with present oil film thickness results. It can be seen that thermal theory shows better agreement with experimental results than isothermal theory.

Present experimental oil film thickness results are compared with theories of Cheng [17] which considered thermal consideration in the inlet zone and Chui [24] in the case of a model for starvation in a rolling-element bearing, Fig. 11. It is clear from present results that the oil could maintain a film separating disks without failure and is unaffected by oil film replenishment even at contact flow number 6×10^{-7}, while experimental results at contact flow number greater than 6×10^{-7} do not show starvation compared to the starvation model [24]. The results compared favorably with thermal solution [17] and that considering ellipticity parameter [25].

Conclusions

The experimental results presented herein show that the EHD behavior under rolling situation is qualitatively similar to that under pure sliding or combined sliding and rolling. The existence of a spike past the maximum Hertzian pressure at the contact and the presence of a constriction at the exit of the oil film are experimentally confirmed. Compared to available EHD theories, it is expected that thermal solutions, though complicated, are in better agreement with the experimentals than isothermal ones.

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DISCUSSION

L. Houpert[1]

Many calculations of the EHD film, pressure, and surface temperature distributions have been done in the past, but only a few experimental results are available. There is indeed a serious need for experimental verification, and the authors should be congratulated for their efforts. The potential of the technique used by the authors is enormous and the discusser hopes that the authors will be able to tell us in the near future about the transient effects of roughness and lubricant rheology on film, pressure, and temperature. This type of information has not even been provided theoretically because of the complexity of the calculations. Experimental measurements seem presently to be the only tool and also the more realistic way to understand the such previously mentioned effects.

1) Looking at Figs. 2 and 3, the discusser would like to ask the authors if they have any explanations for the pressure in the outlet zone of the contact? The measured pressure is rather large and drops very slowly to zero. One could even suspect that there is a mistake in the rolling direction for the pressure. The inlet and outlet zones should be reversed, leading to a well accepted pressure build up in the inlet zone, and a sharp pressure drop in the outlet zone.

2) Concerning now the pressure maximum of Fig. 3, could the authors explain why it moves from the left side to the right side and then back to the center of the contact, as the load increases?

3) Do the authors have any comments for Fig. 5 at high loads where, as opposed to the low load case of 3000 N, the film constriction does not move towards the center of the contact as the speed increases, but in the other direction?

4) At 7000 N, it is also difficult to accept that the measured pressure is twice as large as the maximum Hertzian pressure. Why would the surfaces not deform elastically in order to reduce the pressure? It would be very interesting to check from surface roughness and shape measurements whether or not the

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transducer perturbs the initial smooth cylindrical shape, and therefore also the pressures.

5) The discusser also has the feeling that the simulated conditions are very close to the Piezo-Viscous-Rigid regime (PVR) where elastic deformations are neglected in the calculations. This can already be seen on Fig. 3, where the film thickness shape is close to the parabolic one, showing that the elastic radial deformations are small, smaller than the film thickness itself. The PVR regime was fully described by Houpert using the dimensionless parameter A, reference [27]. For A smaller than 2.5, PVR formulas are valid. Using the geometrical data given in reference 1, the discusser calculated an equivalent radii ratio k ($k=Ry/Rx$) equal to 4.3 and an equivalent radius Rx in the rolling direction equal to 0.035 m. With these data, a film thickness equal to 4 micrometers, and with the dimensionless parameter GU equal to $5\ 10^{-7}$ (taken from Fig. 11), the parameter A is calculated to be equal to 0.735, i.e., the lubrication regime is described by PVR formulations.

6) This could also explain why only one pressure maximum only is measured. The words "pressure maximum" are according to the discusser more appropriate than "pressure spike" when describing Figs. 2 and 3. In the PVR regime, there is one pressure maximum in the inlet zone. As the load (or the parameter A) increases, this maximum moves toward the center and then toward the outlet zone of the contact. At a certain load, this maximum can be called pressure spike, when a second broader maximum is found at the center of the contact.

7) By plotting the film thickness versus speed in a log - log scale, the film thickness is seen to vary with speed to the power exponent 1.3 approximately. This is another indication that the lubrication regime is PVR, since the speed exponent is 0.7 for the EHD regime, 2 for the Isoviscous Rigid (IVR) regime and between these two values for the PVR regime.

8) Another way of predicting that the authors' experimental conditions are in the PVR/EHD transition is to notice on Fig. 8 that the Hamrock and Dowson calculated values are close to the measured ones at low speed and high load only. These calculated values get much lower than the measured ones as the speed increases or the load decreases.

9) It seems inappropriate to add on Fig. 10 (extracted from Murch and Wilson's reference [22]) the authors' experimental points since the authors' experimental conditions (geometry, viscosity, load, ...) and the ones in reference [22] are different.

Furthermore, thermal correction factors, established in the true EHD regime and at large k values (or line contact), as well as starvation correction factors, should be applied with caution since the lubrication regime is more likely to be PVR.

If these correction factors are still applied, it would be instructive to apply them in a consistent way on the Hamrock and Dowson calculated films only.

10) Last but not least, it would also be of great interest to compare the experimental results (minimum film thickness and maximum pressure) to the calculated PVR values using the relationships described in reference [27].

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J. B. Medley[2]

The authors are to be congratulated on their efforts to obtain experimental data on elastohydrodynamic contacts which are thermally influenced. Extensive numerical analysis has been performed recently on thermal elastohydrodynamic lubrication [28-30] and tribology as a whole would benefit from detailed comparisons between theory and experiment.

The present paper could perhaps have made this and other points but, unfortunately, the authors seemed to have forgotten to include an Introduction. However, in their Abstract, they do state that the present paper is "a complimentary work to a previously published work" and upon consulting their reference list, the previously published work was inferred to be references [1-3]. The explanations in two of these previous works were read and helped clarify some aspects of the present paper but a question arose. What additional purpose did the present paper serve? In their Abstract, the authors explain that the purpose of the present paper was "to cover all aspects of EHD lubrication."

In my opinion, the present paper would have covered all aspects of EHD lubrication far better and been much more useful to tribologists if it had included the actual values of the disk radii of curvature in the x and y-directions, lubricant viscosity and its variation with temperature and pressure, the thermal conductivity of the lubricant and, to be complete, the reduced elastic modulus of the contact. Can the authors supply these values in their written response to this review? In order to further establish the actual contacts examined by the authors, it might also be useful to calculate and list the values of the Johnson parameters.

The next question concerns the apparent high pressure levels in the exit zone of the contact as shown in Fig. 3. Can the authors discuss the physical reasons for these pressures?

Finally, the peak pressure in Fig. 3 for a load of 7000. N is about 3.2 GPa, yet this value is not included in Fig. 7. Why was this peak pressure omitted?

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Authors' Closure

The authors wish to thank Dr. Houpert and Dr. Medley for their kind and interesting discussions to this paper and would like to comment on some of them.

The direction of rolling shown in Fig. 3 for the pressure distribution, is correct. It is possible, however, that there are effects of temperature on the CRO signal. In particular, the voltage does not return to the value at the inlet after the transducer has obviously left the junction.

For comparison between present experimental data and PVR regime [27], the test disks geometrical data are as follows: $R_x = 0.0375$ m, $R_y = 0.300$ m and $K = 8$. Hence, the theoretical film thickness H_{PVR} can be calculated. Values of film thickness as attained experimentally $H_{\exp}$ and predicted theoretically [27] are shown in Table 1. In general, it can be seen that the experimental film thickness values are higher than the theoretical.

Adopting equation (38) proposed in reference [27], Table 2 shows the calculated values of g_e, g_{e2}, and g_{e4}. From which table, the values of g_e are shown to be higher than that of g_{e4} but slightly less than g_{e2}. This could conclude that experimental results herein presented are critically within PVR and EHD lubrication regimes.

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Table 1

Applied load Q	Nondimensional Speed parameter U	Predicted H_{PVR}	Experimental $H_{\exp}$
3000 N	10.4287×10^{-11}	9.5138×10^{-5}	9.07×10^{-5}
	13.227×10^{-11}	11.1827×10^{-5}	12.267×10^{-5}
	18.4413×10^{-11}	14.018×10^{-5}	18.67×10^{-5}
4000 N	10.4287×10^{-11}	9.4593×10^{-5}	8.533×10^{-5}
	13.227×10^{-11}	11.1187×10^{-5}	12.8×10^{-5}
	18.4413×10^{-11}	13.9377×10^{-5}	16.532×10^{-5}
7000 N	10.4287×10^{-11}	9.3541×10^{-5}	6.933×10^{-5}
	13.227×10^{-11}	10.995×10^{-5}	10.133×10^{-5}
	18.4413×10^{-11}	13.6127×10^{-5}	15.467×10^{-5}

Table 2

Q	u(m/s)	6.44	8.168	11.388
3000 N	g_{e2}	8.9613×10^{6}	5.5987×10^{6}	2.9079×10^{6}
	g_e	3.8952×10^{6}	2.4214×10^{6}	1.2456×10^{6}
	g_{e4}	1.2316×10^{10}	0.7729×10^{10}	0.4029×10^{10}
4000 N	g_{e2}	21.1424×10^{6}	13.1937×10^{6}	6.8358×10^{6}
	g_e	8.3728×10^{6}	5.2049×10^{6}	2.6776×10^{6}
	g_{e4}	2.8693×10^{10}	1.8007×10^{10}	0.9387×10^{10}
7000 N	g_{e2}	112.6269×10^{6}	70.1631×10^{6}	33.6908×10^{6}
	g_e	37.0983×10^{6}	23.0618×10^{6}	11.8640×10^{6}
	g_{e4}	14.8702×10^{10}	9.3322×10^{10}	4.8651×10^{10}

Nomenclature: [27]

A = nondimensional parameter = $12GU$

$$\frac{0.177\mathrm{K}}{K+0.778} H_{IVR}^{-1.5}$$

C = correction factor to predict H_{PVR} ($C = A^{2/3}$ $(1 - \exp(-A)^{-2/3})$.

G = nondimensional material parameter = αE (m^2/N)

$$g_e = \frac{W^{8/3}}{U^2}$$

$$g_{e2} = \left[\frac{C\hat{H}_{IVR}}{(3.4\mathrm{g}^{0.49}(1-\exp(-0.68 \times 1.03\mathrm{K}^{0.64})))}\right]^{\frac{1}{0.17}}$$

$$g_v = \frac{GW^3}{U^2}$$

$$g_{e4} = 0.1545\mathrm{g}_v{}^{0.98}$$

$$\left[\frac{(1-\exp(0.68 \times 1.03\mathrm{K}^{0.64}))}{(1-0.85\exp(-0.31 \times 1.03\mathrm{K}^{0.64}))}\right]^2$$

$\hat{H}_{IVR}$ = nondimensional minimum lubricant film thickness

$$\frac{h}{R_x}\,\frac{W^2}{U^2}$$

H_{IVR} = isoviscous rigid nondimensional minimum film thickness

$$= \left[\frac{\left(1+\frac{2}{3\mathrm{K}}\right)\frac{W}{U}}{\left(0.131\ \mathrm{arctg}\left(\frac{K}{2}\right)+1.683\right)\sqrt{128\mathrm{K}}} + 2.6511\right]^{-2}$$

H_{PVR} = piezoviscous rigid nondimensional minimum film thickness = CH_{IVR}

$H_{\exp}$ = experimental nondimensional minimum film thickness

$$= \frac{h_{\min}}{R_x}$$

K = equivalent radius ratio

$$= \frac{R_y}{R_x}$$

R_x = equivalent radius,

$$\left(\frac{1}{R_x} = \frac{1}{R_x} + \frac{1}{R_{x2}}\right)(\mathrm{m})$$

R_{x1}, R_{x2} = radius of curvature of upper and lower disks in x direction (m)

R_y = equivalent radius,

$$\left(\frac{1}{R_y} = \frac{1}{R_{y1}} + \frac{1}{R_{y2}}\right)(\mathrm{m})$$

R_{y1} = radius of curvature of upper disk = 0.300 m

R_{y2} = radius of curvature of lower disk = ∞

u = average tangential velocity in the contact (m/s)

U = nondimensional speed parameter

$$= \frac{\eta_0 u}{ER_x}$$

W = nondimensional load parameter

$$= \frac{Q}{ER_x}$$

Q = normal load (N)