Phase-Shift Analysis of Proton-Proton Scattering at $P_L=12$ GeV/c

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An attempt is made to construct the proton-proton scattering amplitude at $P_L=12$ GeV/c by a semi-phenomenological phase-shift analysis in which the phase-shifts of peripheral waves are represented by the one-boson-exchange type amplitudes. A solution having $\chi^2=86$ is obtained for the 168 experimental data points.

In previous papers\(^1,2)\) we have presented the results of the phase-shift analysis of proton-proton scattering at $P_L=6$ GeV/c. The purpose of the present analysis is, as an extension of these previous investigations, to test the feasibility of the amplitude-determination program by the phase-shift analysis up to $P_L=12$ GeV/c and to obtain some information useful for further study.

At 12 GeV/c the number of free parameters to be floated in the $\chi^2$-minimizing process will increase to about 1.4 of that at 6 GeV/c, if a naive impact-parameter argument is made. This together with still scanty experimental data will constrain us to take some phenomenological approach and we follow the one taken in Ref. 1) where the peripheral phase shifts are represented by the one-boson-exchange type amplitudes.

In the phase-shift-reflection-parameter representation of the partial wave amplitudes, we float the phase shifts of states with angular momentum $J<J_0$ and the phase shifts for $J\geq J_0$ are calculated by the one-boson-exchange type amplitudes of which parameters are also floated,\(^1\) whereas the reflection parameters are freed for $J<J_1$ and are fixed at unity for $J>J_1$. We take $J_0=13$ and $J_1=26$ in the present analysis. The number of floating parameters is 97. Here $J=13$ roughly corresponds to the impact parameter 1.1 fm.

The experimental data used in the present analysis are given in Ref. 3). At 12 GeV/c only a limited number of spin correlation parameters have been measured. We have $l_\alpha, l_\beta, l_\gamma, l_\delta, l_\sigma, d\sigma/dt, 65P, 25A, A_{LL}$ as well as $Re F_2$ and $Re F_3$ of dispersion-relation calculation. The total number of data points is 168. Since we have a sizable data only for three observables out of nine independent ones, the obtained solution should be interpreted only for its some gross features.

In finding a best-fitting solution, we have taken the $B$ solution at 6 GeV/c\(^3\) as the starting values for the floating parameters. This gives, however, a formidable large initial $\chi^2$-value to which main contributors are the differential cross-sections at large momentum transfers. A long computing time has been required for fine tuning of the partial waves to produce small amplitudes at large $-t$. Here is an irritating aspect of phase-shift analysis at high energy: The main features of the partial wave amplitudes are

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\(^1\) The boson-exchange amplitude assumed here is $g^2(1/m_s^2)(nA^2/(nA^2+t))^{\sigma}$, $g$, $n$ and $A$ being floated. The normalization mass $m_s$ is taken to be the mass of the observed meson. The $\pi$, $\rho$, $\omega$ and $\phi$ are included.
determined by the experimental data in the forward peak region and the fine tuning gives relatively minor influence on the individual partial waves. This difficulty will be considerably overcome if we go up higher energies by steps of smaller momentum intervals unlike the present case; \( dP_\text{lab} = 6 \text{ GeV}/c \). If the analysis is aimed at the determination of individual partial waves rather than the construction of the full amplitudes at the whole region of \( t \), we may use only the experimental data for \( t \geq t_c \) imposing constraint \( \frac{d\sigma}{dt} \leq \frac{d\sigma}{dt} \text{ at } t \), there, by choosing an appropriate \( t_c \).

At the stage of \( x^2 \approx 10^3 \), we have observed incompatibility between \( \sigma_0 \) and \( \sigma/dt \) at small angles which has been removed by introducing the normalization factors to the differential cross section data. Final outcome is a solution with \( x^2 = 86.2 \).

We give the partial wave parameters (only for \( J \leq 16 \) by space reason) together with other floating parameters and the forward predictions in Table I.

The predictions to some observables are given in Fig. 1. We note, however, that the predictability of the present solution is very limited and is unlikely to reach beyond \( t \geq 2 \text{ (GeV}/c)^2 \).

Only one analysis at a single energy point cannot give much interesting information.

| State | \( \delta \) (degree) | \( \eta \) | State | \( \delta \) (degree) | \( \eta \) | Normalization parameters
|-------|-----------------|------|-------|-----------------|------|-------------------|
| \( ^1S_0 \) | -19.24 | 0.2297 | \( ^1L_0 \) | -6.182 | 0.6760 | \( N_1 = 0.8835 \)
| \( ^1P_1 \) | 34.55 | 0.3108 | \( ^1L_1 \) | -4.879 | 0.7383 | \( N_0 = 0.8531 \)
| \( ^2P_1 \) | -33.68 | 0.2733 | Re \( \rho_\pi \), Im \( \rho_\pi \) | 0.0960 | (0.0) \( ^1 \) | \( N_3 = 1.084 \)
| \( ^3P_1 \) | -33.60 | 0.2546 | \( ^1M_0 \) | -4.382 | 0.7712 | \( \alpha_1 = (3.14) \text{ MeV}, \quad A_0 = 128.0 \text{ MeV} \)
| Re \( \rho_\pi \), Im \( \rho_\pi \) | 0.0153 | (0.0) \( ^1 \) | \( ^1N_1 \) | -5.316 | 0.7684 | \( \alpha_0 = 448.0 \text{ MeV} \)
| \( \alpha_1 \) | -19.18 | 0.2730 | \( ^1N_1 \) | -4.186 | 0.7506 | \( n_e = (1.05) \), \( n_\pi = 0.67 \)
| \( ^1F_1 \) | -27.55 | 0.3306 | \( ^1N_2 \) | -2.871 | 0.8471 | \( n_e = n_\pi = 8.43 \)
| \( ^2F_1 \) | -23.92 | 0.2912 | Re \( \rho_\pi \), Im \( \rho_\pi \) | -0.0030 | (0.0) \( ^1 \) | \( \alpha_0 = 6.5 \pm 0.2 \text{ mb} \)
| \( ^3F_1 \) | -22.25 | 0.2625 | \( ^1L_1 \) | 2.563 | 0.8397 | \( \alpha_0 = 448.0 \text{ MeV} \)
| Re \( \rho_\pi \), Im \( \rho_\pi \) | 0.0549 | (0.0) \( ^1 \) | \( ^1L_2 \) | -3.415 | 0.8471 | \( n_e = (1.0) \), \( n_\pi = 0.67 \)
| \( ^2G_1 \) | -16.14 | 0.3583 | \( ^1L_2 \) | -2.569 | 0.8654 | \( n_e = n_\pi = 8.43 \)
| \( ^2H_1 \) | -18.17 | 0.4022 | \( ^1L_2 \) | -1.308 | 0.9217 | \( \alpha_0 = 6.5 \pm 0.2 \text{ mb} \)
| \( ^2H_2 \) | -14.24 | 0.3863 | \( ^2L_2 \) | 0.0104 | (0.0) \( ^1 \) | \( \alpha_0 = 448.0 \text{ MeV} \)
| \( ^2H_2 \) | -12.08 | 0.3897 | \( ^2L_2 \) | -1.025 | 0.9917 | \( \alpha_0 = 6.5 \pm 0.2 \text{ mb} \)
| Re \( \rho_\pi \), Im \( \rho_\pi \) | 0.0491 | (0.0) \( ^1 \) | \( ^2L_2 \) | -2.037 | 0.9017 | \( \alpha_0 = 6.5 \pm 0.2 \text{ mb} \)
| \( ^1J_1 \) | -11.14 | 0.4897 | \( ^2L_3 \) | -1.301 | 0.9254 | \( \alpha_0 = 448.0 \text{ MeV} \)
| \( ^2J_0 \) | -11.54 | 0.5233 | \( ^2L_3 \) | 0.0820 | 0.9674 | \( \alpha_0 = 6.5 \pm 0.2 \text{ mb} \)
| \( ^3J_1 \) | -9.096 | 0.5347 | \( ^2L_3 \) | -0.0090 | (0.0) \( ^1 \) | \( \alpha_0 = 448.0 \text{ MeV} \)
| \( ^3J_2 \) | -7.435 | 0.5796 | \( ^2L_3 \) | -0.2253 | 0.9374 | \( \alpha_0 = 448.0 \text{ MeV} \)
| Re \( \rho_\pi \), Im \( \rho_\pi \) | 0.0235 | (0.0) \( ^1 \) | \( ^2L_3 \) | -1.014 | 0.9386 | \( \alpha_0 = 448.0 \text{ MeV} \)
| \( ^1K_0 \) | -8.931 | 0.6393 | \( ^2L_3 \) | -0.0491 | (0.0) \( ^1 \) | \( \alpha_0 = 448.0 \text{ MeV} \)
| \( ^2L_0 \) | -7.749 | 0.6546 | | | | |
Combined with the phase-shift solutions at lower energies, the present solutions are found on a smooth extrapolation of lower-energy points. We observe that there is a general tendency for lower partial waves approaching to zero phase shifts with increasing energy.

On the helicity amplitude basis, the present solution considerably retain features of B solution at 6 GeV/c for some amplitudes. The dominant $\text{Im } N_0$ amplitude resembles well that at 6 GeV/c with zero around $-t \sim 1.4 \text{ (GeV/c)}^2$, while $\text{Re } N_0$ has much changed from 6 GeV/c and does not fit to the calculation by dispersion relation by Grein et al. We have forced the solution as for $N_0$ to behave like that of Grein et al. at the last stage of data fitting, but the solution has rejected it. It seems to be necessary to include any data from the beginning at this energy. The predicted differential cross sections at small $-t (\leq 0.3 \text{ (GeV/c)}^2)$ have a curved structure as found at higher energies.

Our conclusion is that the phase-shift-analysis construction of nucleon-nucleon scattering amplitude will be feasible at least up to 12 GeV/c. The crucial factor to the success of such an analysis is a fair accumulation of experimental data. If the analysis is made at small momentum intervals starting from the lower energy, then the slow convergence in the $\chi^2$-minimizing process will be considerably avoided and the uniqueness of the solution will be likely guaranteed. The search with random initial parameters are practically forbidding.

The importance of the phase-shift analysis should be stressed particularly in connection with the dibaryon resonance problem. The observation of resonance spectrum covering a wide energy region is essential for
establishing a proper model of the resonances and the current analysis in the 1–2 GeV/c region should be pushed up to higher energies to uncover the whole picture of resonance structure.

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3) Most of the experimental data used in the present analysis are found in the compilation by J. Bystricky et al., Landolt-Börnstein I/9 (1980), 38.
   c) $\Delta\sigma$: A. Yokosawa, ANL-HEP-CP-80-01.
   (2) J. V. Allaby et al., CERN 68-7 (1968), 580.
   g) $A_{\gamma}$: A. Yokosawa, private communication.