We investigate the possibility that the perturbative vacuum is unstable against gauge boson pair condensation in non-Abelian gauge theories without fundamental scalar fields. The force law between gauge boson pairs is studied in the Coulomb gauge. Assuming that the pairs condense in channels of attractive gauge force, we investigate the patterns of spontaneous symmetry breakdown in the \( SU(N) \) and \( SO(2N) \) gauge theories. As a possible application of this symmetry breaking scheme, we consider \( SU(5) \) and \( SO(10) \) gauge models of grand unification without fundamental scalar fields. It is found that the desirable breaking patterns \( SU(5) \to SU(3) \times SU(2) \times U(1) \) and \( SO(10) \to SO(6) \times O(4) \to SU(3) \times SU(2) \times SU(2) \times U(1) \) arise as a result of gauge boson pair condensation. Our mechanism is also applicable to GUTs with massless fundamental scalars, in which supercooling of the Universe would last up to extremely low temperature.

§ 1. Introduction

Both electro-weak and strong interactions are described most probably by non-Abelian gauge theories. It is attractive to suppose further that both interactions can be unified in a theory based on a larger simple group which is broken spontaneously (GUTs).\(^1,2\) In the standard approach of spontaneously broken gauge theories one introduces a set of fundamental scalar fields. The number of arbitrary parameters which characterize the scalar couplings is so many that we feel reluctant to accept them as fundamental ingredients of the theory.\(^3\) We are therefore tempted to look for alternative approaches.

It was asserted that in the scheme of dynamical symmetry breakdown\(^4\) for the electro-weak gauge theory\(^5\) a variety of new kinds of strongly interacting gauge fields and fermions are required to make possible the formation of bound states out of two fermions, which play the rôle of Higgs scalars. Raby, Dimopoulos and Susskind have recently made an interesting suggestion that it is the attractive gauge forces between massless fermions that is responsible for fermion pair condensation and consequent symmetry breakdown of the original gauge group.\(^5,6\) An important advantage of their scheme is that no new interactions need to be invoked in order to break down the symmetry. A popular notion in these approaches is that fermion pair condensation\(^7\) is the origin of breakdown.

\(^*)\) We will not consider the possibility that all parameters related to scalar fields are determined in theories possessing higher symmetry such as super-symmetry.
of gauge symmetries, while gauge field configurations are assumed to be irrelevant to the occurrence of symmetry breaking.

In this paper we shall investigate the possibility that gauge field configurations play an important rôle as fermion pair condensation in the determination of symmetry breaking phases of non-Abelian gauge theories without fundamental scalar fields. To be specific, we shall consider the possibility that gauge boson pair condensates in attractive channels are responsible for the vacuum instability and subsequent symmetry breakdown. This mechanism of dynamical symmetry breaking, like that of Raby et al., has two desirable features. It does not invoke new super-strong interactions nor new free parameters. It also implies that asymptotic freedom is guaranteed unless we introduce too many fermions. This nice feature is known to be difficult to implement in theories with fundamental scalar fields.\(^8\)

The suggestion that spontaneous symmetry breakdown occurs dynamically in non-Abelian gauge theories may seem to be incompatible with what is popularly believed to occur in the case of \(SU(3)\) (QCD). However, no general arguments are known to rule out the possibility of self-breaking of gauge symmetries. Evidently, it is worth pursuing to study from a general viewpoint which phases are the real ones in a general class of non-Abelian gauge theories, including grand unified theories. As a matter of fact, there exist a couple of literatures in which the authors discuss dynamical symmetry breaking in non-Abelian gauge theories in several different contexts.\(^9\)

It has been known for some time that attractive gauge forces operate between gauge bosons\(^10,11\) as well as between fermions. The lowest order gauge forces between two gauge bosons can be shown to be attractive in some channels and sometimes much stronger than those between two fermions. Therefore, it does not seem unreasonable to suspect that gauge boson pairs will form bound states and eventually start to condense.

The criterion of attractive versus repulsive channels can be formulated by considering the Cooper equation for two gauge boson systems in the Coulomb gauge. As for the nature of the condensates, for the reasons explained later, we will not take the criterion of Raby et al. that the condensation occurs in the most attractive channel (MAC) in the sense of one gauge boson exchange.\(^5\) We will instead consider all attractive channels as candidates for symmetry breaking condensates.

We investigate the symmetry breaking patterns in non-Abelian gauge theories with gauge groups \(SU(N)\) and \(SO(2N)\) induced by the gauge boson condensation mechanism. The condensate direction can be determined by assuming that the symmetry breaking properties are described effectively in terms of Higgs field with the same quantum number as the condensate channel.

As a possible application of our scheme we consider the possibility that the
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symmetry breakdown in the grand unified theory is of the dynamical origin and occurs in the manner described above. We shall study what kinds of symmetry breaking patterns arise in the $SU(5)$ and $SO(10)$ models.

The breakdown of the weak $SU(2)_L$ symmetry is known to be caused by a doublet of complex scalar fields. Obviously, a doublet of scalar bound states cannot be constructed out of gauge bosons. Hence, for the breakdown of $SU(2)_L$ symmetry one has to consider some other mechanism than the gauge boson condensation. Such breaking mechanism will not be discussed in this paper.

The remainder of this paper is organized as follows. In § 2, we discuss the force law between two gauge bosons, and derive the criterion for attractive versus repulsive channels. In § 3, we find the attractive channels for two-gauge boson systems in pure $SU(N)$ and $SO(2N)$ gauge theories and the patterns of symmetry breakdown induced by the gauge boson pair condensate. Section 4 is devoted to the application of our mechanism to dynamical symmetry breakdown in GUTs. In § 5, we discuss a few important questions regarding the possibility of dynamical symmetry breakdown in gauge theories. Unfortunately, at the moment we cannot offer convincing answers to these questions.

§ 2. Gauge boson pair condensation

In order to investigate the dynamical properties of a gauge theory, we first have to define its vacuum. Several different approaches, invoking different physical pictures, have been proposed to investigate this difficult problem. Here we shall study the symmetry properties of the possible vacuum in a rather intuitive approach of gauge boson pair condensation.

In an attempt dealing with the aforementioned problem in the $SU(3)$ theory (QCD), Fukuda and Kugo suggested that the attractive gauge forces between two octet gauge bosons in the singlet channel are likely to produce tachyon bound states and lead to vacuum instability against gluon pair condensation. They discussed this pairing phenomenon both in the Coulomb gauge and in the covariant $a^*$ gauge, and reached the same conclusion.

Here we shall recapitulate their lowest order calculation in the Coulomb gauge and extend the result to all possible two gauge boson channels in general pure gauge theories with simple groups. Our basic assumption can be stated as follows: Pairs of two gauge bosons with opposite spins and momenta will form scalar bound states in channels where the gauge forces are attractive and eventually condense.

The trial "Cooper states" are

$$|r⟩ = \frac{1}{N} \sum_{\alpha, \beta, c, r} \sum_{a,b} D(\alpha, \beta) \int d^3 p \bar{a}_c(p) a^a_{\alpha, r} a^{\dagger b}_{\beta, p} |0⟩ . \quad (2.1)$$
Here \( r \) denotes the representation of the system of two gauge bosons. \( D(r)_{a,b} \) are the Clebsch-Gordan coefficients. \( a_{a,\lambda}^* \) is the creation operator of a gauge boson with spin \( \lambda \) and momentum \( p \). \( N \) is a normalization constant. The Coulomb energy is given, in lowest order, by

\[
\langle r | H_{\text{Coulomb}} | r \rangle = \frac{g^2}{16\pi^2 N^2} \{ C_2(r) - 2C_2(\text{adj}) \}
\times \int d^3p d^3q \sum \alpha_c^*(p) \alpha_c(q) \frac{(p_0 + q_0)^2}{|p - q|^2 p_0 q_0} (1 + \cos^2 \theta),
\]

where \( C_2(r) \) denotes the quadratic Casimir operator for \( r \). “adj” means adjoint representation which corresponds to the representation of the gauge boson.

The “Cooper equation” for the pair condensation can be derived by considering the variation of the energy expectation value \( \langle r | H | r \rangle \) for the trial state (2.1) with respect to \( \alpha_c(p) \) under the condition

\[
\sum \alpha_c^* \int d^3p |\alpha_c(p)|^2 = 1.
\]

The energy expectation value is given by the sum of the free part and the Coulomb energy part (2.2). We thus obtain the following equation for the ground state energy \( E \), after integrating over the direction of \( q \).

\[
(E - 2p) \alpha_c(p) = \Delta C_2(r) \frac{g^2}{64\pi^2} \int dq \alpha_c(q) K(p, q) p q
\]

where

\[
K(p, q) = (p + q)^2 \left( \frac{p^2 + q^2}{2p^2 q^2} + \frac{p^4 + q^4 + 6p^2 q^2}{4p^2 q^2} \ln \left| \frac{p + q}{p - q} \right| \right),
\]

and we have defined

\[
\Delta C_2(r) = C_2(r) - 2C_2(\text{adj}).
\]

The Cooper equation (2.4) has the same form as that considered by Fukuda and Kugo\(^{10}\) and can be shown to have a negative energy \( (E < 0) \) solution if (and only if)

\[
\Delta C_2(r) < 0.
\]

It implies that the perturbative vacuum is unstable against the gauge boson pair condensation in the channel \( r \) for which \( \Delta C_2 \) is negative. It is therefore appropriate to call the channel \( r \) attractive channel (AC) if the sign of its \( \Delta C_2 \) is negative. This definition of AC coincides with the intuitive definition and is the same as that of Raby et al. for two fermion channels.

We have to add to (2.2) the contributions from transverse gauge boson
exchange and the contact four point interaction to the order of \( g \) under consideration. These two contributions can be shown not to change the sign and the group factor on the r.h.s. of (2·2). Following Ref. 5), we shall assume that the gauge forces between gauge bosons are described at least qualitatively by the lowest order approximation (2·2) and extract the quantum number dependence from this expression as stated above.

The same criterion as above can be obtained in the covariant \( \alpha \) gauge by the use of the ladder-approximated B-S equation for the bound states of a gauge boson pair. To see this, we only have to extend Fukuda’s argument on the existence of a tachyonic bound state in the singlet amplitude\(^{11)}\) to the non-singlet case. The group factor \( C_2(G) \) appearing in the B-S equation for the singlet channel is to be replaced by \( -\frac{1}{2} \beta C_2 \) for the B-S equation for non-singlet channels. We can then easily convince ourselves that there exist tachyon poles in non-singlet B-S amplitudes as well, if \( \beta C_2 \) is negative. It is thus implied in the covariant \( \alpha \) gauge\(^{7)}\) that the perturbative vacuum is unstable against the condensation of non-singlet tachyonic bound states of gauge boson fields.

§ 3. Breaking patterns in \( SU(N) \) and \( SO(2N) \) gauge theories

We are now in a position to examine the patterns of symmetry breakdown in the pure gauge theories induced by the gauge boson condensation. Raby et al. assumed the criterion that the condensate occurs in the most attractive channel (MAC) in the sense of the approximation of one gauge boson exchange.\(^{3)}\) It is conceivable, however, that one boson exchange forces describe the law of gauge forces only qualitatively. The true condensate channel may be determined by a more complicated criterion. Therefore, we shall consider here all attractive channels as candidates for symmetry breaking condensates.

There is another reason why we take all attractive channels into consideration. Even if the MAC defined by one gauge boson exchange forces coincides with the true most attractive channel, it is still possible that the real world corresponds to one of the other local minima with sufficiently long life time, i.e., the so-called false vacua.\(^{**)}\) If there exist several attractive channels, each of the channels will be a candidate for the local minimum corresponding to the real world.

We consider two classes of symmetry groups, \( SU(N) \) and \( SO(2N) \), discussed most commonly in SBGT. Representation \( r \) of the two gauge boson

\(^{*)\) This conclusion holds true for \( \alpha \geq -1 \). For \( \alpha \leq -1 \) the lowest order calculation does not make sense.\(^{11)}\)

\(^{**)\) The real world can be in a false vacuum, provided its lifetime is much longer than the age of the Universe. This possibility has been discussed including finite temperature effects in some different contexts.\(^{23)}\)
channels can be obtained by decomposing the Kronecker product of two adjoint representations into a direct sum of irreducible representations. We only have to consider channels which are symmetric under interchange of the two gauge boson indices; antisymmetric channels cannot form scalar bound states. We identify the representation by its dimension.

In \( SU(N) \) gauge group, the dimension of the adjoint representation is \( N^2 - 1 \). Irreducible decomposition for \( N \geq 3 \) is

\[
\begin{align*}
N^2 - 1 \otimes N^2 - 1 &= 1 - 2N \\
&\oplus N^2 - 1 - N \\
&\oplus \frac{1}{4} N^2(N - 3)(N + 1) - 2 \\
&\oplus \frac{1}{4} N^2(N + 3)(N - 1) + 2 \\
&\oplus \left\{N^2 - 1 \oplus \frac{1}{4} (N^2 - 1)(N^2 - 4) \oplus \frac{1}{4} (N^2 - 1)(N^2 - 4)^*\right\}_{\text{antisym.}}, (3.1)
\end{align*}
\]

(For \( N = 3 \) the third term is absent.)

where the first four representations are symmetric with respect to the two gauge boson indices and the last three in curly brackets antisymmetric. We have given values of \( \Delta C_2 \) for these four symmetric channels. Only first three channels are ACs.

In \( SO(2N) \) gauge group, the adjoint representation (second rank antisymmetric tensor) has a dimension \( N(2N - 1) \).

Irreducible decomposition of the Kronecker product and \( \Delta C_2 \) for \( N \geq 4 \) are

\[
\begin{align*}
28 \otimes 28 &= 1(-12) \oplus 35(-4) \oplus 35(-3) \oplus 35^*(-3) \\
&\oplus 300(+2) \oplus \{28 \oplus 350\}_{\text{antisym.}}, (N = 4)
\end{align*}
\]

where the numbers in parentheses are \( \Delta C_2 \), and

\[
\begin{align*}
N(2N - 1) \otimes N(2N - 1) &= 1 - 4(N - 1) \\
&\oplus (2N - 1)(N + 1) - 2(N - 2) \\
&\oplus \frac{1}{6} N(N - 1)(2N - 1)(2N - 3) - 4 \\
&\oplus \frac{1}{3} N(N + 1)(2N + 1)(2N - 3) + 2 \\
&\oplus \left\{N(2N - 1) \oplus \frac{1}{2} N(N + 1)(2N - 3)(2N - 3)\right\}_{\text{antisym.}}, (N \geq 5) (3.2)
\end{align*}
\]
Only first three of the four symmetric channels are ACs.

We shall now derive the breaking patterns of $SU(N)$ and $SO(2N)$ induced by each of the condensates. In both $SU(N)$ and $SO(2N)$ cases, the singlet channel is AC and hence is a candidate for condensation. The singlet condensation will presumably lead to confinement of fermions, as advocated by several authors in the context of quark confinement in QCD. The original gauge symmetry will remain unbroken. We do not have much to add to their results.

The breaking patterns induced by non-singlet condensates can be determined by assuming that the symmetry breaking properties are described effectively by Higgs fields. To this end, we extend the analysis of Ling-Fong Li in such a way that not only the absolute minimum of the effective potential but also its local minima are taken into account; as remarked in the previous section, local minima should also be considered as candidates for the real vacuum.

For the condensate of the adjoint representation $(N^2 - 1)$ in $SU(N)$ and that of the second rank symmetric tensor $(2N - 1)(N + 1)$ in $SO(2N)$, the breaking patterns can be found by a straightforward extension of the results of Ref. 16). The result is:

\begin{align}
&\rightarrow SU(N - 1) \otimes U(1) \\
&\rightarrow SU(N) \otimes SU(N - M) \otimes U(1); \quad N \geq 2M > N - M. \quad (3.3)
\end{align}

\begin{align}
&\rightarrow SO(2N - 1) \\
&\rightarrow SO(M) \otimes O(2N - M); \quad 2N \geq 2M > 2N - M. \quad (3.4)
\end{align}

The second pattern of $SO(2N)$ breaking corresponds to false vacua and has not been considered in Ref. 16). The breaking patterns induced by the fourth rank tensors in $SU(N)$ and $SO(2N)$ have not been analysed in the Higgs model and are not worked out here either.

\section*{§ 4. Applications to grand unified theories}

Neither super-strong interactions nor super-heavy fermions are introduced in our self-breaking scheme of non-Abelian gauge symmetries. The symmetry breaking pattern of a gauge theory is fixed without any freedom of adjustable parameters, once the gauge symmetry group is chosen (though we do not know at present a rigorous way of solving this dynamical problem). It is a highly non-
trivial problem whether the desirable breaking pattern of gauge symmetries, e.g., in GUTs, is realized in our scheme. We take $SU(5)^{(1)}$ and $SO(10)^{(2)}$ gauge theories without fundamental scalar fields and investigate the symmetry breaking patterns arising from gauge boson condensation.

Grand unified models contain fermion multiplets as well as a gauge boson multiplet. In this paper we have been discussing the rôle of gauge boson pair condensation, instead of fermion pair condensation suggested by Raby et al., in spontaneous breakdown of non-Abelian gauge symmetries. For comparison, $\Delta C_2(r)$ for two fermion channels are given in the case of $SU(5)$ and $SO(10)$ models in the Appendix.

A. Symmetry breakdown in $SU(5)$ GUT

The gauge boson belongs to the adjoint representation 24. The two gauge boson system can be decomposed into a sum of channels of irreducible representations, as

$$24 \otimes 24 = \{1 \oplus 1 \oplus 75 \oplus 200\}_\text{sym} \oplus \{24 \oplus 126 \oplus 126^* \}_\text{antisym}. \quad (4.1)$$

As mentioned in the case of $SU(N)$, the first three of the four symmetric channels are attractive. The symmetry breaking patterns caused by each of these three condensates are

$$SU(5) \rightarrow SU(4) \times U(1) \quad (4.2)$$

As is well known, by the condensation of 24 channel $SU(5)$ symmetry breaks down to either $SU(4) \times U(1)$ or $SU(3) \times SU(2) \times U(1)$. The latter is obviously the direct product of the color $SU(3)$ and $SU(2) \times U(1)$ of Glashow-Weinberg-Salam model. Breaking patterns induced by condensation of the fourth rank tensor 75 bound state have not been worked out in the Higgs model. The pattern for 75 channel in (4.2) only indicates one possibility.

B. Symmetry breakdown in $SO(10)$ GUT

In the $SO(10)$ gauge theory, the gauge boson belongs to 45 dimensional representation. The Kronecker product of two 45 representations is

$$45 \otimes 45 = \{1 \oplus 54 \oplus 210 \oplus 770\}_\text{sym} \oplus \{45 \oplus 945\}_\text{antisym}. \quad (4.3)$$

The first three of the four symmetric channels are attractive. The analysis in the case of general $N$ in §3 tells us that the $SO(10)$ symmetry can be broken down to $SO(6) \times O(4)$ symmetry by the 54 dimensional scalar. As mentioned there,
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this symmetry corresponds to a false vacuum. The breaking patterns due to gauge boson pair condensation are

\[
\begin{array}{c}
SO(10) \\
\downarrow 54 \\
SO(5) \times O(5) \\
\downarrow 210 \\
SU(5)
\end{array}
\rightarrow
\begin{array}{c}
SU(4) \text{ (confinement)} \\
\downarrow 15 \\
SU(3) \times SU(2) \times U(1)
\end{array}
\]

\[
SO(6) \times O(4)
\]

Notice in particular the breaking pattern

\[
SO(10) \rightarrow SO(6) \times O(4) \rightarrow SU(3) \times SU(2) \times U(1),
\]

caused first by the 54 condensate and then by the 15 condensate in \( SO(6) \). We find it quite gratifying that the breakdown of \( SO(10) \) to the left-right symmetric gauge theory\(^{17,18} \) through the Pati-Salam\(^{17} \) type symmetry emerges as one of the possible breaking paths in our scheme. The symmetry breaking due to the fourth rank tensor 210 has not been worked out in the Higgs model. We can only suggest that the breakdown to \( SU(5) \) or \( SO(6) \times O(4) \) is consistent with the 210 condensate.

The self-breaking of gauge symmetries considered above may also be applicable to the usual GUTs containing fundamental scalars and with the Coleman-Weinberg type symmetry breaking.\(^{19,20} \) Phase transitions in such theories have recently been studied in connection with the big-bang cosmology. It has been shown that the phase transition is of first order and that the Universe supercools at the perturbative vacuum to extremely low temperatures.\(^{21} \) Our argument for gauge boson condensation should be applicable to GUTs with zero-mass fundamental scalars in such supercooled states. It implies an interesting possibility that the Universe is driven to a broken phase by non-singlet gauge boson condensation at a much earlier stage.

§ 5. Discussion

In this paper we have considered the possibility that in non-Abelian gauge theories, perturbative vacuum gets unstable against gauge boson pair condensation in non-singlet channels due to attractive gauge forces. The possible patterns of consequent spontaneous breakdown of gauge symmetries have been obtained. Of particular interest is the application to GUTs. It is interesting that our scheme of dynamical symmetry breaking has been found to offer an explanation for the symmetry breaking patterns commonly assumed in GUTs.

At a more theoretical level, there arise a couple of questions regarding this self-breaking mechanism of gauge symmetries. At the present time we know no
tools to give convincing answers. We shall instead try to clarify the questions and offer some suggestions to which we should look for the answers.

The dynamics of pure non-Abelian gauge theories has recently been the focus of great interest. The main theme is of course to see whether QCD is in the confinement phase. The possibility of phases other than the absolute confinement, e.g., symmetry breakdown to a non-Abelian subgroup, has been mostly left out of consideration. 't Hooft has taken a more general view and tried to classify possible phases of SU(N) gauge theories in terms of two kinds of dual loop operators. According to his analysis, spontaneous breakdown to a non-Abelian subgroup via composite (or elementary) Higgs field is one of the possible phases. More explicitly, in the SU(5) gauge theory, SU(3) x SU(2) x U(1) has been found to satisfy 't Hooft's duality equation. Of course, whether dynamically broken phases we would like to have in GUTs, e.g.,

\[ SU(5) \rightarrow SU(3)_C \times SU(2)_L \times U(1), \]
\[ SO(10) \rightarrow SO(6) \times O(4) \text{ or } SU(5) \]

are realized or not is a dynamical question to be solved by explicit calculation. It is a problem of the same kind as that of how quark confinement is realized in QCD.

Suppose that our picture of dynamical symmetry breakdown in pure non-Abelian gauge theories is basically correct. There still remains an important question: Is the symmetry breakdown caused by gauge boson pair condensation as asserted in this paper? As remarked in § 2, our argument on attractive gauge forces is not entirely gauge independent and cannot be taken at a quantitative level. We believe, however, that our intuitive picture is qualitatively reliable to get indication of the vacuum instability. It is of course possible that more complicated field configurations are also involved in the self-breaking of gauge symmetries.

We now come to a somewhat disturbing question. Suppose that the broken phases (5·1) and (5·2) are the real phases in SU(5) and SO(10) theories respectively. We immediately face the following question: QCD is most likely to be in the confinement phase without SU(3)_C breaking. How can the confinement phase being realized in QCD be reconciled with the broken phase being realized in SU(5)? We are not able to offer a persuasive answer to this question; we can only suggest that the resolution to this puzzle might lie among the following possibilities:

i) The phase structure of non-Abelian gauge theories with SU(N) and SO(2N) may have a non-trivial dependence on N, at least for finite small values of N. Then the realized phase has some dependence on N.*

*) Such N dependence would not be in good accord with the general philosophy of the 1/N expansion.
ii) Fermions may have something to do with the distinction between the phase structure of $SU(3)_c$ and those of $SU(5)$ and $SO(10)$. Recall in this respect that quarks belong to a real representation $3 + 3^*$ in $SU(3)_c$ while fermions belong to complex representations $5^*$ and $10$ in $SU(5)$ (16 in $SO(10)$) (Fermion pairs can condensate without affecting the quark confinement in QCD, whereas only symmetry-broken phases are compatible with condensation of fermion pairs in $SU(5)$).

iii) It does not appear to be excluded that quark confinement in $SU(3)_c$ is insured by virtue of the unbroken $Z(3)$ symmetry, the centre of the $SU(3)$ gauge group, while dynamical Higgs mechanism takes place for $SU(3)/Z(3)$ symmetry via Higgs scalars composed of gluon fields.\(^{24}\) It remains to be shown, however, how quark confinement by $Z(3)$ symmetry can offer an explanation for the apparent success of the symmetric quark model in low-mass baryon spectroscopy.

Acknowledgements

The authors would like to thank R. Fukuda and T. Yoneya for their careful reading of the manuscript and enlightening discussions. They also acknowledge a useful conversation with Z. F. Esawa.

Appendix

Group factors for two fermion channels\(^3\) are given in the $SU(5)$ and $SO(10)$ models.

i) $SU(5)$. Fermions belong to $5^*$ and $10$ representations.

\[
10 \otimes 10 = 5^* \left( -\frac{24}{5} \right) \oplus 45 \left( -\frac{4}{5} \right) \oplus 50 \left( \frac{6}{5} \right),
\]

\[
5^* \otimes 5^* = 10^* \left( -\frac{6}{5} \right) \oplus 15^* \left( \frac{4}{5} \right),
\]

\[
5^* \otimes 10 = 5 \left( -\frac{18}{5} \right) \oplus 45^* \left( \frac{2}{5} \right).
\] \hspace{1cm} (A.1)

ii) $SO(10)$. Fermions belong to a 16 representation.

\[
16 \otimes 16 = 10_8 \left( -\frac{27}{4} \right) \oplus 120_4 \left( \frac{51}{4} \right) \oplus 126_8 \left( \frac{45}{4} \right).
\] \hspace{1cm} (A.2)

The values in parentheses are $C_2(r_c) - C_2(r_1) - C_2(r_2)$, where $r_c$ denotes the representation of the fermion pair channel.

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