

Optimized operation of water distribution system using multipurpose fuzzy LP model

Y. Arai, A. Koizumi, T. Inakazu, A. Masuko and S. Tamura

ABSTRACT

This research is aimed at multiple-objective optimization of water operations in a water supply and distribution system. These objectives include reducing energy use while at the same time meeting water quality needs. The first objective is to propose water operations aimed at minimizing energy consumption. The second is to optimize water supply and distribution from the standpoint of water quality based on total organic carbon and the third is to attempt optimization that satisfies the first two objectives through multipurpose fuzzy linear programming (LP). This study mathematically formulates water operation planning issues focusing on reducing energy consumption and improving water quality in a water distribution system. Estimates show that a reduction in energy use of around 10% can be expected. Fuzzy LP is applied to achieve a balance among multiple objectives. The research demonstrates the effectiveness of the proposed multipurpose optimization when applied to trade-offs in water operation.

Key words | energy reduction, fuzzy linear programming model, improved water quality, water distribution system

Y. Arai (corresponding author)

A. Koizumi

T. Inakazu

Department of Civil and Environmental Engineering,

Tokyo Metropolitan University (TMU),

1-1 Minami-Osawa,

Hachioji-shi,

Tokyo 192-0397,

Japan

E-mail: y-arai@tmu.ac.jp

A. Masuko

S. Tamura

Bureau of Waterworks,

Tokyo Metropolitan Government (TMWB)/Graduate

School of TMU,

Tokyo,

Japan

INTRODUCTION

Environmental problems represent a significant issue for Japan in various fields (MOE 2011). The Tokyo Metropolitan Government has announced its intention to become the city with the world's lowest environmental impact (TMEB 2011). The Bureau of Waterworks is thus seeking to formulate policies effective from the perspective of energy-saving. One objective is the creation of a new water distribution system that takes into account not only the control of water pressure and water quantity, but also improved energy efficiency and water quality.

The Bureau of Waterworks is striving to reduce CO₂ emissions and energy usage. In this context, attention is focused on electricity usage at treatment plants and supplying stations, which account for the bulk of energy use. The ratio of electricity usage for raw water, water treatment and water distribution is 1:3:6. The bureau is committed to initiatives in areas such as photovoltaic power generation and small-scale hydroelectric power

generation (TMWB 2011a). However these have resulted in a reduction of only around 3% in annual CO₂ emissions (of approximately 300,000 tons). It must be conceded that there are limits to how much such conventional energy-saving measures can achieve. Thus dramatic improvements must be made from a long-term perspective. Initiatives targeted at sectors with the highest electricity usage are required in order to gain the greatest possible reductions (see Figure 1).

Walski confronted the problem of saving energy in pumping operations (Walski 1993). Furthermore, Wu *et al.* analyzed the optimal trade-offs for water distribution systems (Wu *et al.* 2012). Their argument concerns the relationship between cost and greenhouse gas emissions. Although it is beyond the scope of the present work to describe the leakage level, it has been shown in various researches that leaks are costly in terms of energy consumption (Colombo & Karney 2002).

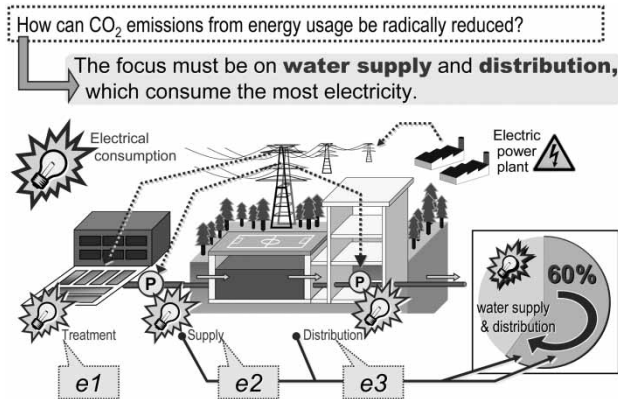


Figure 1 | The water supply system and electricity usage. To reduce electricity usage in the system as a whole, we should aim for greater efficiencies in the most power-hungry processes, namely water supply and distribution.

The purpose of this study is to propose water operations that will minimize power consumption within the water distribution system. The second objective is to devise water supply methods from a water quality perspective, focusing on TOC (total organic carbon: a value indicating the total quantity of organic substances in water). Optimized solutions for various objectives are calculated using linear programming (LP). The third objective is to attempt multi-objective optimization using fuzzy LP (Koizumi *et al.* 1997), applying optimized solutions to the first two objectives. The key topic of this study is to determine from which treatment plants water should be supplied, and through which supplying stations it should be distributed. Where various indicators such as energy minimization and water quality improvement exist, how can water operations be designed to achieve a balance among the different considerations?

METHODS

Network model

This research uses the type of network model illustrated in Figure 2, showing the flow of water from treatment plants to distribution areas. Since this sample is a simple one, it consists of four levels, marked 01 to 04. The relationships among treatment plants, supply stations, and distribution areas can be represented by a simple structural diagram using nodes (points) and links (lines).

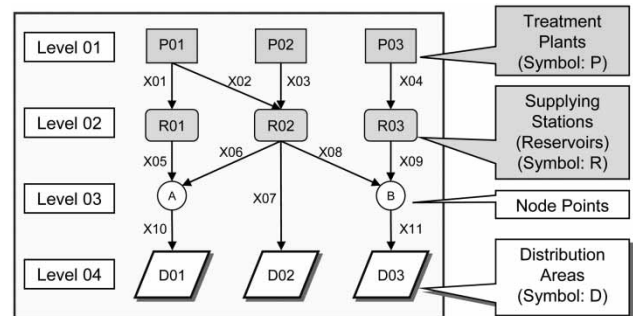


Figure 2 | Network model. Treatment plants (P), supplying stations (R), and distribution areas (D) are shown as nodes and the lines connecting them as links.

Hierarchization using interpretive structural modelling (ISM)

The relationships among facilities must be fully and consistently represented on a mathematical basis. A water supply and distribution system, however, is of course extremely complex. Interpretive structural modelling (ISM) was therefore used to model such an intricate structure. ISM is valued for its capacity to represent hierarchical structures (Warfield 1976). The various steps involved in the ISM technique are as follows: (1) identifying elements; (2) establishing a contextual relationship between elements; (3) developing a structural self-interaction matrix (SSIM) of elements; (4) developing a reachability matrix from the SSIM; (5) partitioning of the reachability matrix into different levels; (6) drawing a directed graph; (7) reviewing the model to check for conceptual inconsistency and making the necessary modifications.

Using this method, the 'level' of each facility as indicated in Figure 3 was determined by devising and calculating a matrix in which 1 and 0 represent the presence or absence of a connection relationship.

Mathematical formulation

Minimizing energy consumption (Objective function 1)

LP (Dantzig 1963) was applied with the function of minimizing electricity usage. Electricity usage was calculated by multiplying the volume of water flowing from treatment plants to distribution areas by the energy consumption

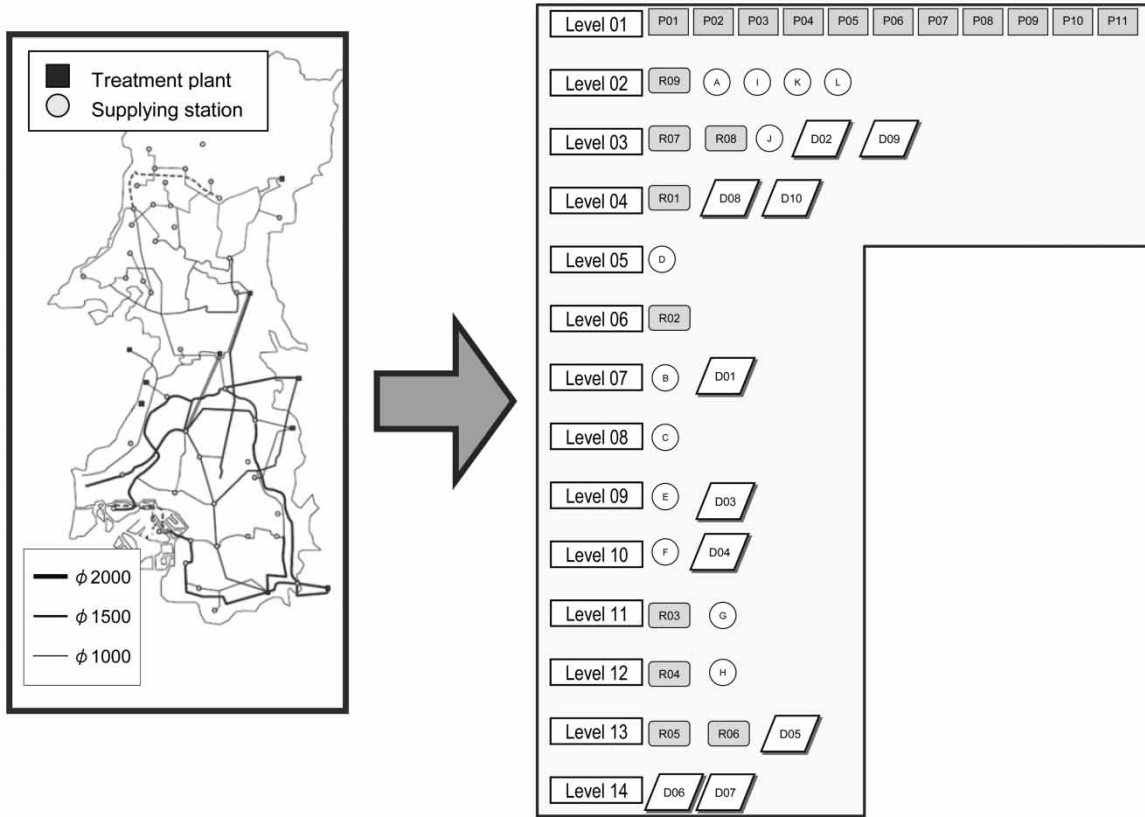


Figure 3 | Target area and result of applying the ISM method. The system examined comprises 11 water treatment plants ($j = 1, 2, \dots, 11$), nine water supply points ($k = 1, 2, \dots, 9$) and 10 water distribution districts ($l = 1, 2, \dots, 10$) (TMWB 2011b). ISM was applied to stratify the relationships among all the facilities, obtaining a composition model ranging from Level 1 to Level 14. Basic data on the water supply as follows: Service area; 1,233.99 [km²]. Population served; 12,822,722 [people]. Pervasion; 100 [%]. Number of service connections; 6,939,984 [cases]. Total length of distribution pipes; 26,219 [km]. Total capacity of facilities; 6,859,500 [m³/day]. Total distribution amount per year; 1,569,336 [$\times 10^3$ m³]. Maximum distribution amount per day; 4,803,400 [m³/day] (Service area, population served, pervasion and number of service connections are numbers as of October 1st, 2010). In addition to water loss, the leakage rate for FY 2010 was 2.7%.

rate of the facilities on each route, as indicated by Equation (1):

$$TE = \sum_i \left(\sum_n e_{n@Yj\sim k} \times Q_i \right) \rightarrow \min \tag{1}$$

where TE is the total energy consumption (electricity usage) in all route, Q_i is the water volume in route

i [m³], and $e_{n@Yj\sim k}$ is the energy consumption rate of each facility Y (treatment plant j /reservoir k) in route i [Wh/m³].

The energy consumption rates (e_n) consist of energy used for water treatment at treatment plants ($n = 1$), energy for supplying water from treatment plants ($n = 2$), and energy for water distribution from supply stations ($n = 3$). Energy consumption differs at each facility (cf. Table 1). The purpose

Table 1 | Energy consumption rate [Wh/m³] (2006 actual values)

Water treatment plant		P01	P02	P03	P04	P05	P06	P07	P08	P09	P10	P11
For water treatment	e1	13	33	23	147	224	188	20	196	80	154	170
For supplying water	e2	54	173	0	147	179	263	0	168	169	152	198
Supply station		R01	R02	R03	R04	R05	R06	R07	R08	R09		
For water distribution	e3	127	100	87	79	129	126	199	115	244		

of this research is to find water volume Q_i at the minimum value of Equation (1).

Evaluation indicator for water quality (Objective function 2)

A second LP problem was then introduced. Using TOC as the indicator of water quality, optimization for minimizing its value weighted by the distance water travels from the treatment plant to the distribution areas was studied. Here WL represents the water quality distance. The presence of large amounts of TOC in drinking water affects the consumption of residual chlorine in water pipes. Two optimization requirements were adopted to account for water quality; the amount of organic substances contained in water should be minimal and the distance traveled by water containing TOC should be as short as possible. Based on this approach, we proposed 'water quality distance WL ' as in Equation (2).

$$WL = \sum_i (C_i \times Q_i \times L) \quad (2)$$

where WL is the water quality distance, C_i is the TOC concentration in route i [mg/m^3], Q_i is the water volume in route i [m^3], and L_i is the route length in route i [km]. The smallest possible value for WL is desirable. Table 2 shows the TOC concentration [mg/L].

Logical constraints for LP

In order to perform the optimization calculation in LP, the logical constraints need to be determined. They can be written as follows:

$$\sum_{i \in J} Q_i \leq P_j \quad (3)$$

$$\sum_{i \in K} Q_i \leq R_k \quad (4)$$

$$\sum_{i \in L} Q_i \geq D_l \quad (5)$$

where P_j is the treatment plant capacity [m^3], R_k is the reservoir permissible volumes [m^3], and D_l is the distribution area demand volumes [m^3].

The total water volume Q_i originating from one treatment plant cannot exceed the treatment capacity of that facility (cf. Equation (3)). The total water volume Q_i flowing into a supply station must be no greater than the permissible reservoir volume of that supply station (cf. Equation (4)). The total water volume flowing into a distribution area must be sufficient to satisfy the demand volume of that area (cf. Equation (5)). Taking the limitation in each link into consideration, the maximum flow in this model is assumed as follows: both P_j and R_k are assumed to be the maximum observed value between 1999 and 2006 respectively; D_l is supposed to be the observed data for 2006. Table 3 shows the water volume of treatment plants/supply stations and the demand volume of distribution areas.

Expanding multipurpose fuzzy LP

When analyzing the results of applying LP, the optimal water operation will differ for each of the objectives. How can outcomes successfully balance differing requirements? To solve this problem, the methodology was extended to multipurpose fuzzy LP (Zimmermann 1976), introducing linear membership functions. Figure 4 shows the linear membership functions.

In the case of energy consumption (left), for example, μ_1 can be normalized within the range of 0–1. The smaller the value of TE , the closer the membership function value moves to 1. Similarly, for water quality distance (right), minimizing WL to the furthest extent possible increases the value of μ_2 .

Using linear membership functions can normalize TE and WL , which have different units, between 0 and 1. Moreover, introducing supplementary variable ' λ ' enables moving from this multipurpose optimization problem to a standard LP problem with the maximization of λ as its objective

Table 2 | TOC concentration [mg/L] (Average monthly value of FY 2006)

Treatment plant	P01	P02	P03	P04	P05	P06	P07	P08	P09	P10	P11
TOC [mg/L]	0.65	0.39	0.44	0.63	0.34	0.46	0.53	0.61	0.88	0.65	0.65

Table 3 | Water volume of treatment plants/supply stations and the demand volume of distribution areas [million m³/year]. Min., Max. and Avg. were based on the actual values from 1999 to 2006

Water treatment plant	P01	P02	P03	P04	P05	P06	P07	P08	P09	P10	P11
2006	289	59	49	1	16	10	54	371	68	280	303
Min.	276	58	15	1	15	10	52	322	62	280	265
Max.	347	72	49	1	25	15	80	395	97	332	319
Avg.	306	63	28	1	21	13	67	358	82	307	292
Supply static	R01	R02	R03	R04	R05	R06	R07	R08	R09		
2006	270	363	127	194	47	102	19	38	26		
Min.	268	288	96	181	34	95	13	0	16		
Max.	352	363	127	213	65	123	21	51	32		
Avg.	299	320	110	193	47	111	18	26	22		
Distribution area	D01	D02	D03	D04	D05	D06	D07	D08	D09	D10	
2006	360	42	173	48	353	47	102	251	96	31	
Min.	339	37	173	36	331	34	95	225	84	28	
Max.	361	104	197	48	353	65	123	257	131	37	
Avg.	350	62	183	41	346	47	111	245	102	33	

function (Zimmermann 1978). Thus, it can be written:

maximize λ
 subject to $\lambda \leq \mu_i, \quad i = 1, 2$

$$(i = 1) \quad \lambda \leq 1 - \frac{TE - TE_L}{TE_U - TE_L} \tag{6}$$

$$(i = 2) \quad \lambda \leq 1 - \frac{WL - WL_L}{WL_U - WL_L}$$

Logical constraints require closer examination of the relationship between λ and μ . In relation to μ_1 and μ_2 , λ is

governed by the smaller μ . Thus a solution should be sought where λ is as large as possible. Even if one μ is large, as long as the other μ is small, the result is that λ will still be small (Bellman & Zadeh 1970). Multipurpose fuzzy LP works by selecting a solution that is not slanted too far to either extreme, with reasonably good levels for both μ_1 and μ_2 (Arai et al. 2009).

RESULTS AND DISCUSSION

Results of applying LP

The results of the LP calculations are shown in Figure 5. On the left is TE-min, where energy consumption is minimized. On the right is WL-min, where water quality distance is minimized. The shading columns stand for more than three times as much as 2006 actual value.

Figure 6 shows estimates from optimizations of the expected improvements in energy consumption and water quality distance respectively. TE-min, the minimum value for energy consumption, is shown on the left. WL-min, the minimum value for water quality distance, is shown in the

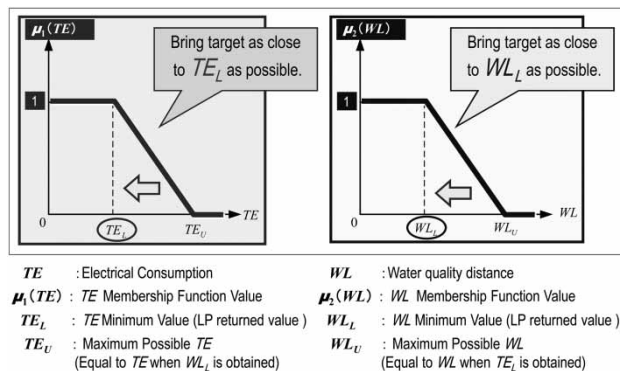
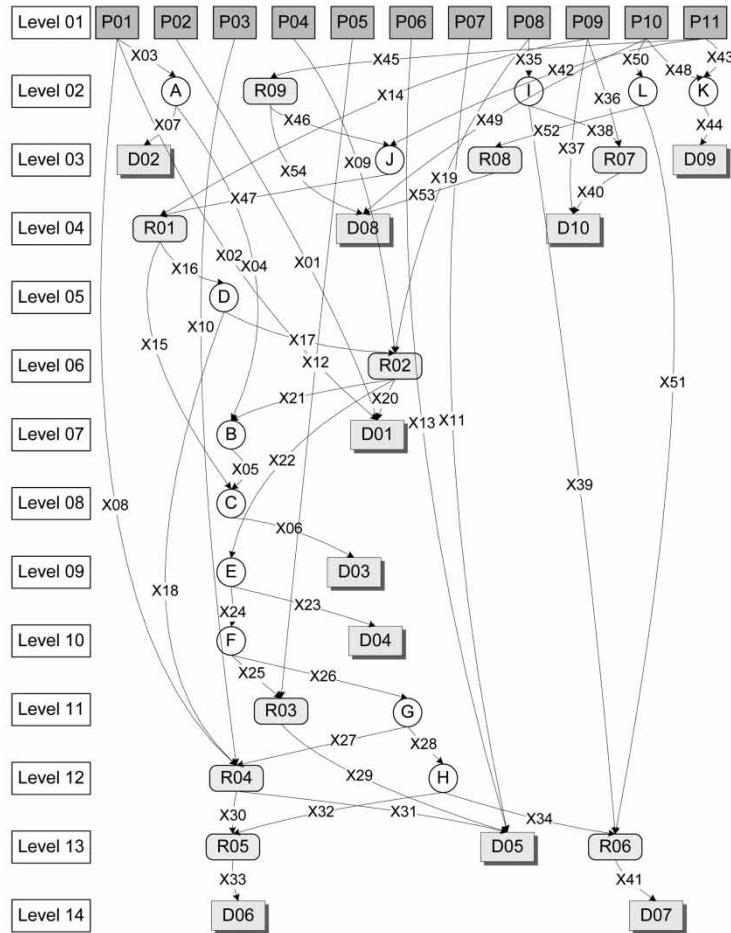


Figure 4 | Linear membership functions for TE and WL.



	Origin -> Terminal	TE-min	WL-min	2006
X01	P02 -> D01	72 (122%)	57 (97%)	59
X02	P01 -> D01	40 (19%)	303 (144%)	210
X03	P01 -> nodeA	209 (354%)	42 (71%)	59
X04	nodeA -> nodeB	167 (1044%)	0 (0%)	16
X05	nodeB -> nodeC	167 (477%)	86 (246%)	35
X06	nodeC -> D03	173 (100%)	173 (100%)	173
X07	nodeA -> D02	42 (100%)	42 (100%)	42
X08	P01 -> R04	98 (490%)	2 (10%)	20
X09	P04 -> R02	1 (100%)	1 (100%)	1
X10	P03 -> R04	49 (100%)	49 (100%)	49
X11	P07 -> D05	80 (148%)	80 (148%)	54
X12	P05 -> R03	25 (156%)	25 (156%)	16
X13	P06 -> D05	15 (150%)	15 (150%)	10
X14	P09 -> R01	66 (138%)	87 (181%)	48
X15	R01 -> nodeC	6 (4%)	87 (64%)	135
X16	R01 -> nodeD	66 (49%)	0 (0%)	135
X17	nodeD -> R02	0 (0%)	0 (0%)	33
X18	nodeD -> R04	66 (64%)	0 (0%)	103
X19	P08 -> R02	362 (110%)	362 (110%)	329
X20	R02 -> D01	248 (273%)	0 (0%)	91
X21	R02 -> nodeB	0 (0%)	86 (478%)	18
X22	R02 -> nodeE	115 (45%)	277 (109%)	253
X23	nodeE -> D04	48 (100%)	48 (100%)	48
X24	nodeE -> nodeF	67 (33%)	229 (112%)	205
X25	nodeF -> R03	20 (18%)	20 (18%)	111
X26	nodeF -> nodeG	47 (50%)	209 (222%)	94
X27	nodeG -> R04	0 (0%)	162 (736%)	22

	Origin -> Terminal	TE-min	WL-min	2006
X28	nodeG -> nodeH	47 (65%)	47 (65%)	72
X29	R03 -> D05	45 (35%)	45 (35%)	127
X30	R04 -> R05	0 (0%)	0 (0%)	32
X31	R04 -> D05	213 (131%)	213 (131%)	162
X32	nodeH -> R05	47 (313%)	47 (313%)	15
X33	R05 -> D06	47 (100%)	47 (100%)	47
X34	nodeH -> R06	0 (0%)	0 (0%)	57
X35	P08 -> nodeI	33 (79%)	21 (50%)	42
X36	P09 -> R07	0 (0%)	0 (0%)	8
X37	P09 -> D10	31 (258%)	10 (83%)	12
X38	nodeI -> R07	0 (0%)	21 (210%)	10
X39	nodeI -> R06	33 (106%)	0 (0%)	31
X40	R07 -> D10	0 (0%)	21 (111%)	19
X41	R06 -> D07	102 (100%)	102 (100%)	102
X42	P11 -> nodeJ	6 (3%)	0 (0%)	209
X43	P11 -> nodeK	84 (124%)	85 (125%)	68
X44	nodeK -> D09	96 (100%)	96 (100%)	96
X45	P11 -> R09	0 (0%)	32 (123%)	26
X46	R09 -> nodeJ	0 (0%)	0 (0%)	13
X47	nodeJ -> R01	6 (3%)	0 (0%)	222
X48	P10 -> nodeK	12 (43%)	11 (39%)	28
X49	P10 -> D08	251 (126%)	168 (84%)	200
X50	P10 -> nodeL	69 (133%)	153 (294%)	52
X51	nodeL -> R06	69 (493%)	102 (729%)	14
X52	nodeL -> R08	0 (0%)	51 (134%)	38
X53	R08 -> D08	0 (0%)	51 (134%)	38
X54	R09 -> D08	0 (0%)	32 (246%)	13

Figure 5 | Results of applying LP for TE-min and WL-min.

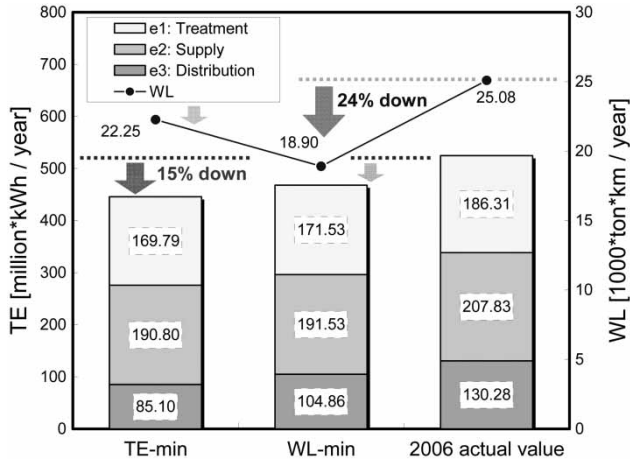


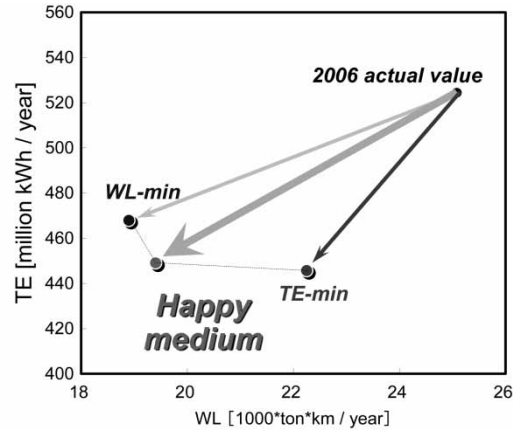
Figure 6 | Estimates of the expected improvements in energy consumption and water quality distance from the LP solutions.

middle column. On the right are actual results for fiscal 2006, as estimated based on water volume. Looking at energy consumption on the vertical axis, a reduction in *TE* of around 15% can be expected. When measuring water quality distance represented by the line plot, these estimates point to an expected improvement in *WL-min* of more than 20%.

Results of applying fuzzy LP

The results of applying multipurpose fuzzy LP are shown in Figure 7. At the top are the results plotted on the two coordinates of energy consumption and water quality distance. At the bottom is the water supply and distribution pattern for λ -max (resulting from multipurpose optimization). ‘Same’ means that the flow obtained by multipurpose fuzzy LP equals ‘TE-min’ and/or ‘WL-min’.

By applying multipurpose fuzzy LP, a value exactly intermediate to the optimal solutions to the two individual objectives was obtained. This can truly be called a ‘happy medium’ solution to balancing the different objectives of water operation, which enables reasonably good levels for both energy consumption and water quality distance. This result was affected by the shape of membership function. Definition of linear membership function is simple as shown in Figure 4; however, it is commonly employed in many fields to solve the multi-objective problem since it is the clear-cut definition of function.



	λ -max	TE-min	WL-min		λ -max	TE-min	WL-min
X01	72	same		X28	47	same	
X02	288			X29	45	same	
X03	42		same	X30	0	same	
X04	0		same	X31	213	same	
X05	101			X32	47	same	
X06	173	same		X33	47	same	
X07	42	same		X34	0	same	
X08	17			X35	8		
X09	1	same		X36	0	same	
X10	49	same		X37	31	same	
X11	80	same		X38	0	same	
X12	25	same		X39	8		
X13	15	same		X40	0	same	
X14	66	same		X41	102		
X15	72			X42	6	same	
X16	0		same	X43	96		
X17	0	same		X44	96	same	
X18	0		same	X45	13		
X19	362	same		X46	0	same	
X20	0		same	X47	6	same	
X21	101			X48	0		
X22	262			X49	238		
X23	48	same		X50	94		
X24	214			X51	94		
X25	20	same		X52	0	same	
X26	194			X53	0	same	
X27	147			X54	13		

Figure 7 | Results of applying fuzzy LP. The optimized solution obtained with multi-objective fuzzy LP represented a ‘happy medium’ between the two single-objective alternatives.

CONCLUSION

This study mathematically formulated water operation planning issues, focusing on reducing energy consumption and improving water quality in a water supply and distribution system. These estimates show that a reduction in energy use of around 10% can be expected. Fuzzy LP was applied to achieve a balance among multiple objectives. The research demonstrates the effectiveness of the proposed multipurpose optimization when applied to trade-offs in water operation.

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