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Friction, force chains, and falling fruit FREE

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Jacqueline Krim; Robert P. Behringer

Physics Today **62** (9), 66–67 (2009);
https://doi.org/10.1063/1.3226776

quick study

Friction, force chains, and falling fruit

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Why is it that stacked apples seem so stable, but removing the “wrong” apple can cause the whole pile to tumble down?

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In a bad dream or in reality, nearly all of us have witnessed or personally endured moments of embarrassment as a stacked structure of store merchandise collapses while we helplessly stand by. How is it that such seemingly stable structures can completely disintegrate when just one constituent is moved? The answer lies in a delicate balance of friction, shape, packing density, and rotational inertia. In brief, the greater the friction, the fewer contacts are required to stabilize particles within a collection of objects. The shape and density of particles control how many contacts will be formed, and the rotational inertia, which determines how quickly a particle will start or stop rolling, plays a key role in how readily a pile will recover upon removal of a weight-bearing constituent.

Jamming

In the past few years, investigators have intensely studied the mechanical stability of spherical and circular packings, and also the density of particle packings—a topic that dates to the time of Johannes Kepler. The renewed interest has been spurred by a suggestion from Andrea Liu and Sidney Nagel that diverse systems of granular materials—including sands, glasses, foams, and colloids—might all be describable in terms of a common “jamming” phase diagram. As the density of the system is increased above a critical jamming value, Liu and Nagel’s theory predicts that the system will develop mechanical rigidity via a discontinuous jump in the average number of contacts Z per particle and a power-law increase in the pressure. In actual experimental systems, the transition in Z is continuous, albeit abrupt. Intuitively, one might expect jamming to occur when Z reaches the so-called isostatic value Z_I at which the number of force and torque constraints on the system’s particles equals the number of independent contact-force components, or degrees of freedom, acting among the particles. For densities above the jamming value, some forces remain unconstrained—as a result, the jamming phenomenon is history dependent. Multiple trials must thus be performed if one is to adequately characterize a jammed state.

For frictionless spheres, Z_I is 6; for frictionless disks in two dimensions it’s 4. When friction is in play, Z_I drops to a lower value. For example, disks experiencing infinite frictional forces (or, experiments confirm, even moderate ones) have a Z_I of 3. Much has been learned about jamming under relatively simple conditions—for example, for spherical particles or isotropic stresses. Scientists, however, have only a limited understanding for nonspherical particles. Some recent works have probed the stability of elliptical particles and thin rods, and an amusing study in 2004 showed that M&M candies pack more densely than spheres. But overall, as soon as one leaves the realm of simple particle shapes or isotropic stress conditions, one is entering uncharted territory. (For a discussion of heterogeneous granular materials, see the article by Anita Mehta, Gary Barker, and Jean-Marc Luck in PHYSICS TODAY, May 2009, page 40.) Undaunted, we decided to apply our own ideas on packing stability to everyday objects, in particular to stacked displays of fruits and vegetables. We couldn’t find any videos of fruit-pile collapse online, so we ran our own experiments and filmed them expressly for this article.

Stroll through a local market or peruse still lifes by Paul Cezanne and it will become apparent that pieces of fruit on open, uncontained surfaces rarely rise higher than two layers, and stacks held within the confines of a retaining barrier are rarely more than one level higher than the edge of the tray that holds the produce stack. Although a high-friction pad is

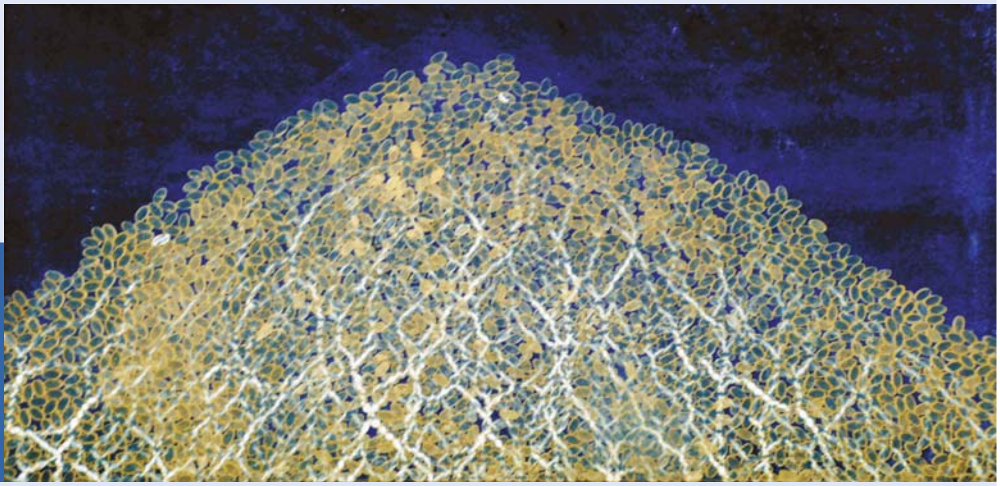

Figure 1. Photoelastic ellipses simulate a two-dimensional fruit pile. When viewed through polarizing filters, the ellipses manifest force chains (white) composed of weight-supporting particles. Removal of such particles poses the greatest danger for pile collapse.

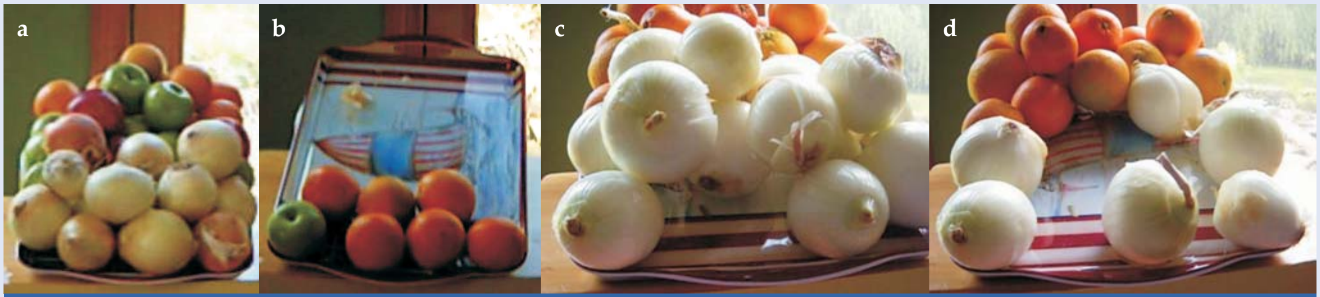

Figure 2. Unpeeled onions (a) provide a weak foundation for the apples and oranges higher up in a tilted fruit tray. After removal of just one onion from the base, almost all the fruits fall off **(b)**. A pile whose base is made of peeled onions is difficult to collapse, an effect directly attributable to friction. Pile is shown after removal of one **(c)** and six **(d)** onions.

sometimes placed at the bottom of a pile and the lowest layer frequently forms a well-ordered array, the ordering rapidly degrades with height because of variations in the size and arrangement of the fruit. Although a structure may be stable, it is virtually impossible to know by visual inspection whether it will collapse when a particular constituent is moved.

Illuminating contact forces

The relationship between structure and friction is certainly not a new topic. Charles Augustin de Coulomb, formally trained in mathematics and engineering, had already achieved prize-winning successes in the areas of structural stability, soil mechanics, and tribology before his mid-career venture into electrostatics and physics at the age of 41.

Coulomb, who did not have the benefit of atomic-scale knowledge of surface morphology—or even the awareness that atoms existed—was particularly interested in the fundamental origins of friction and whether it might arise from interlocking asperities and surface roughness. He was able to rule out that mechanism, however, since the energy gained in lifting a surface over an asperity can be completely regained when dropping it back down. Nonetheless, the idea that increased roughness means increased friction persisted well into the 1970s. It was only definitively discounted when surface-science experiments demonstrated that one-molecule-thick films can substantially change friction but have minimal impact on surface roughness. Macroscopic tribological behavior is evidently highly sensitive to details of interfacial contact at the atomic scale; friction coefficients arise from complicated processes. The nature of the atomic-scale mechanisms that dominate the dissipative process by which mechanical energy is transformed into heat and, indeed, the very definition of contact roughness at atomistic length scales are topics of much current and exciting work.

Knowledge of contact roughness provides only limited information about contact forces. Insight about those forces, however, has emerged from ingenious experiments that use photoelastic techniques to literally illuminate both the normal and tangential components of granular contact forces in two-dimensional systems. The earliest practitioners of the technique were Takao Wakabayashi in 1950 and Pierre Dantu in 1957. As seen in figure 1, running through the photoelastic disks are illuminated force chains, quasi-linear groups of particles with above-average stress that support granular structures. Far more subtle is the role of the surrounding weaker chains that appear almost ghostlike in the image. Theoretical modeling has revealed that the ghost chains also help support the overall stability of the structure. The seemingly innocuous ghosts might also turn out to be forbidden fruit, not to be touched lest disaster ensue.

Fruit market disaster: Read it and weep

With the aid of a middle-school volunteer and a collection of apples, oranges, and onions, we authors, Krim and Behringer, made some videos (see figure 2), as part of a study of real-world piles. Krim had measured the friction coefficients of the apples, onions, and oranges in contact with a tray to be roughly 0.3, 0.2, and 0.5, respectively; the rank ordering remained intact for fruit-to-fruit contact. We piled fruit on tilted trays and then filmed events in which a fruit that was thought most likely to cause collapse was removed from the pile. After more than 20 takes, it became apparent that the sequence in which the fruit was stacked had a major influence on whether a collapse would occur. Our student fruit stacker hypothesized that unpeeled onions form a solid base of support for the pile, so removing one would make it more likely to collapse. Not so, countered Behringer. The onions provide a *weak* foundation at the base, and that's why removal of one is more likely to trigger collapse. Their low friction means that onions require more contact points in order to be stable. If their friction were to increase somehow, they would then provide a strong foundation and fewer contacts would be required to create a stable structure. More onions at the base would then have to be removed before the pile collapsed.

An onion's friction coefficient can increase sixfold or more if the outer skin is removed. Many tears later, and with far fewer trials than one might expect, Krim personally observed Behringer's prediction to be true. Bottom line for fruit lovers: You may never know exactly when or where the apples will fall, but watch out for how they are waxed.

The online version of this Quick Study links to an essay deriving the isostatic values of Z_I for disks interacting with and without friction. Also available are further resources and links to some of the videos of collapsing fruit piles mentioned in the text. The full set of videos is available at http://www.dukefruit.info.

Additional resources

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▶ J. Krim, "Surface Science and the Atomic-Scale Origins of Friction: What Once Was Old Is New Again," *Surf. Sci.* **500**, 741 (2002).

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