$), and a slide/roll ratio of 2.0 (simple sliding). The simple sliding case was chosen since it shows definite non-Newtonian effects on film thickness. The minimum film thickness and central film thickness, as functions of the Eyring stress, are shown in Fig. 17. As expected, the film thickness values approach an asymptotic limit as the Eyring stress increases.

The effect of τ_0 on pressure is shown in Fig. 18. Note that the pressure in the inlet region is more pronounced in Fig. 18 than in Figs. 5 or 7 due to the increased shear effects of simple sliding. Also the surface shear stress, τ_1, and the Sigma function, shown in Figs. 19 and 20, respectively, have significant values in the inlet region, which are indicativve of non-Newtonian effects. Note that as the Eyring stress increases, the

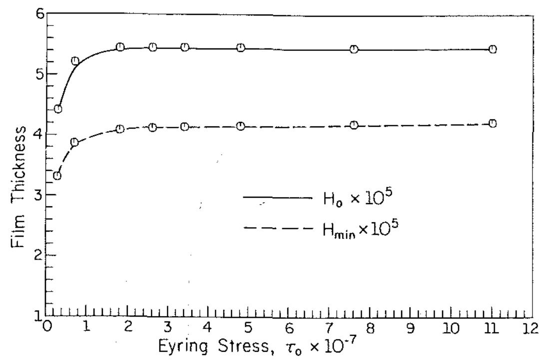

Fig. 17 Dimensionless film thickness versus Eyring stress for U_o = 3.5 m/s, g_1 = 1320, T = 60°C and ξ = 2

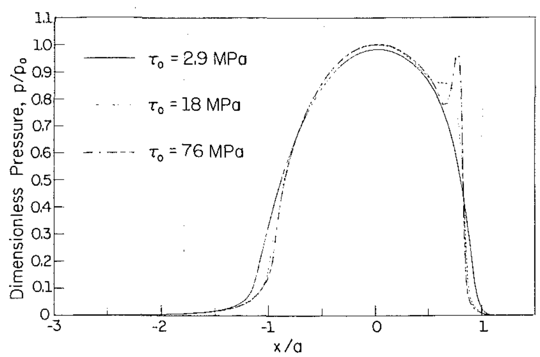

Fig. 18 Effect of Eyring stress on dimensionless pressure for U_o = 3.5 m/s, g_1 = 1320, T = 60°C and ξ = 2

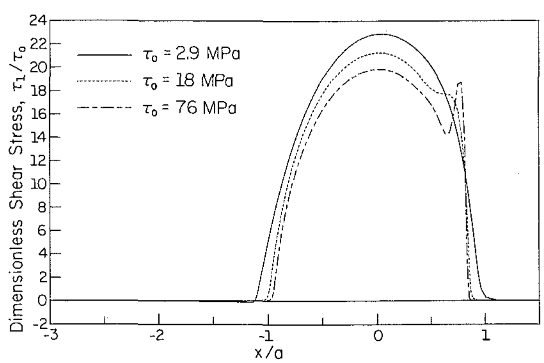

Fig. 19 Effect of Eyring stress on dimensionless shear stress for U_o = 3.5 m/s, g_1 = 1320, T = 60°C and ξ = 2

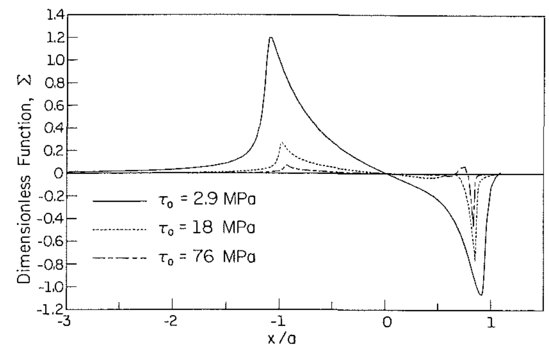

Fig. 20 Effect of Eyring stress on dimensionless function, Σ, for U_o = 3.5 m/s, g_1 = 1320, T = 60° and ξ = 2

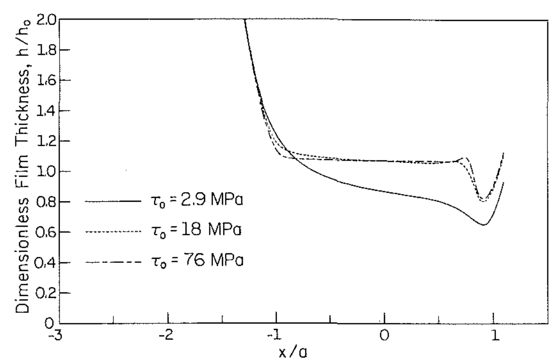

Fig. 22 Effect of Eyring stress on modifying factors, *S(x)*, for U_o = 3.5 m/s, g_1 = 1320, T = 60°C and ξ = 2

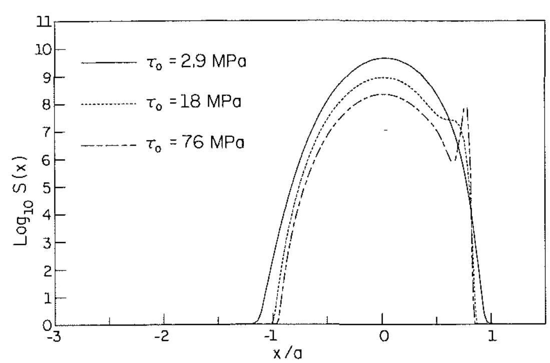

Fig. 21 Effect of Eyring stress on dimensionless film thickness for U_o = 3.5 m/s, g_1 = 1320, T = 60°C and ξ = 2

pressure distribution develops a spike, so often seen in Newtonian solutions, and the pressure and shear stresses in the inlet zone ($x < -a$) diminish.

The film shape, Fig. 21, undergoes a transition from a converging shape with a "gentle" dimple at the outlet to an almost flat shape with an abrupt constriction near the outlet as the value of Eyring stress increases. The central film thickness increases to a value approximately equal to that predicted at a slide/roll ratio of 0.3 as the Eyring stress increases.

The value of the modifying function S(x), Fig. 22, is significant in the inlet region for τ_0 = 2.9 MPa. It decreases to a function with a value of 1.0 in the inlet region as the Eyring stress increases. This indicates that the Newtonian Reynolds equation governs in the inlet for moderately high values of the Eyring stress, which implies that the film thickness should approach the Newtonian values as the value of Eyring stress increases.

The differences between Eyring stress, τ_0, and limiting shear stress, τ_L, are discussed in Evans and Johnson (1986). They note that the Eyring stress, τ_0, is associated with the flow properties of the lubricant, and that the Eyring equation is no longer valid when the work done by the shear stress in promoting flow approaches the value of the activation energy of the fluid. When the shear stress reaches a critical value, the mechanism of flow changes from the Eyring thermally activated motion to a formation of a shear band. This stress value, on the order of G/C ($C \cong 30-45$), may be considered as the limiting shear stress of the material, τ_L. The limiting shear stress, τ_L, and the Eyring stress, τ_0, should be considered as independent fluid properties since it is believed that their physical bases are different.

The issue that must now be addressed is whether the results presented in the paper are valid when the lubricant cannot sustain a shear stress above a limiting value. The peak values of τ_1/τ_0, for all the cases considered, (slide/roll ratio greater than 0.3) were in the range of 20 to 23 for a range of Eyring stress values which varied from 2.9 MPa to 110 MPa. For an Eyring stress of 2.9 MPa, the maximum value of τ_1 would be about 67 MPa, while the corresponding limiting shear stress for LVI-260 oil at a peak pressure of 1 GPa is on the order of 69 to 100 MPa. Thus the limiting value of shear stress was not reached for this case. If an Eyring stress of 8 MPa were used, a peak value of shear stress would be predicted that would exceed the limiting shear stress of the lubricant. This points to the need for more study in this area. If a limiting shear stress is imposed, a large unconstrained value of the velocity gradient at either of the two surfaces could exist under isothermal assumptions, which would contribute to local heating and thus a change in viscosity of the lubricant in the vicinity of the surfaces would be expected. A simultaneous solution of the energy equation would be needed. At lower values of slide/roll ratio, the shear stresses should be lower than what have been presented here. We are presently working to develop computationally efficient solution schemes that will permit calculation of the pure rolling case and cases of very low slip.

The independent dimensionless parameters g_1 and g_2 were chosen for the presentation of data because they, together with a dimensionless film thickness parameter, completely define the EHD conditions for a Newtonian Reynolds equation. A fourth independent parameter needed to completely define the EHD conditions for a Reynolds-Eyring equation would include the slide/roll ratio or the velocity difference. The proposed dimensionless parameter L' has the sliding effect in it, however, we would not recommend the use of a value of H_0 based on a Newtonian constitutive equation because it is not an intrinsic parameter associated with the Reynolds-Eyring equation.

The suggestion that the sudden jump in film thickness ratio in Figs. 3 and 4 is due to a mesh effect or numerical error seems plausible but, even some of our more recent results have not contradicted the general trends given in these figures. The mesh size near the outlet was varied until there was no noticeable change in solution results. The solution scheme sought the largest area of contact for which the pressure was positive, which resulted in a numerical solution that satisfied the outlet boundary conditions of $p = dp/dx = 0$. The outlet conditions have a dominant effect on the solution. The value of the minimum film thickness was very sensitive to the location of the outlet point in the contact. Further, the value of the mass flux was monitored at each point in the contact and one of the convergence criteria was to have the largest difference between mass flux at a point and mass flux at the outlet be less than some very small number.

Houpert and Hamrock (1985) derive a Reynolds-Eyring equation which is equivalent to the equation presented in this paper. As often happens, two sets of investigators have worked on a problem, each unaware of the other's efforts. We wish to point out, however, that the solutions presented in their paper are not valid for the cases of scratches or bumps because they do not consider a case of simple sliding where the surface with the bump or scratch is stationary. In their formulation, both surfaces are moving and, to be correct, a *transient* solution of the governing equations should have been found as the geometry of the problem is changing with time.