Magnetic Effects on Spinodal Decomposition

Tatuo KAWASAKI

Physics Department, College of Liberal Arts
Kyoto University, Kyoto 606

(Received October 22, 1981)

Effects of magnetic interaction and magnetic field on phase separation of a magnetic alloy are studied from the point of interrelation between the clustering of atoms and the growth of magnetization. The work is based on a phenomenological model which is equivalent in all static properties to the microscopic model within the mean-field approximation. Numerical analysis is performed on a one-dimensional version of the model to get more clearly the time dependent behavior of the interrelation. A soliton-like spatial pattern is obtained from the coupled equations governing the evolution of clustering and magnetizing and has turned out to be stable against small perturbations. However it became clear that a proper noise source is important for the system to attain a true equilibrium state.

§ 1. Introduction

Rapid quenches into the coexistence region of binary alloys are followed by a coarsening into two phases. The qualitative understanding is based on the approach of Cahn,\(^1\) Hillert\(^3\) and Hilliard.\(^3\) In addition, previous papers revealed that the magnetic interaction plays an important role in the process of coarsening.\(^4\)–\(^6\) Shift of the critical temperature is already convinced by many authors.\(^7\)–\(^12\) Interrelation between the clustering of atoms and the growth of magnetization was also studied by computer simulation though the execution seems unsatisfactory in that varieties examined there are too scarce for us to get a thorough understanding of the magnetic effect on spinodal decomposition.\(^4\),\(^6\)

In this paper, therefore, we will extend the previous works by presenting not only a simple phenomenological theory but also an exhaustive numerical analysis based on it. The phenomenological model introduced here is assured to have all static properties inherent in the usual microscopic model within the mean-field approximation. Numerical study is performed on a one-dimensional version of the new model. The coupled equations governing temporal and spatial dependence of atomic configuration and magnetization show that development of clusters of atoms correlates intimately with the growth of magnetic domains and vice versa.

In the next section, § 2, a phenomenological model is introduced to describe the spinodal decomposition of a magnetic binary alloy and its static property is discussed by comparing with that due to the usual, microscopically derived model. In § 3 a one-dimensional version of the model is analyzed numerically by laying...
§ 2. Theory

2.1. A phenomenological model

Thermodynamic behaviour of random magnets composed of movable, magnetic atoms is expected to be described well by a model with magnetic interaction between magnetic spins and interatomic interaction between movable atoms. The occupation operator of atoms is able to be replaced by the Ising operator for a binary alloy. When the magnetic interactions between similar atoms are common to both species, the system is reduced to the double Ising system, whose Hamiltonian is defined by

$$\mathcal{H}_0 = - \sum J_{ij} S_i S_j - \sum V_{ij} \sigma_i \sigma_j - \sum I_{ij} S_i S_j \sigma_i \sigma_j - \Delta \sum \sigma_i + h \sum S_i . \quad (2-1)$$

Here $S$ and $\sigma$ stand for the magnetic Ising spin and the pseudo Ising spin connected with the occupation number operator $x(0 \text{ or } 1)$ by the relation $2x=1+\sigma$. $J$ and $V$ denote magnetic and atomic coupling constants between nearest neighbours respectively. The parameter $I$ denotes a magnetic interaction depending on atomic configuration. The last two terms express effects of chemical potential difference and an external magnetic field. In the previous paper this Hamiltonian was used to study phase separation of a magnetic binary alloy.

Here in this paper to consider a simple prototype of a magnetic system which shows phase separation, we apply the usual technique of continuum approximation to the Hamiltonian (2-1), to get a Ginzburg-Landau Hamiltonian

$$\mathcal{H} = \int d\mathbf{r} \left[ \frac{K}{2} (\nabla \cdot \mathbf{S})^2 + \frac{M}{2} (\nabla \cdot \sigma)^2 + f_0(S, \sigma) \right] , \quad (2-2)$$

where

$$f_0(S, \sigma) = \frac{\gamma}{2} S^2 + uS^4 + \frac{\lambda}{2} S^2 \sigma^2 + \frac{a}{2} \sigma^2 + b\sigma^4 - \Delta \sigma - hS \, . \quad (2-3)$$

Here the expansion is terminated up to the fourth order and force range of the interaction $I$ is assumed to be of $\delta$-type. Therefore temperature dependence is included only by parameters $r$ (magnetic) and $a$ (atomic) near the critical region.

2.2. Static properties

Since static properties of the original Hamiltonian (2-1) have already been known well, we first examine whether our new Hamiltonian (2-3) shows an

*) In this subsection configuration averages $\overline{\sigma}$ and $\overline{S}$ of $\sigma(r)$ and $S(r)$ are simply rewritten by $\sigma$ and $S$ respectively.
Magnetic Effects on Spinodal Decomposition

equivalent character to the old one. Spontaneous magnetization below the magnetic critical temperature \( T_M \) is derived by the solution of \( \partial f_0 / \partial S = 0 \) as

\[
S^2 = (|\sigma| - \lambda \sigma^2) / 4 \mu .
\]  

Thus \( T_M(\sigma) \) is determined by the equation

\[
|\sigma| - \lambda \sigma^2 = 0
\]  

which determines concentration dependence of \( T_M \). \( T_M(\sigma = 0) \) is extremum with respect to concentration \( \sigma \), independent of \( \lambda \). Form of the magnetic critical line \( T_M(\sigma) \) is shown in Fig. 1, where two types of curves exist dependent of the sign of \( \lambda \). This classification corresponds to that based on the sign of the coupling constant \( I \) in the original Hamiltonian. It is also without saying that the index \( \beta \) is equal to 1/2 within the approximation \( r \propto (T - T_M) \). When the magnetic critical line is below the spinodal line, the line is modified by fluctuations of concentration, too.

The coexistence line \( T_c(\sigma) \) and the spinodal line \( T_{sp}(\sigma) \) associated with phase separation, are obtained respectively by the following relations:

\[
\frac{\partial f_0}{\partial \sigma} = (a + \lambda S^2) \sigma + 4 b \sigma^3 = 0 , \quad (A = 0)
\]  

and

\[
\frac{\partial^2 f_0}{\partial \sigma^2} = (a + \lambda S^2) + 12 b \sigma^2 = 0 .
\]  

Above the magnetic critical temperature these lines are expressed as a function \( \sigma \) by two parabolas tops of which coincide with each other as is seen in Fig. 2. Here it is important to note that phase transition associated with \( \sigma \) is not the first order, but the second order within the present approximation (upto the fourth order term of \( \sigma \)). Below the magnetic critical temperature, both coexistence and

![Fig. 1. Two types of magnetic critical lines \( T_M(\sigma) \), defined by Eq. (2.5).](https://academic.oup.com/ptp/article-abstract/67/4/1015/1842327)

![Fig. 2. Shifts of coexistence line and spinodal line below the magnetic critical line, given by Eqs. (2.9) and (2.10). Dashed lines are new ones.](https://academic.oup.com/ptp/article-abstract/67/4/1015/1842327)
spinodal lines shift upward or downward depending on the sign of the coupling \(\lambda\), as is seen in Fig. 2. Namely, \(T_c(\sigma)\) and \(T_{sp}(\sigma)\) are modified by the existence of magnetization into the following forms

\[
\bar{T}_c(\sigma) = T_c(\sigma) - \lambda' S^2,
\]

\[
\bar{T}_{sp}(\sigma) = T_{sp}(\sigma) - \lambda' S^2.
\]

(\(\lambda'\) differs from \(\lambda\) only by a positive numerical constant) and are shown by the dotted lines (\(\lambda<0\)) in Fig. 2. Such behavior indicates that spinodal decomposition is enhanced due to the magnetic interaction of the system with \(\lambda<0\) while it is suppressed in the system with \(\lambda>0\). The same character as this is seen in the original Hamiltonian.\(^9\)

Intersections among three lines (coexistence, spinodal and magnetic critical lines) are obtained, from Eqs. (2·5)~(2·7), as

\[
\omega \lambda = 4br,
\]

\[
\omega \lambda = 12br,
\]

respectively. When solutions of (2·11) and (2·12) are denoted by \(T_c-M\) and \(T_{sp-M}\), it is apparent that

\[
T_c(\sigma=0) > T_{c-M} > T_{sp-M} > T_M(\sigma=0) \quad \text{for} \quad \lambda<0,
\]

\[
T_c(\sigma=0) > T_M(\sigma=0) > T_{sp-M} > T_{c-M} \quad \text{for} \quad \lambda>0.
\]

To discuss the atomic system below the magnetic critical temperature, it is sometimes useful to reduce \(f_0(S, \sigma)\) into \(\tilde{f}_0(\sigma)\) by eliminating \(S\) by means of Eq. (2·4),

\[
\tilde{f}_0(\sigma) = \frac{\tilde{a}}{2} \sigma^2 + \frac{\tilde{b}}{4} \sigma^4 - \Delta \sigma.
\]

(\(\tilde{a} = a + |r|/4\mu\) and \(\tilde{b} = b - \lambda^2/16\mu\) for \(h=0\)). However in the following discussion, this form is not used since it is not convenient for studying the system in which the average value of \(\sigma\) is kept constant.

### 2.3. Dynamical properties

To treat dynamics of the magnetic binary alloy at a fixed mean concentration, we will first write down the phenomenological Langevin equations on \(S\) and \(\sigma\),

\[
\dot{S} = -\Gamma_S \delta \frac{\partial \mathcal{H}}{\partial S} + \xi_S
\]

\[
= -\Gamma \{-K \mathcal{P}^2 S + rS + 4\mu S^3 + \lambda \sigma^2 S - h\} + \xi_S,
\]

\[
\dot{\sigma} = \Gamma_\sigma \mathcal{P} \frac{\partial \mathcal{H}}{\partial \sigma} + \xi_\sigma
\]
where $\xi_s$ and $\xi_\sigma$ are assumed as usual to be uncorrelated Gaussian white noise random forces with zero means. In the following discussion in this section, these terms are neglected for the time being.

Stationary States The stationary states are described by the solutions of

$$\frac{\delta \mathcal{H}}{\delta S} = 0 \quad \text{and} \quad \frac{\delta \mathcal{H}}{\delta \sigma} = 0 .$$  \hspace{1cm} (2.17)

If the adiabatic approximation is applicable to the evolution of magnetization, the stationary profile of $\sigma(r)$ is given by the equation

$$-M V^2 \sigma + \tilde{a}_s + 4 \tilde{b} \sigma^3 - \Delta = 0 ,$$  \hspace{1cm} (2.18)

where $\tilde{a}$ and $\tilde{b}$ are defined in Eq. (2.14). The solution of the one-dimensional version of (2.18) (with $\Delta = 0$ and $\tilde{a} < 0$) is known to be

$$\sigma(x) = \sigma_0 \tanh((x - x_0)/\xi_0),$$  \hspace{1cm} (2.19)

where $\sigma_0 = (|\tilde{a}|/4 \tilde{b})^{1/2}$ and $\xi_0 = (2M/|\tilde{a}|)^{1/2}$, and $x_0$ is an arbitrary constant. Substituting this into Eq. (2.4), we get an approximate space-dependence of $S$ in the following form

$$S(x)^2 = S_0^2 \left[ 1 - \frac{\lambda}{|r|} \sigma(x)^2 \right].$$  \hspace{1cm} (2.20)

This seems reasonable as for the effect of the coupling $\lambda$. Namely, magnetization grows larger in cases with negative $\lambda$ while it becomes smaller for cases with positive $\lambda$ than for those with vanishing $\lambda$.

Early stage of evolution We will first consider the early time behavior of evolution after the system is quenched from a high temperature state to a two-phase region. For the stability analysis we linearize Eqs. (2.15) and (2.16) around an initial homogeneous equilibrium state characterized by $S = S_0$ and $\sigma = \sigma_0$. Denoting $\delta S = S - S_0$ and $\delta \sigma = \sigma - \sigma_0$, the linearized equations in Fourier space are

$$\frac{d}{dt} \begin{bmatrix} \delta S_k(t) \\ \delta \sigma_k(t) \end{bmatrix} = - \begin{bmatrix} \Gamma_5 & 0 \\ 0 & \Gamma_6 k^2 \end{bmatrix} \begin{bmatrix} K k^2 + f_{11} \\ f_{12} \end{bmatrix} \delta S_k(t)$$

$$= - N \begin{bmatrix} \delta S_k(t) \\ \delta \sigma_k(t) \end{bmatrix},$$  \hspace{1cm} (2.21)

where

$$f_{11} = \left. \frac{\partial^2 f_0}{\partial S^2} \right|_{S_0, \sigma_0} = r + 12 u \sigma_0^2 + \lambda \sigma_0^2,$$  \hspace{1cm} (2.22)
The state with $S = S_0$ and $\sigma = \sigma_0$ becomes unstable when an eigenvalue of the matrix $N$ changes its sign. When the initial state lies in the paramagnetic region (with $S_0 = 0$), then Eq. (2·21) becomes decoupled and the equation $\det|N| = 0$ gives

$$Kk^2 + r + \lambda \sigma_0^2 = 0 \quad \text{and} \quad k^2(Mk^2 + a + 12b\sigma_0^2) = 0,$$

while in the mean-field-approximation (where the gradient terms $Kk^2$ and $Mk^2$ are neglected) these equations give nothing but boundary of the unstable region, Eq. (2·25) itself expresses the maximum wave-vector below which the coarsening proceeds at the quenched temperature,

$$k^2_s = (|r| - \lambda \sigma_0^2)/K, \quad \propto (T_M(\sigma) - T),$$

and

$$k^2_s = (|a| - 12b\sigma_0^2)/M, \quad \propto (T_{sp}(\sigma) - T)$$

(the wave-vector at which the growth rate of $\sigma$ is maximum is determined as an extremum of the latter equation, being $k_s/\sqrt{2}$). Equation (2·26) also means that lines with constant $k$-values compose a family of similar curves on the $T - \sigma$ plane. This is already pointed out in the different approach.\textsuperscript{15} Even in the case of finite $S$ ($= S_0$) $\det|N|$ can be diagonal as far as $|\lambda| \ll 1$. Therefore in the mean-field approximation the equation $\det|N| = 0$ becomes

$$(r + \lambda \sigma_0^2 + 12uS_0^2) = 0 \quad \text{or} \quad (a + 12b\sigma_0^2 + \lambda S_0^2) = 0$$

or, reminding the relations $(r + \lambda \sigma_0^2) \propto (T - T_M(\sigma))$ and $(a + 12b\sigma_0^2) \propto (T - T_{sp}(\sigma))$, they are reduced to

$$T - T_M + p^2 S^2 = 0 \quad \text{or} \quad T - T_{sp} + \lambda' S_0^2 = 0$$

($p^2, \lambda'$ are numerical constants). The second relation is nothing but a version of Eq. (2·10) while the first relation means only that magnetization will grow upto its equilibrium value from $S_0$.

§ 3. One-dimensional model

To study dynamical interrelation of the coarsening processes in more detail, we wish to consider numerically a one-dimensional version of the model though it contains a weakness in that the diffusion process is limited in low-dimensional
systems. In this section Eqs. (2·15) and (2·16) are numerically traced concentrating our attention to interrelation between the growth of magnetic domains and atomic clusters.

**Theoretical Predictions** Before presenting numerical data, we would like to summarize our predictions on behavior of the one-dimensional model. a): The phenomenon spinodal decomposition is clearly limited within the region bounded by the spinodal line defined by Eq. (2·7). Therefore the boundary may be detected numerically by varying the temperature variable \(a\) at a given concentration in Eq. (2·16). b): As for the sign of the coupling constant \(\lambda\) (c.f., \(\lambda S^2\sigma^2\) in \(\mathcal{H}\)), spinodal decomposition in the case with negative \(\lambda\) is enhanced and accelerated in phase with the growth of magnetization while that of positive \(\lambda\) is suppressed and decelerated out of phase with the magnetic domain. Namely, in the latter case \((\lambda >0)\) it is unfavorable for atomic clusters, owing to the coupling form \(\lambda S^2\sigma^2\), to coincide their boundaries with those of magnetic domains. The case with large \(|\lambda|\) of course gives a remarkable effect on their interrelation. c): Previous numerical results with no magnetic interaction show that in the course of phase separation the density of one species inevitably fluctuates with large amplitude owing to particle number conservation and sometimes has a quasi-periodic form.\(^{16,17}\) Since particle density fluctuations strongly depend on the magnitude of the diffusion constant \(M\) in Eq. (2·16), there is a possibility for cases with small diffusion constants to have a periodic pattern of density with an appropriate boundary condition\(^*\) d): At the even concentration \((\bar{\sigma}=0)\) the system has no metastable region and is always unstable below the critical temperature \((a<0)\). Since it means even small clusters can grow larger, the system will fluctuate more frequently in space than that with \(\bar{\sigma} \neq 0\) at an early stage of evolution. To the contrary the domain should grow monotonically when the system is quenched at a concentration \(\bar{\sigma}=0\); Rate of the growth should be favorable to the rich atoms since one of the species dominates the other in number. e): Both initial growth rate and stationary profile of the density are determined sensitively by the coefficients \(a\) and \(r\) (temperature variables). In Eqs. (2·15) and (2·16) linear growth rates are given by \(a\) and \(r\), terms associated with which dominate others while the density \(\sigma\) and the magnetization \(S\) are small at the early stage of evolution. And large values of \(|a|\) and \(|r|\) should give a sharp space variation of density and magnetization because space derivatives also become large in the evolution equations. f): Effect of a magnetic field may be transferred only by virtue of the term \(\lambda S^2\sigma^2\) to the process of phase separation. Spinodal decomposition is accelerated in the system with \(\lambda <0\); the critical temperature is effectively raised through the coupling \(\lambda S^2\sigma^2\) by the existence of magnetization induced by the field as was discussed around Eq. (2·8).

\(^*\) Computer simulation on a two-dimensional system also showed a periodic behavior in the density profile when the system is not a square but a prolonged rectangular in its form (unpublished).
When $\lambda > 0$, the opposite behavior is expected. g): The original coupled equations (2-15) and (2-16) include the random forces $\xi_s$ and $\xi_\sigma$ which were neglected in the above numerical calculation. After we get a stationary configuration of $S$ and $\sigma$, we first wish to examine whether this is stable to introduction of random forces. Effect of random forces is also studied when they are present through the course of evolution. These examinations may teach us the degree of importance of random forces in considering the process of phase separation.

**Numerical Results** To evaluate the coupled nonlinear equations (2-15) and (2-16) numerically, a set of 150 local concentration variables $\sigma(x)$ and spin variables $S(x)$ are used to discretize the continuous space variables. Time increment is controlled so that it is small enough for the linear approximation to time evolution to be valid, but large enough for us to save computer time. The evolution of equations is traced in principle up to eight times as long as a relaxation time $(T_s MT_0)^{-1}$ by the step of time increment $10^{-4}$. Periodic boundary condition is imposed on the ends. Several parameters in the equations are fixed, unless stated otherwise, in the following numerical analysis: $K=M=1.0$, $u = b = 2$ and $h = A = 0$. One of the phenomenological constants $\Gamma_s$ is assumed to be five times as large as the other one, $\Gamma_\sigma$, by taking into account the slowness of atomic motion compared with the growth of magnetization in the course of phase separation. Then the remaining, unfixed parameters are temperature variables $a$ and $r$, and the coupling constant $\lambda$. a): The spinodal line was firstly examined at $\bar{\sigma} = -0.75$. The effective spinodal temperature variable $\bar{\sigma}^*$ is determined by the equation $\bar{\sigma}^* + 12b\sigma^2 + \lambda S^2 = 0$ and is expected to be between $-11$ and $-10$ when $\lambda = -2$ and $r = -5$ which makes the domain magnetization $S = 1.1 \sim 1.3$. This feature is clearly demonstrated in Fig. 3. In the figures shown below, pairs of evolutions for $\sigma$ and $S$ are plotted versus position $x$ at each time. The parameter $\bar{\sigma}^*$ shifted upward from $-13.5$ for a nonmagnetic case to approximately $-10.5$. b): Before discussing the effect of the coupling $\lambda$, we wish to show in Fig. 4 the data with $\lambda = 0$ for the case $\bar{\sigma} = 0$ (number of $A$-atoms is equal to that of $B$-atoms). As the quenched temperature is set lower from the critical temperature, $\sigma(x)$ and $S(x)$ grow larger in their amplitudes. As is clearly seen in the figure, there is of course not any relations between their phases. In Fig. 5 four varieties with $\lambda$ are shown where $\bar{\sigma} = 0$ and temperature variables $a$ and $r$ are set $-10$. In Fig. 6 the same behavior is traced for cases with $\bar{\sigma} = -0.75$, $a = -20$ and $r = -10$. It is apparent that coarsening is suppressed in cases with positive $\lambda$ while it is enhanced for negative $\lambda$. Boundaries of atomic clusters are in phase with those of magnetic domains for negative $\lambda$ while they have a phase shift with each other by $90^\circ$ that decreases contribution to the total energy from term $\lambda S^2 \sigma^2$. c): Effect of the diffusion constant $M$ is summarized in Fig. 7, where the longest execution was performed in the present calculation. As the constant $M$ increases, the separation between domains becomes wider and seems to be propor-
Magnetic Effects on Spinodal Decomposition

Fig. 3. Coarsening behavior below and above the spinodal line at several temperatures. That $a=0$ corresponds to a point at $\sigma=0$ on the coexistence line. Symbols $c$ (upper) and $M$ (lower) in each figure denote the concentration of one of the species and magnetization respectively. Abscissa is used for space coordinate which is discretized into 150 points with periodic boundary condition. Marks on ordinate for concentration denote equilibrium value obtained by minimizing the free energy with no magnetic interaction. Time $t$ counts frequency of iteration proceeding by $\Delta t$. The initial configuration is prepared by means of random sampling technique with magnitude of one-fourth equilibrium value.

Fig. 4. Evolution profiles for $\sigma(x)$ and $S(x)$ at $\sigma=0$ in the one dimensional model (no magnetic coupling ($\lambda=0$) exists between them.) As the temperature goes down from the critical temperature, $\sigma(x)$ and $S(x)$ grow larger in their amplitudes. Moreover their cluster and domain boundaries become sharper and have a clear periodicity in their space profiles. At $a=r=-20$, the last configuration is not yet stationary within the present execution.
Fig. 5. Effect of the magnetic coupling $\lambda S^2 \sigma^2$ at $\sigma = 0$ and $a = r = -10$.

Fig. 6. Effect of the magnetic coupling $\lambda S^2 \sigma^2$ at $\sigma = -0.75$, and $a = -20$ and $r = -10$.

Suppression and enhancement of spinodal decomposition and magnetization depend clearly on the sign of the coupling $\lambda$. For $\lambda < 0$ atomic concentration behaves in phase with magnetization amplitude while they have a phase shift with each other by $90^\circ$ for $\lambda > 0$.

tional to the square root of $M$. It is seen from Fig. 6 that this separation is independent of $\lambda$, the stability of which will be examined later. Furthermore this periodicity is independent of temperature as is seen in Fig. 8. d): The coarsening process at $\sigma = 0$ is shown in Fig. 8. Though final patterns are similar to
Fig. 7. Evolution profiles associated with variation of the diffusion constant $M$ in Eq. (2.16); $\sigma = -0.75$, $\lambda = 2$, $a = -20$ and $r = -5$. Separation between domains becomes wider with the increase of $M$. The longest execution was performed to find the stationary state.

Fig. 8. Temperature dependence of coarsening behavior at $\sigma = 0$. Profiles at an earlier stage have the more frequent variation in their amplitudes than those at $\sigma = -0.75$ shown in Fig. 6.

each other in cases with $a = r = -5$ and $-10$, initial coarsening behavior differs not only in its amplitude but also in its cluster size, which is related with Eq. (2.26). To the contrary it seems from Fig. 6 at $\tilde{\sigma} = -0.75$ that odd concentration forces the system to form a well-behaved pattern at an early stage of coarsening process though the cluster size is not different from that at $\tilde{\sigma} = 0$. e): Temperature variations in evolution patterns are seen in Figs. 4 and 8. As the temperature decreases, their amplitudes grow larger and their domain boundaries become sharper owing to the large linear growth rate governed by temperature variables $a$ and $r$. f): Effect of the magnetic field is seen in Figs. 9 and 10 where the field is adjusted to yield a proper average value of magnetization. In Fig. 9 the field is on through the course of evolution while in Fig. 10 it is switched on at $t = 5000$ when well-defined patterns are already formed. Since the critical temperature of the system with positive $\lambda$ decreases due to the existence of magnetization, the field has a definite effect of weakening the coarsening process in this case. When $\lambda$ is negative, there seems to be no remarkable differences, within the present investigation, between systems with and without fields. g): Effect of introducing noise terms on the evolution equations is studied for several cases. In cases (a) and (b) in Fig. 11 the noise is introduced after fairly well-defined stationary patterns are obtained. Irrespective of the existence of magnetization, the stationary states are fairly stable against the noise whose integrated intensity ($\xi \Delta t$) is controlled at the level of half of the equilibrium amplitude ($2|\xi \Delta t| \leq S_{eq}$).
Fig. 9. Effect of magnetic fields on coarsening processes. Fields were on through the course of evolution to keep the magnetization at a given value, which is indicated at the top of each column. \( \sigma = -20 \) and \( \lambda = 2 \). The coarsening is suppressed by the fact that the critical temperature decreases due to the uniform, large magnetization. The case with \( \lambda = -2 \) is shown for reference.

Fig. 10. Effect of magnetic fields on coarsening processes when the fields are applied on the stationary states. The fields were on for \( t > 5000 \). Other parameters are the same as in Fig. 9. The last column is given for reference.

or \( \sigma_{eq} \). In the case (c) stability is examined for an arbitrarily constructed pattern. The result also shows that the pattern once formed is stable against small perturbations. In Fig. 12 the noise source is introduced at the beginning of evolution as it is in the original equations. The formation of periodic patterns depends critically on the intensity of noises. At the same level of the noise as in Fig. 11, no periodic pattern is formed and in addition no phase separation is seen to proceed. To the contrary two-phase separation is attained at the half level of
Magnetic Effects on Spinodal Decomposition

Fig. 11. Stabilities of the obtained stationary profiles against introduction of noise sources. The noise terms in Eqs. (2.15) and (2.16) are added for \( t > 5000 \) to the evolution equations in cases (a) and (b) while it is considered for an arbitrarily constructed domain in (c). Noise amplitude \( \xi dt \) is kept at the level of half of the equilibrium amplitude \( S_{eq} \) or \( \sigma_{eq} \). These three examples suggest that the stationary state once reached may be fairly stable against small perturbations (\( a = -20 \) and \( r = -10 \)).

Fig. 12. Stabilities of pattern formation. The noise terms are introduced at the beginning of evolutions (\( a = -20 \) and \( r = -10 \)). Time sequence of figure is from up to down line and from left to right column. The last figure is for \( t = 300000 \). Two-phase separation is realized by means of the noise.

the above noise intensity. As far as the present calculation is concerned, only the proper noise may be helpful for the two-phase separation at the later stage of evolution. This suggests that the stationary state may be long-lived though it should not be true equilibrium state.

§ 4. Summary and discussion

Magnetic effects on spinodal decomposition was discussed on the basis of the phenomenological Ginzburg-Landau-type model. The magnetic interaction has turned out to be significant in the process of coarsening. Whether spinodal
decomposition is enhanced or suppressed, accelerated or decelerated by the magnetic interaction depends essentially on its sign as well as temperature depth from the critical line. The critical size of atomic clusters was determined as usual by linearized equations of evolution. Numerical analysis on the one-dimensional version of the equations revealed that well-defined, stationary, spacially periodic patterns were obtained over the wide range of parameters and were found stable against small perturbations. Introduction of the proper noise sources may be favorable for coarsening processes. Looking more closely at the figures, we see that the coarseness of the final configuration predicted by the equations is fairly independent of the initial configuration against the previous result. The last statement might depend strongly on the dimensionality because computer simulations on three-dimensional system have already revealed that phase separation was always complete. In consonance with the effect of the magnetic interaction, magnetic fields were also effective in the coarsening process. Mathematical analysis of the soliton-like behavior of the stationary pattern will be given in a future paper.

Acknowledgements

The work is partially supported by the Grant-in-Aid for Scientific Research from the Ministry of Education, Science and Culture. Numerical calculations are carried out at the Data Processing Center of Kyoto University.

References