

Experimental investigation of leak hydraulics

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ABSTRACT

In recent decades the hydraulics of leaks, i.e. the definition of the relationships linking the hydraulic quantities in pipes with leaks, has received increasing attention. On the one hand, the definition of the relationship between the leak outflow and the relevant parameters – e.g. the leak area and shape, the pressure inside the pipe and outside the leak, and the pipe material – is crucial for pressure control and inverse analysis techniques. On the other hand, if the effect of the leakage on the governing equations is not taken into account, i.e. the loss of the flow axial momentum is not considered, significant errors can be introduced in the simulation of water distribution systems. In this paper, the governing equations for a pipe with a leak are derived. The basic equations, obtained within different approaches, are presented in a consistent formulation and then compared with the results of some experimental tests. The leak jet angle and other major features of the results are analysed. The estimated values of the parameters can be used in the water distribution network models when pipes with a diffuse leakage are considered.

Key words | jet angle, leak, momentum equation

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INTRODUCTION

In recent decades, due to the changing scenarios in the use and availability of water, more attention has been paid to the efficiency of water distribution system management, particularly in leakage control and detection. Leaky pipeline systems are costly in terms of both water wastage and increasing cost of pumping (Colombo & Karney 2002). To reduce leakage and to improve the water system efficiency, the simulation of the pipe system behaviour within the extended period simulation (EPS) approach has become crucial. For this reason, two issues related to the hydraulic modelling of the leakage have been recently considered in the literature: the head-discharge relationship definition and the effects of the axial momentum loss due to the leak on the pipe momentum equation.

With regard to the head-discharge relationship, many authors have focused their attention on the link between the leakage and the pipe functioning conditions, discussing the proportionality of the leak discharge to the square root of the pressure head inside the pipe (May 1994; Lambert & Thornton 2005; Walski *et al.* 2006; Greyvenstein & van Zyl

2007; van Zyl & Clayton 2007; Cassa *et al.* 2010). The correct definition of this relationship is fundamental in the techniques for leakage control by pressure reduction as well as for leak detection by inverse analysis (e.g. Ferrante *et al.* 2009). In previous papers, the contribution of the writers to this issue mainly concerned the effects of pipe materials (Ferrante *et al.* 2011; Ferrante 2012; Massari *et al.* 2012). Since the head-discharge relationship can be affected by the pressure induced leak area deformation, which in turn depends on the pipe material rheology, a strong correlation exists between the pipe material stress-strain and the leak head-discharge. As shown also in this paper, in a steel pipe the leak area can linearly depend on the pipe inner pressure.

With regard to the effects of the axial momentum loss, many authors considered the effect in the momentum equation of lateral outflow due to junctions and lateral branches but few of them focused the attention on the effect of leaks. McNown (1954) analysed experimentally the variation in piezometric and total head in a circular conduit with lateral outflow at a right angle. He analysed the

change of the piezometric head due to the lateral flow for the case of combining and dividing flow, with different values of the ratios d and r with $d = D_3/D$, being the ratio between the diameters of the lateral pipe, D_3 , and the main conduit, D , and $r = Q_3/Q_1$, being the ratio between the outflow discharge, Q_3 , and the discharge in the pipe, Q_1 , upstream of the leak. The main result for the case of dividing flow was that the gain in piezometric head is significantly larger and tends to increase for d approaching 1 and r approaching 0, leading in some circumstances to negative quantities. *Acrivos et al. (1959)* investigated experimentally the combined effects of the pressure drop in the flow direction due to the friction, with the pressure rise due to the momentum of the main fluid stream flowing into the branch section. To account for the axial component of the force introduced by the flow leaving the main conduit, *Bajura (1971)* experimentally and analytically investigated the case of dividing flow by introducing a pressure recovery coefficient in the momentum equation. The author showed that such a coefficient can range from 0 to 1 according to the value of d and r .

The definition of the momentum equation for a leaky pipe is important for the water distribution network simulation models. After *Todini (1979)* and *Todini & Pilati (1988)* introduced the global gradient algorithm (GGA), several authors tried to improve its capability to simulate the presence of leaks and demands at the nodes of the network (e.g. *Giustolisi et al. 2008*). *Giustolisi & Todini (2009)*, *Berardi et al. (2010)* and *Ferrante et al. (2012)* analysed how, under some circumstances, a diffuse outflow can affect the calibration of the numerical models. In fact, the use of equivalent pipe elements that transform the diffuse outflow into lumped demand at the end nodes is often based on approximate or inadequate equations. In such a case, a correction in the friction term is recommended to balance the introduced errors. Since the proper definition of the correction term requires an experimental activity, in the following, some equations linking pressures and discharges in a pipe upstream and downstream of a leak, are discussed and compared with the results of some laboratory tests. The procedure outlined in the present paper allows the experimental calibration of the parameters and of the friction correction term introduced and discussed by *Ferrante et al. (2012)*.

THE MOMENTUM EQUATION FOR A PIPE WITH A LEAK

Considering the control volume defined in grey in *Figure 1* and according to *McNown (1954)*, *Bajura (1971)* and *Bajura & Jones (1976)*, assuming that a gradually varied flow occurs at sections 1 and 2 and that the cross-sectional values completely define the flow field, the momentum balance along the horizontal pipe axis direction yields

$$\frac{p_2}{\gamma} - \frac{p_1}{\gamma} = \frac{V_1^2}{g} - \frac{V_2^2}{g} - \gamma_d V_1 V_3 \frac{A_3}{A_1 g} \quad (1)$$

where p = pressure in the centre of the cross-section, V = mean velocity, γ = unit weight, g = gravity acceleration and A = pipe constant cross-section area, and subscripts refer to the section number. In Equation (1), the last term on the right hand side takes into account the resultant of the unbalanced force in the direction of the pipe axis due to the inclination of the flow leaving the pipe at section 3'. In fact, the pressure regain coefficient, $\gamma_d = V_x/V_1$, is introduced and the continuity equation $A_3 V_y = A_3 V_3$ is used, with V_x (V_y) being the component along (normal to) the pipe axis of the mean velocity of the flow exiting through the leak (*Figure 1*). Following *Bajura (1971)*, in Equation (1) the Boussinesq coefficient, β , has been considered as unity but its variation along the pipe axis can be also introduced (*Jaumouillé 2009*; *Jaumouillé et al. 2010*).

By introducing a modified Euler number,

$$E = \frac{p_2 - p_1}{\frac{\gamma}{2g} \frac{V_1^2}{g}} \quad (2)$$

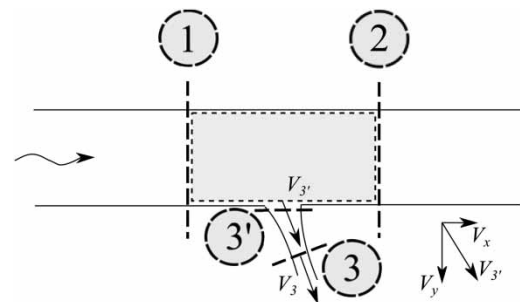


Figure 1 | Definition sketch of the flow at the leak.

and considering that $r = Q_3/Q_1$, Equation (1) can be written as:

$$E = 2r(2 - \gamma_d - r) \quad (3)$$

Another equation, similar to Equation (3), can be derived considering the difference h_f in the total head between section 1 and 2:

$$\frac{p_2}{\gamma} - \frac{p_1}{\gamma} = \frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_f \quad (4)$$

In terms of E and r it is:

$$E = r(2 - r) - \gamma_f \quad (5)$$

With:

$$\gamma_f = \frac{h_f}{\frac{V_1^2}{2g}} \quad (6)$$

It is worth noting that, strictly speaking, Equations (4) and (5) are not an application of the Bernoulli's theorem but they simply assume that there is a difference in the total head from section 1 to 2. As a matter of fact, the Bernoulli's theorem for a stream flow cannot be applied from section 1 to 2 since the volume in between is not a stream tube (at the leak the velocities have a component normal to the tube) and then the discharge is not constant along the flow. This is not a formal distinction but a physical one: the term h_f does not simply represent a difference in the energy content of the flow from one section to the other but it takes into account also the effects of mass variation and momentum outflow. On the contrary, assuming that the Bernoulli's approach of Equations (4) and (5) has a physical meaning, and h_f represents the head losses and not just a head difference, for a frictionless fluid it is $h_f = 0$ and Equation (5) becomes:

$$E = r(2 - r) \quad (7)$$

Similarly, considering h_f as a head loss due to the sudden decrease of the flow mean velocity due to the

discharge variation, a Borda (or Borda-Carnot) minor loss formula can be used and it is:

$$h_f = \frac{(V_1 - V_2)^2}{2g} \quad (8)$$

By introducing Equation (8) in Equation (4), with

$$\gamma_f = \frac{(V_1 - V_2)^2}{V_1^2} = \frac{(Q_1 - Q_2)^2}{Q_1^2} = r^2 \quad (9)$$

the following equation is derived:

$$E = 2r(1 - r) \quad (10)$$

To summarise the above equations, on the one hand Equation (3) is derived by the global momentum equation and it takes into account the effects of the leak discharge leaving the pipe by means of the coefficient γ_d . On the other hand, following a Bernoulli's approach and explaining the head differences in the pipe from upstream to downstream of the leak as head losses, other equations can be derived, by considering: (i) a generic head loss as in Equation (5); (ii) no head losses as in Equation (7); or (iii) Borda minor head losses as in Equation (10).

Some correspondence between the global momentum and the Bernoulli's approach can be found. As an example, when $\gamma_d = 1$, Equation (3) corresponds to Equation (10). In other words, when $\gamma_d = 1$ the effect of the mass variation in the flow can be neglected ($V_x = V_1$) and the usual Bernoulli's theorem can be used if the head losses are evaluated by Borda. This is a borderline condition because we expect that $V_x \leq V_1$ and $\gamma_d \leq 1$ since the fluid loss causes a discharge - and a mean velocity - reduction along the pipe. Furthermore, assuming that both Equations (3) and (5) hold, it can be written:

$$\gamma_f = r(r + 2\gamma_d - 2) \quad (11)$$

Hence, when $\gamma_d = 1 - (r/2)$, $\gamma_f = 0$, i.e. the total head is constant at sections 1 to 2.

In Ferrante *et al.* (2012), the writers emphasised that the head differences between the end nodes of a leaky pipe

element of a water distribution network model cannot be explained as losses in the usual Bernoulli's approach. Similarly, Equation (11) confirms that the term γ_f of the Bernoulli's approach depends on γ_d . In Ferrante et al. (2012), this relationship is explored and the parameter γ_d (there denoted as C_β) is used to evaluate the friction correction term introduced in the GGA framework.

EXPERIMENTAL SET-UP

Tests were carried out at the *Water Engineering Laboratory* of the University of Perugia, Italy, on a HDPE (high density polyethylene) pipe, with a total length of 16.7 m, an internal diameter of 93.3 mm and a wall thickness of 8.1 mm (Figure 2). The pump (P) supplied the needed discharge to the upstream air vessel (AV), from the recycling reservoir (R). At the downstream end section of the pipe, there was a hand-operated ball valve (DV) discharging into the air. An automatically controlled butterfly valve (MV) was placed immediately upstream of DV. At a distance of 10.28 m from the air vessel, two steel trunks 1.05 m long, of different thickness, were placed one at time. The same longitudinal leak of 2×90 mm with rounded edges (L) was machined on each trunk, discharging into atmosphere.

Two electromagnetic flow meters were used to measure the discharge upstream (UD) and downstream (DD) of the leak, with an accuracy of 0.2% of the measured value. Two piezoresistive pressure transducers, with a 7 bar full scale (f.s.) and an accuracy of 0.1% f.s., measured the pressure upstream (UP) and downstream of the leak (DP).

A differential pressure transducer was used, with an accuracy of 0.25% f.s. and a full scale of ± 50 mbar, to measure the small pressure differences between UP and DP. The pipe between UD and DD was horizontal. Since the pressure transducers are both at more than 1 m from the leak, in order to estimate the values of p_1 and p_2 , the head losses between UP and L and between L and DP are evaluated by means of the Colebrook-White formula. The calibration on some tests on an intact trunk with the same geometry (not shown) provided a relative roughness of 0.0005.

Three tests are shown in the following: in Tests 1 and 3 the leak was on a steel trunk with a thickness of 1.5 mm (TS); in Test 2 the leak was on a steel trunk with a thickness of 0.5 mm (FS).

In Tests 1 and 2, the pressure and discharge measurements are used to experimentally evaluate E and r for different flow conditions and then to compare these data with the proposed relationships. Furthermore, assuming that Equations (3) and (5) hold, the same pair of E and r values are used to evaluate

$$\gamma_d = 2 - \frac{E}{2r} - r \quad (12)$$

and

$$\gamma_f = r(2 - r) - E \quad (13)$$

During Test 3, photographs of the outflowing jet on TS have been taken to estimate the angle of the jet with respect

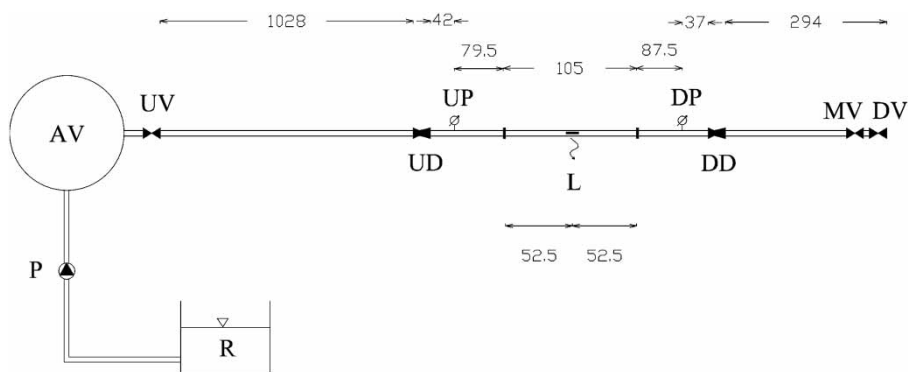


Figure 2 | Layout of the experimental set-up. R = recycling reservoir; P = pump; AV = air vessel; UV = upstream valve; UD (DD) = upstream (downstream) flowmeter; UP (DP) = upstream (downstream) pressure transducer; L = leak; MV = automatically controlled butterfly valve; DV = end valve. Lengths are in centimetres.

to the pipe axis, α , and to relate these data to the measured pressures and discharges.

The same leak was machined on the trunks TS and FS but a plastic deformation took place before the beginning of tests on FS, with a noticeable enlargement of the leak area (Ferrante et al. 2011). This event explains the larger values for FS of the leak non-dimensional area

$$l_k = \frac{C_l A_{3'}}{A} = \frac{Q_3}{A \sqrt{\frac{2p_1}{\rho} + V_1^2}} \quad (14)$$

shown in Figure 3, with C_l = discharge coefficient. For TS (Tests 1 and 3) the leak area does not show an appreciable

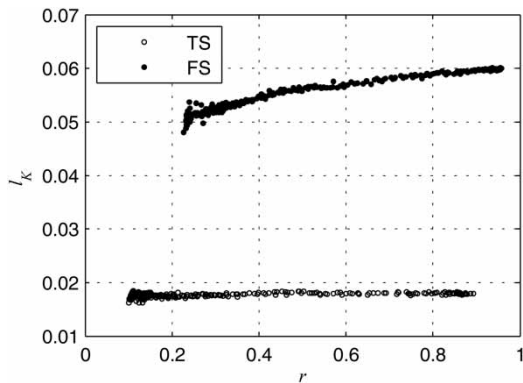


Figure 3 | Variation of the non-dimensional area of the leak, l_k , with r , for TS (Test 1) and FS (Test 2).

variation with r while the same does not apply for FS (Test 2). This effect can be explained by the smaller thickness of the trunk FS compared with the trunk TS and by the increase in the pressure that follows the increase of r . In fact, in the tests the increase of r corresponds to an increase of the pressure which causes an elastic enlargement of the leak and the increase of l_k (Ferrante 2012).

THE RELATIONSHIPS BETWEEN γ_D , E AND r

Figure 4 shows the variation of E with r for Test 1 (a) and Test 2 (b). As proposed by McNown (1954), in this figure the experimental data can be compared with Equations (7) and (10) which correspond to the no head loss and Borda head loss conditions, respectively. As previously shown, none of these cases is appropriate for the considered phenomenon since the changes in the discharge contradict the hypothesis of the Bernoulli's theorem applied to a stream tube. Nevertheless, the experimental data lie in between the two curves. For this reason, in an attempt to fit the experimental data, the relationship

$$E = 2r - kr^2 \quad (15)$$

is used, where k is a fitting parameter. In fact, Equation (15) corresponds to Equation (7), i.e. the no head losses case, for $k = 1$ and to Equation (10), i.e. the Borda head loss case, for

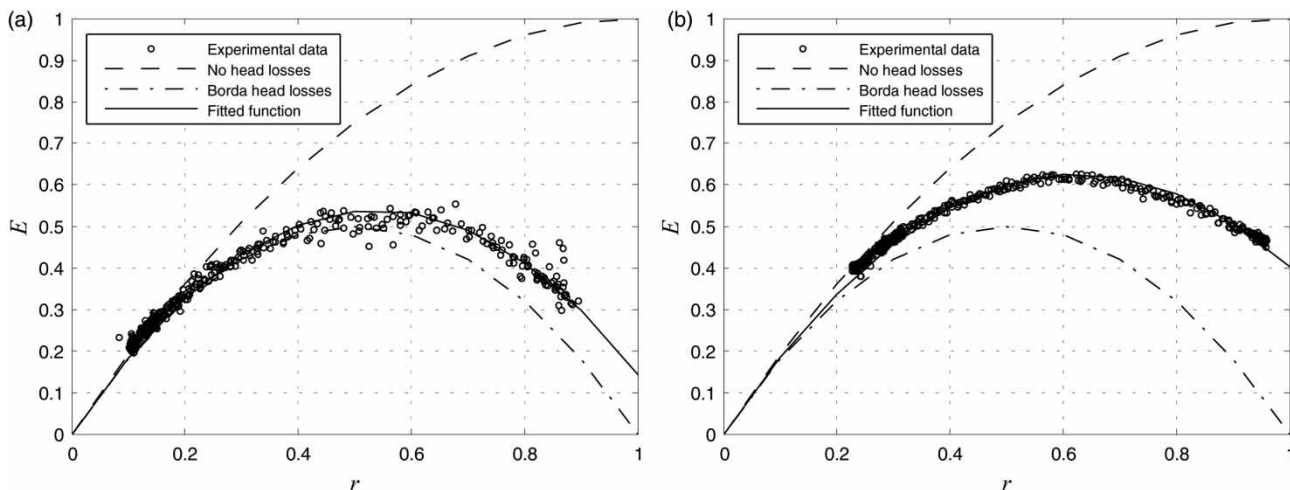


Figure 4 | Experimental values of the modified Euler number compared with those evaluated with Equations (7) (no head losses) and (10) (Borda head loss) for (a) Test 1 and (b) Test 2.

$k = 2$. The curves obtained by the fitting for Tests 1 and 2 are shown in Figures 4(a) and 4(b), with $k = 1.856$ and 1.598 , respectively. In both cases, the agreement of the fitting to the data seems to be satisfactory (with R^2 being 0.986 and 0.953 respectively).

Introducing Equation (15) in Equation (12), the fitting function

$$\gamma_d = 1 + \left(\frac{k}{2} - 1\right)r \tag{16}$$

is obtained, which approximates the dependence of γ_d on r with a linear relationship, and reproduces both the no

head losses and the Borda head loss solutions ($k = 1$ and 2 , respectively). Similarly, for γ_f , introducing Equation (15) in Equation (13), we can write:

$$\gamma_f = (k - 1)r^2 \tag{17}$$

In Figures 5 and 6, the fitting functions defined by Equations (16) and (17) are plotted along with the experimental values of γ_d and γ_f evaluated by Equations (12) and (13), respectively; the same k values obtained by the fitting of Equation (15) to the experimental data are used.

As shown in Figure 5, in Test 1 γ_d is close to 1 and slightly varies with r , while in Test 2, it shows a reduction

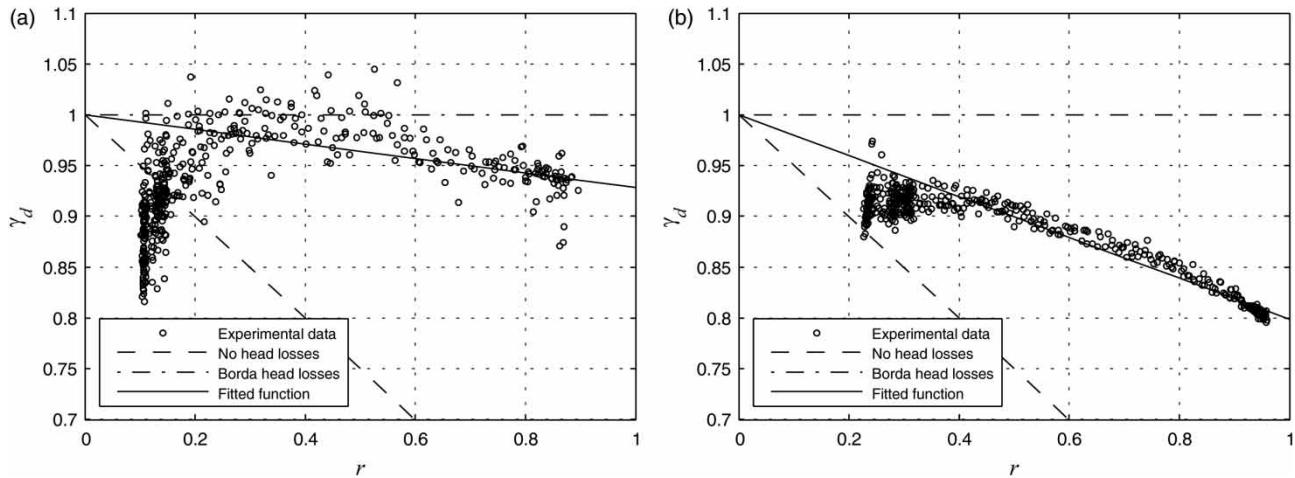


Figure 5 | Experimental values of γ_d and the theoretical expression for no head losses and Borda head loss for (a) Test 1 and (b) Test 2.

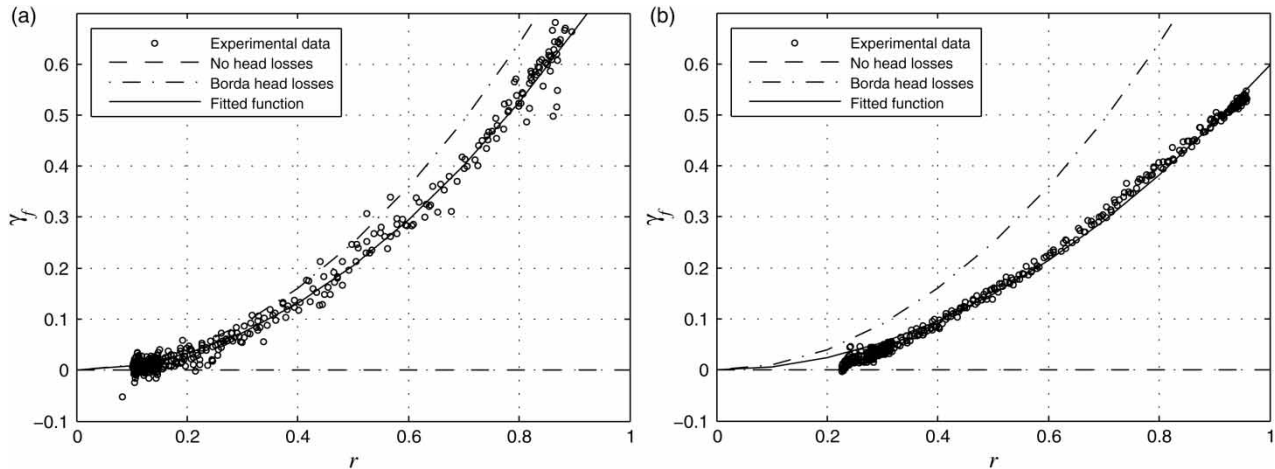


Figure 6 | Experimental values of γ_f and the theoretical expression for no head losses and for Borda head loss for (a) Test 1 and (b) Test 2.

from about 0.9 to 0.8. This finding partially contradicts the results of Bajura (1971) suggesting that such a coefficient is insensitive to the flow ratio r . Our result can be explained, at least for Test 2, considering that the increase of the leak area with pressure likely affects the trend of $\gamma_d(r)$.

In Figure 6, some negative values of γ_f can be found for small values of r . Although the spreading of the data could explain such a behaviour, it is worth noting that negative values of γ_f – i.e. an increase of the total head along the pipe from upstream to downstream of the leak – lead to an apparent paradox if they are analysed in the Bernoulli's theorem framework. That is, due to the loss of mass and axial momentum by the leak, an increase in the total head does not imply an increase in the energy of the flow.

THE JET ANGLE VARIATION WITH E AND r

The 15 photographs of Figure 7 show the jet from the leak TS for different flow conditions (Test 3). The jet flow is angled with respect to the perpendicular to the pipe axis by an angle, α , from about 0° to almost 14° , confirming how the work hypothesis of a leak outflow perpendicular to the pipe axis can be wrong. The dependence on r of the angles α measured with the aid of 94 photographs is shown in Figure 8. In this figure as well as in Figure 9, filled symbols and photograph numbers correspond to the photographs in Figure 7.

Based on the definition of γ_d and assuming that $\tan \alpha = V_x/V_y$, it is:

$$\gamma_d = \frac{V_x}{V_1} = \tan \alpha \frac{Q_3 A_1}{Q_1 A_3} \quad (18)$$

and hence

$$\alpha = \arctan\left(\frac{\gamma_d l_k}{r}\right) \quad (19)$$

As a result, introducing in Equation (19) the expression for γ_d of Equation (16), both the limit conditions of no head losses and Borda head loss, as well as the fitting function, can be evaluated. All these functions, with the fitted function between the other two, are close to each other and in good agreement with the experimental data, as shown in Figure 8.

The variation of α with r was clearly appreciated during the experiments by the manoeuvre of the downstream valve. The opening manoeuvre, increasing the discharge downstream of the leak and correspondingly decreasing r , causes a clear rotation of the jet around the leak towards the downstream pipe.

Figure 9 shows the variation of α with E . If Equation (16) is introduced in Equation (19) with the proper value of k and then r is expressed as a function of E by Equations (7), (10), and (15), the limit conditions of no head losses, Borda head loss, and the fitted function can be derived, respectively. These curves are plotted in Figure 9 along with the experimental data.

CONCLUSIONS

Within an experimental activity carried out at the *Water Engineering Laboratory* of the University of Perugia, data on pressures and discharges in a pipe upstream and downstream of a leak were collected to investigate the effects of a leak on the momentum equation. In fact, the classical approach based on the explanation of the total head variation inside the pipe with head losses can be misleading. On the contrary, if the momentum equation is used, it cannot be generally assumed that the jet through the leak leaves the pipe in a perpendicular direction with respect to the pipe axis. As a result, a term due to the axial momentum of the jet in the momentum equation has to be considered.

The experimental investigation of the variation of the pressure regain coefficient, γ_d , with the pipe functioning conditions leads to an approximate relationship linking this quantity with the leak discharge ratio r . This result is relevant for the water distribution network simulation when pipes with diffuse outflows are modelled by equivalent pipes with a lumped demand at the end nodes. In this case, as shown in Ferrante et al. (2012), the values of γ_d , providing the values of the friction correction term can be directly implemented in the GGA.

The shown photographs and the experimental data confirm that the leak jet is not necessarily normal to the pipe axis. The analytical relationship linking the leak jet angle with the functioning conditions of the pipe are provided and tested by means of the collected data.

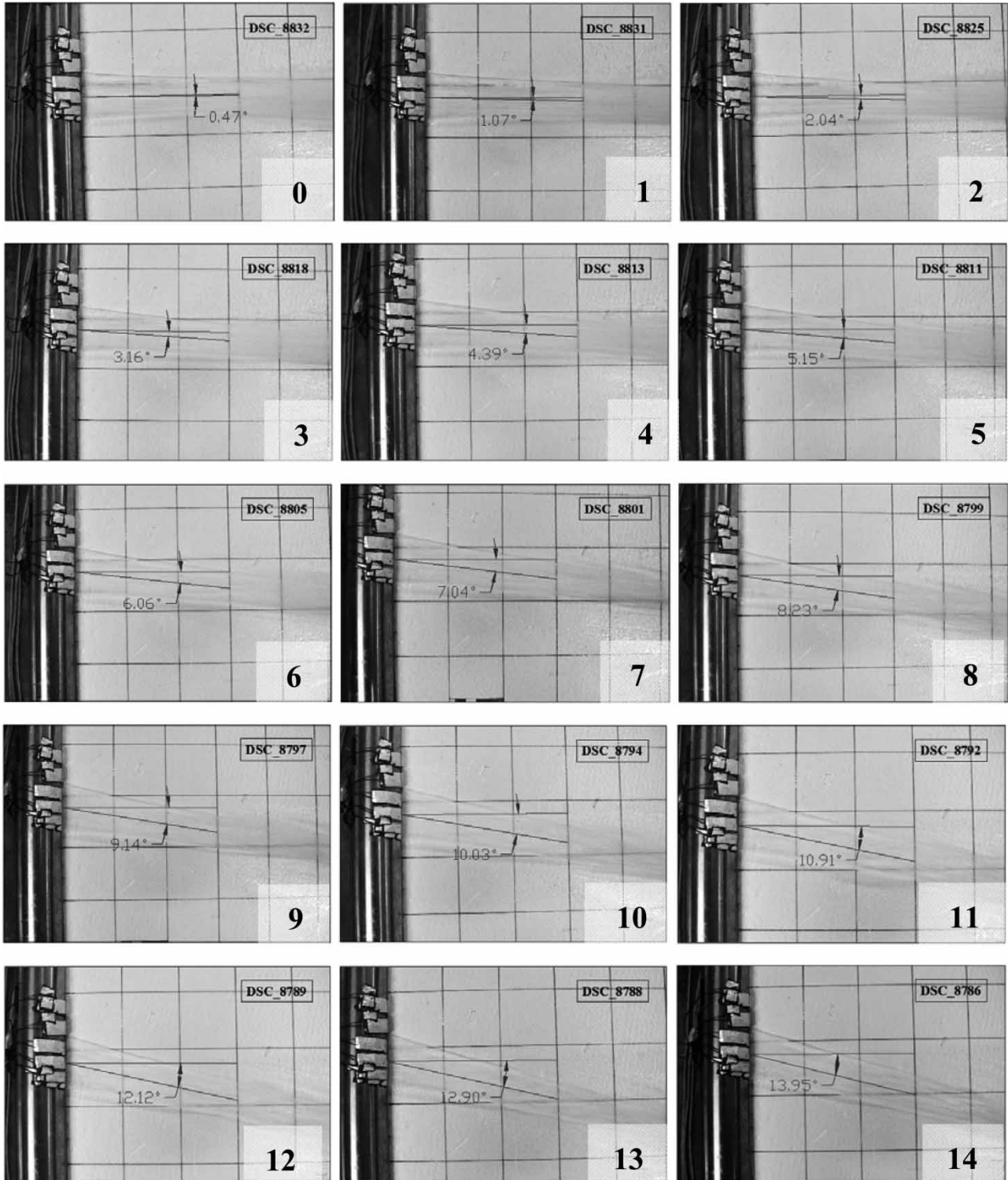


Figure 7 | Photographs of the jet for Test 3.

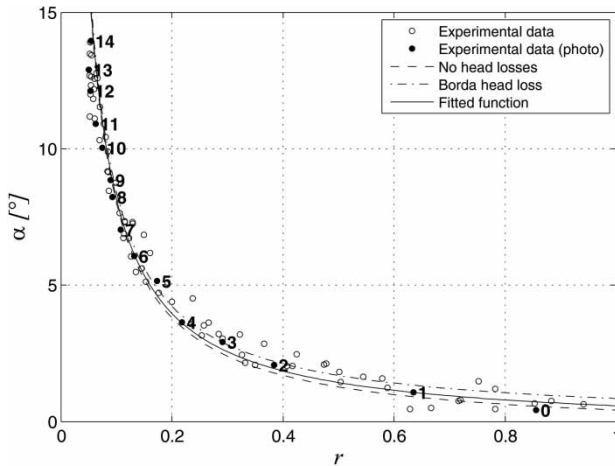


Figure 8 | The experimental variation of α with r for Test 3 (numbers refer to the photographs of Figure 7).

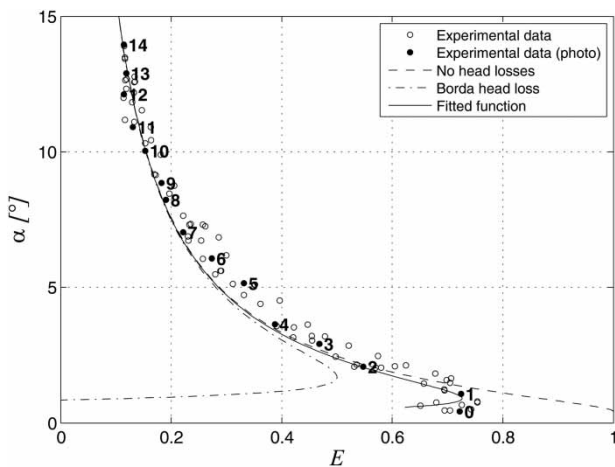


Figure 9 | The experimental variation of α with E for Test 3 (numbers refer to the photographs of Figure 7).

The approximation due to the hypothesis of a free efflux from the leak could limit the application of the laboratory results to buried pipes (e.g. Walski *et al.* 2006; van Zyl & Clayton 2007), but further experiments are needed to explore such a complex phenomenon in more detail and in different hydraulic conditions.

ACKNOWLEDGEMENTS

This research has been supported by *Fondazione Cassa Risparmio Perugia* under the Project ‘Leaks and blockages

detection techniques for reducing energy and natural resources wastage’.

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First received 15 February 2012; accepted in revised form 2 September 2012. Available online 23 November 2012