## Conclusions

For zero pressure gradient and a range of free-stream turbulence intensity of 0.8 to 2.3 per cent at the test station, the following conclusions are drawn regarding incompressible flow in a corner of a rectangular channel:

1 Isotach patterns are essentially independent of free-stream turbulence intensity. (This cannot be the case for all ranges of turbulence intensity because in laminar flow ($I = 0$) secondary flows do not occur and the isotachs do not have the characteristic shape which is associated with secondary flows.)

2 Prandtl's hypothesis ($w'/v' > 1$) is verified in regions of isotach curvature.

3 The ratio $w'/v'$ increases with increasing isotach curvature in the vicinity of the bisector of the corner angle, but this is not the case elsewhere in the flow field.

4 At any point the ratio of turbulence components in orthogonal directions in a plane normal to the mean flow direction is a maximum for directions tangent and normal to the isotach at that point.

The last conclusion indicates that the isotachs may form a natural co-ordinate system which is more promising for analysis than a co-ordinate system based on the channel boundaries.

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# DISCUSSION

## H. A. Einstein[2]

This very well prepared paper describes the empirical results of velocity measurements in a straight duct of rectangular cross section. The resulting lines of constant velocity are following the boundary much closer near concave corners of the cross section in turbulent flow than in laminar flow. Following the intuitive arguments of Prandtl this distortion is explained as a consequence of secondary currents which supposedly occur only in turbulent flows.

It may be mentioned that no attempt was made to actually measure or observe the circulations or the vorticity in the flow. Even a qualitative observation proving the existence of the circulations would have been very valuable.

A theoretical description of the turbulent flow in such a duct would probably use the vorticity form of the Navier-Stokes equations. Einstein and Li [5] used these equations to show under which conditions vorticity in the direction of flow is spontaneously generated in a flow in which this component of vorticity is zero. This approach greatly reduces the number of terms in the equation, but only indicates a trend, rather than permits the calculation of an equilibrium flow. No method is known today for the theoretical derivation of such equilibrium flows including secondary currents.

## A. W. Marris[3]

It was a pleasure for this discusser to read this account of an extremely fundamental investigation very well prosecuted. The authors are to be congratulated.

The unequivocal verification that the turbulent velocity component tangential to the isotach always exceeds the normal component verifies Prandtl's hypothesis and provides a means whereby fluid in the plane perpendicular to the mean motion could be accelerated toward the corner. These experimental results are a fine commentary on Prandtl's intuitive genius. The accuracy of these difficult measurements is likewise a fine commentary on the authors' experimental ability.

Consider the streamlines of the mean secondary flow shown schematically by the dotted curves of the authors' Fig. 1. As the secondary flow is first created and accelerated these streamlines would converge together, that is, would converge on the bisector of the corner angle. As the secondary flow approaches the corner these streamlines will diverge again due to presence of the walls. This is shown in the authors' Fig. 1. The acceleration and subsequent retarding of the secondary flow may be considered as taking place in an unlimited flow field. By considering the equation of turbulent energy production from the mean secondary flow, it may be shown that the initial acceleration produces a decrease in the turbulence energy while the slowing up of the secondary flow will be accompanied by an increase in the turbulent energy [13]. One would like to ask the authors if they noticed any change in the total turbulence energy along the bisector of the corner angle.

The authors' discovery that the ratio $w'/v'$ increases with isotach curvature in the region about the bisector of the corner angle but not elsewhere in the flow field may perhaps support the following idea concerning the distribution of turbulent energies corresponding to the turbulent velocity components along and perpendicular to the mean secondary flow streamlines

(1) In the creation of the secondary flow and the initial convergence of the secondary flow streamlines, the turbulent intensity in the direction of the mean secondary flow is being increased while that in the cross-flow direction is being reduced.

(2) In the subsequent divergence of the mean secondary flow streamlines, the turbulent intensity in the direction of the secondary flow is decreased while the cross-flow turbulent intensity is correspondingly increased.

This phenomenon of the elongation of the turbulent stress ellipsoid in the direction of an accelerated mean flow, and the


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corresponding flattening of the ellipsoid in the direction of a retarded flow, is met with in the problem of flow in a curved channel of large depth-to-width ratio. Fluid at the inner, convex-to-flow wall is accelerated and the turbulent intensity corresponding to the flow-wise direction greatly exceeds the cross-flow intensity. Near the outer or concave-to-flow wall where the flow is retarded the cross-flow turbulent intensity is large and the turbulence is more nearly isotropic [14].


**Additional References**

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## Authors' Closure

The authors would like to thank Professors Einstein and Marris for their discussions.

No direct observations or measurements of the secondary currents were made in this investigation because the existence of these currents has been well established qualitatively [1, 12] and in the case of fully developed flow they have been measured by Hoagland [7]. Furthermore, for the purpose of verifying Prandtl's hypothesis, no such measurements were necessary, but it would indeed be enlightening to compare secondary flow strengths at different free-stream turbulence intensities. Certainly further quantitative studies of turbulence structure in corner flow should involve complete measurements of the three-dimensional mean flow in conjunction with turbulence measurements.

Along the bisector of the corner angle there was a large variation in turbulence energy. Unfortunately, the causes of the variation cannot be isolated since the turbulence energy distribution along the bisector is the result of combined convective and viscous effects as well as energy transferred from the mean and secondary flows through the turbulence shear stresses. Calculations based on the results of Hoagland [7] and Klebanoff [9] indicate that the variation resulting from acceleration of the secondary flow component along the bisector is of the same order of magnitude as the total variation found along a line normal to the wall in a two-dimensional boundary layer.

The suggestion of Professor Marris regarding the interaction between the mean flow accelerations and the variation in shape of the turbulent stress ellipsoid is interesting. It is based on the view that the directional characteristics of turbulence are established by the mean flow, including the secondary currents, whereas we have followed Prandtl in regarding the secondary currents as results of the turbulence structure which accompanies curved isotachs. The acceleration of the fluid near the convex-to-flow wall of a curved channel (with either laminar or turbulent flow) follows from dynamic considerations of the mean flow only, and undoubtedly the turbulence structure is largely determined by this acceleration. In contrast, the secondary flow in a corner does not follow from dynamic considerations of the mean flow but appears to be the result of the turbulence structure which accompanies curved isotachs, although the presence of the secondary flow must in turn influence the turbulence structure.

Prof. J. W. Delleur has called to our attention the work done by Rodet [15]. Since in both Rodet's investigation and the present one only five of the necessary six components of the turbulent stress tensor were measured, a full comparison of the results of the two investigations cannot be made.


**Additional Reference**

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