Neutral-Current Effects in Electron-Polarized Deuteron Scattering

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The weak neutral-current effects in electron-polarized deuteron scattering are investigated in the Weinberg-Salam, \( SU(3) \times U(1) \) and \( SU(2)_L \times SU(2)_R \times U(1) \) gauge models. We derive the parity-violating asymmetry for the scattered electron. It is expressed in terms of well-known electromagnetic and weak form factors of the nucleon. The asymmetries are expected to be of the order of \( 10^{-6} \) and \( 10^{-5} \) at the incident electron energy of 0.5 GeV and 2 GeV for each gauge model. The experiment at 0.5 GeV gives the strong constraints on these gauge models and at 2 GeV may be useful to discriminate them.

§ 1. Introduction

At present, the standard Weinberg-Salam (W-S) model\(^1\) is successfully applied to neutrino induced neutral current phenomena and to the parity violation in deep inelastic scattering of longitudinally polarized electrons from unpolarized deuterons.\(^2\) Above experimental data, moreover, are also in good agreement with the predictions of the other gauge models such as \( SU(3) \times U(1) \),\(^3\) \( SU(2)_L \times SU(2)_R \times U(1) \)\(^4\) and \( SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R \).\(^5\)

Because the gauge theories are promising candidates for the unified theory of the particle physics, much attention has been focused on the problem to determine the gauge group of weak and electromagnetic interactions. For the W-S model, it is important to determine a more precise value of its single parameter, Weinberg angle, in order to see the model consistency with all the weak neutral current data.

Even though the W-S model predictions are consistent with all the neutral current data, we are not able to reject the other gauge models discussed here. Because they all embed the W-S model in themselves, and then, for the neutral current interactions, their effective Lagrangians are assumed precisely in the form of the W-S model in a certain limit of parameters included in them. Then, we only have strong constraints on the other gauge models when the W-S model works well. The converse, however, is not true. The gauge models discussed here, except for the W-S model, have many adjustable parameters so that there remain possibilities for them to reproduce the experimental data which deviate from the W-S model predictions.
Recently, Komatsu has discussed the asymmetries of polarized electron-proton scattering at very high energy and shown the slight differences between the values predicted from the various gauge models. It is well known, however, in the statistical point of view, the experiment at lower energy region is preferable to find the differences and/or to give the strong constraints on the gauge models. In a previous paper, we showed that the experiment on electron scattering by the polarized deuteron target; \( e + d \to e' + d \) or \((p, n)\), is feasible to determine the weak neutral coupling constants of the electron-quark sector, and the order of the magnitudes of the parity violating asymmetry is given by using the W-S model. In the present paper, we show the proposed experiment is suitable to find the differences among the gauge models (W-S, SU(3) \( \times \) U(1) and SU(2)_L \( \times \) SU(2)_R \( \times \) U(1) models) and/or to give the strong constraints on these models.

In §2, we first derive the general formulas of calculating the differential cross section. We also define the parity-violating asymmetry. It is expressed in terms of the well-known electromagnetic and weak form factors of a nucleon. Numerical results are given in §3 for the various gauge models. We show that the parity-violating effects are of the order of \( 10^{-6} \) and \( 10^{-5} \) at the incident electron energy of 0.5 GeV and 2 GeV for each gauge model. Conclusions are also given in §3.

§2. The differential cross section for electron-polarized deuteron scattering and the definition of the parity-violating asymmetry

In this section, we shall derive first the differential cross section for the process briefly,

\[
\frac{d}{d^4p_1} \frac{d}{d^4p_2} \frac{d}{d^4p_3} \frac{d}{d^4p_4}
\]

where the quantities in the parenthesis express four-momenta of the corresponding particles or \( p, n \) system. \( d \) represents the polarized deuteron target. Relevant Feynman diagrams are shown in Figs. 1(a) and (b). If we observe the hadronic final states of the above reaction separately, we can determine the isoscalar and the isovector neutral current coupling constants individually. It is well known this is one of the important properties of the isoscalar deuteron.
target. The calculations are rather tedious for the complexity of the deuteron wave function and of the non-relativistic reduction of the formulas adopted here. Then, details will be discussed in our succeeding paper.

The neutral current Lagrangian we use is

\[ L = g_\nu \sum_i g_i^i Z_i [g^{\nu i} \gamma^\nu \bar{e} e - g^{A i} \gamma^\nu \gamma^5 e + Q^{\nu i} \bar{d} \gamma^\nu u + Q^{A i} \bar{d} \gamma^\nu \gamma^5 d] + \text{electromagnetic terms}, \]  \hfill (2.2)

where summation is taken over the weak neutral currents to which the weak neutral vector bosons \( Z^i \) are coupled. Coupling constants are dependent upon each gauge model and are tabulated in Table I. \( e, u \) and \( d \) stand for the fields of the electron, up- and down-quarks, respectively and we neglect the contributions from the other quarks in the nucleon. We calculate the differential cross section which is induced by the electromagnetic current and by the interference between the electromagnetic and the weak currents. That induced by the weak current only is safely neglected. We adopt the impulse approximation which is valid in the energy region calculated in the present paper. Then, the nuclear corrections such as the final state interactions are expected to make negligible contributions. After the straightforward calculations, we obtain the sum and the difference of the differential cross sections for the kinematical configurations shown in Fig. 2 as

\[
\frac{d\sigma(\theta)}{d\Omega} + \frac{d\sigma(\pi - \theta)}{d\Omega} = \frac{a q^4}{4M^2E^4(1-\sin^2 \theta)^4} \times \left[ \frac{1}{2} \left( M^2(1+\sin \theta) + E^2(1-\sin \theta)^2 + 2ME(1-\sin^2 \theta) \right) \right] \times \left\{ f_1 \left( f_2 + f_2^2 \right) + f_1 \left( f_1 + f_1^2 \right) \right\} \times 2M^2(1+\sin \theta) \]  \hfill (2.3)

and

\[
\frac{d\sigma(\theta)}{d\Omega} - \frac{d\sigma(\pi - \theta)}{d\Omega} = \frac{a \cos \theta q^4 g_\nu^2}{8M^2E^4(1-\sin \theta)^4} \times \left\{ \frac{g^{\nu i} g_i^A}{M_i^2} \right\} \times \left\{ g^{\nu i} \left[ f_1 \left( f_1 + f_2 + f_2 \right) \left( M + E \right) \right] + g^{\nu i} \left( f_1 + f_2 \right) \left( M + E \right) \right\} \times \left[ g^{\nu i} \left( f_1 \left( f_1 + f_2 + f_2 \right) \left( M + E \right) \right) + g^{\nu i} \left( f_1 + f_2 \right) \left( M + E \right) \right], \]  \hfill (2.4)

where \( q^2 = -2ME(1-\sin \theta)/(M + E(1+\sin \theta)) \). \( M \) is the nucleon mass and \( E \)
**Table I.** Coupling constants of the weak neutral-current.

<table>
<thead>
<tr>
<th></th>
<th>Weinberg-Salam model</th>
<th>$SU(3)\times U(1)$ model</th>
<th>$SU(2)\times SU(2)_R\times U(1)$ model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_\varepsilon$</td>
<td>$e/\sin 2\theta_W$</td>
<td>$e/\sin 2\theta_W$</td>
<td>$e/\sin 2\theta_W$</td>
</tr>
<tr>
<td>$g_\nu^0$</td>
<td>$1/2 + 2 \sin^2\theta_W$</td>
<td>$1/2 + 2 \sin^2\theta_W - A\left(-1/2 + 3/2 \tan^2\theta_W\right)$</td>
<td>$1/2 + 2 \sin^2\theta_W - A\left(-1/2 + 3/2 \tan^2\theta_W\right)$</td>
</tr>
<tr>
<td>$g_\nu^1$</td>
<td>$1/2$</td>
<td>$-1/2 + A\left(-3/2 + 1/2 \tan^2\theta_W\right)$</td>
<td>$-1/2 + A\left(-1/2 + 1/2 \tan^2\theta_W\right)$</td>
</tr>
<tr>
<td>$Q_{\bar{e},\nu}$</td>
<td>$1/2(1 - 8/3 \sin^2\theta_W)$</td>
<td>$1/2(1 - 8/3 \sin^2\theta_W - A\left(-1/2 + 5/6 \tan^2\theta_W\right))$</td>
<td>$1/2(1 - 8/3 \sin^2\theta_W - A\left(-1/2 + 5/6 \tan^2\theta_W\right))$</td>
</tr>
<tr>
<td>$Q_{e,\nu}$</td>
<td>$-1/2$</td>
<td>$-1/2 - A\left(-1/2 + 1/2 \tan^2\theta_W\right)$</td>
<td>$-1/2 - A\left(-1/2 + 1/2 \tan^2\theta_W\right)$</td>
</tr>
<tr>
<td>$Q_{\bar{e},\nu}$</td>
<td>$1/2(1 - 4/3 \sin^2\theta_W)$</td>
<td>$1/2(1 - 4/3 \sin^2\theta_W - A\left(-1/2 + 1/6 \tan^2\theta_W\right))$</td>
<td>$1/2(1 - 4/3 \sin^2\theta_W - A\left(-1/2 + 1/6 \tan^2\theta_W\right))$</td>
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<td>$Q_{e,\nu}$</td>
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</tr>
<tr>
<td>$g_\nu^0$</td>
<td>$-\sin\alpha$</td>
<td>$\sin\alpha$</td>
<td>$\sin\alpha$</td>
</tr>
<tr>
<td>$g_\nu^1$</td>
<td>$\frac{A}{\cos\alpha}$</td>
<td>$\frac{A}{\cos\alpha}$</td>
<td>$\frac{A}{\cos\alpha}$</td>
</tr>
<tr>
<td>$Q_{\bar{e},\nu}$</td>
<td>$\frac{B}{\sin\alpha}$</td>
<td>$\frac{B}{\sin\alpha}$</td>
<td>$\frac{B}{\sin\alpha}$</td>
</tr>
<tr>
<td>$Q_{e,\nu}$</td>
<td>$\frac{B}{\tan\alpha}$</td>
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</tr>
<tr>
<td>$B$</td>
<td>$\cot\alpha \frac{\cos^2\theta_W}{(3 - 4 \sin^3\theta_W)^{1/2}}$</td>
<td>$\cot\alpha \frac{\cos^2\theta_W}{(1 - 2 \sin^3\theta_W)^{1/2}}$</td>
<td>$\cot\alpha \frac{\cos^2\theta_W}{(1 - 2 \sin^3\theta_W)^{1/2}}$</td>
</tr>
</tbody>
</table>

**Fig. 2.** Kinematical variables in the laboratory system.
denotes the energy of the incoming electron. The electron mass is safely neglected. $g_{l,p,n}$, $g_{k,p,n}$, $g_{l,p,n}$, $g_{l,n}$, $f_{l,p,n}$, $f_{2,p,n}$ are written in terms of the electromagnetic and weak form factors. Using the isospin property of the currents,\(^8\) we have the explicit formulas as follows:

\[
\begin{align*}
g_{l,p} &= G_{M,p}(2Q_{l,v} + Q_{l,v}) + G_{M,n}(Q_{l,v} + 2Q_{l,v}), \\
g_{k,p} &= \frac{1}{2M(1-q^2/4M^2)}[(2Q_{l,v} + Q_{l,v})(G_{E,p} - G_{M,p}) \\
&\quad + (Q_{l,v} + 2Q_{l,v})(G_{E,n} - G_{M,n})], \\
g_{l,n} &= GM,p(Q_{l,v} + 2Q_{l,v}) + G_{M,n}(2Q_{l,v} + Q_{l,v}), \\
g_{k,n} &= \frac{1}{2M(1-q^2/4M^2)}[(Q_{l,v} + 2Q_{l,v})(G_{E,p} - G_{M,p}) \\
&\quad + (Q_{l,v} + 2Q_{l,v})(G_{E,n} - G_{M,n})], \\
g_{k,n} &= \frac{G_A(q^2)}{\mu_p - \mu_n} [\mu_p(Q_{l,v} + 2Q_{l,v}) + \mu_n(2Q_{l,v} + Q_{l,v})], \\
f_{l,p} &= G_{M,p}(q^2), \\
f_{2,p} &= \frac{1}{2M(1-q^2/4M^2)}(G_{E,p} - G_{M,p}), \\
f_{l,n} &= G_{M,n}(q^2), \\
f_{2,n} &= \frac{1}{2M(1-q^2/4M^2)}(G_{E,n} - G_{M,n}).
\end{align*}
\]

Here, $G_{E,p}$, $G_{M,p}$, $G_{E,n}$ and $G_{M,n}$ are Sachs' form factors of a nucleon, and are well known experimentally at least for low energy region. Experimentally, they have the same dipole $q^2$ dependence and we have

\[
G_{E,p}(q^2) = \frac{G_{M,p}(q^2)}{1 + \mu_p} = \frac{G_{M,n}(q^2)}{\mu_n} = (1 - q^2/0.71)^{-2}
\]

and

\[
G_{E,n}(q^2) = 0, \quad (2.6)
\]

where $\mu_p$ and $\mu_n$ are the anomalous magnetic moments of the proton and neutron, respectively. We take the axial-vector form factors of the form

\[
G_A(q^2) = G_A(0)(1-q^2/M_A^2)^{-2}, \quad (2.7)
\]
where \( G_A(0) = 1.254 \pm 0.007 \) and \( M_A^2 = 0.89 \pm 0.17 (\text{GeV}/c^2)^2 \). We choose \( G_A(0) = 1.254 \) and \( M_A^2 = 0.89 (\text{GeV}/c^2)^2 \) for numerical calculations. Then, we define the parity-violating asymmetry as

\[
P(\theta) = \frac{d\sigma(\theta) - d\sigma(\pi - \theta)}{d\sigma(\theta) + d\sigma(\pi - \theta)}.
\]

§ 3. Numerical results and conclusions

Now, we present the numerical results. The predictions of the parity-violating asymmetry are given as a function of \( \theta \) (electron scattering angle) used the W-S, \( SU(3) \times U(1) \) and \( SU(2)_L \times SU(2)_R \times U(1) \) models in Figs. 3–5. In each figure, figures labelled (a) and (b) show the calculations at the incident energy of 2 GeV and 0.5 GeV, respectively. For the W-S model, we have used \( 0.238 \leq \sin^2 \theta_W \leq 0.218 \), which is consistent with all the experimental data, to obtain the predicted region.

In the \( SU(3) \times U(1) \) and \( SU(2)_L \times SU(2)_R \times U(1) \) models, we have two massive neutral gauge fields \( Z \) and \( X \). These fields are not necessarily mass eigenstates. We define the mixing angle \( \alpha \) as

\[
\begin{pmatrix}
Z_1 \\
Z_2
\end{pmatrix} =
\begin{pmatrix}
\cos \alpha & -\sin \alpha \\
\sin \alpha & \cos \alpha
\end{pmatrix}
\begin{pmatrix}
Z \\
X
\end{pmatrix}
\]

Fig. 3. The asymmetry \( A \) as a function of \( \theta \) at the incident electron energy of 2 GeV (a) and of 0.5 GeV (b) in the Weinberg-Salam model. Shaded areas are the predicted regions for \( 0.238 \leq \sin^2 \theta_W \leq 0.218 \).
Fig. 4. The asymmetry $A$ as a function of $\theta$ at the incident electron energy of 2 GeV (a) and of 0.5 GeV (b) in the $SU(3) \times U(1)$ model. Shaded areas are the predicted regions for $0.30 \leq \sin^2 \theta_w \leq 0.18$. We choose other parameters as $M_1 = 96$ GeV, $M_2/M_1 = 3$ and $\alpha = 0$.

Fig. 5. The asymmetry $A$ as a function of $\theta$ at the incident electron energy of 2 GeV (a) and of 0.5 GeV (b) in the $SU(2)_L \times SU(2)_R \times U(1)$ model. Shaded areas are the predicted regions for $0.34 \leq \sin^2 \theta_w \leq 0.16$. We choose other parameters as $M_1 = 96$ GeV, $M_2/M_1 = 3$ and $\alpha = -0.02\pi$. 
where $Z_1$ and $Z_2$ are mass eigenstates of the neutral weak bosons and their masses are $M_1$ and $M_2$, respectively. Then in most general case of these models, parameters are Weinberg angle $\theta$, $M_1$, $M_2$ and $a$. We adopt the same values of parameters $M_1$, $M_2$ and $a$ as Ref. 4). We vary the Weinberg angle as shown in each figure to give the predicted regions.

From Figs. 3~5, it is evident that the parity-violating asymmetries are of the order of $10^{-6}$ and $10^{-5}$ for 0.5 GeV and 2 GeV, respectively. From figures (a), it is known these gauge models are distinguishable by the experiment proposed here at the incident electron energy of 2 GeV when we choose the values of parameters shown in each figure caption. This situation is also true at the incident energy higher than about 1 GeV. Moreover, from Fig. (b), lower incident energy is suitable to give the strong constraints on the gauge parameters of each gauge model.

We describe in Fig. 6 the differential cross sections given in Eq. (2.3). It is emphasized, because of the lower energy of the incident electron, the differential cross sections are large enough to be experimentally accessible. Moreover, we know the polarized deuteron targets which have the perpendicular polarization to the incident beam are used in some laboratories. Then, we can expect significantly that it may be possible to observe the parity-violating asymmetry in electron-polarized deuteron scattering. We surely obtain the more precise knowledge of the electromagnetic and weak interactions.

References

2) See, for example, Proceeding of Neutrino 79, Bergen 1979, vol. 1., edited by A. Haatuft and C. Jarlskog, p. 267 and references therein for further references.  
10) Particle Data Group, Rev. Mod. Phys. 52 (1980), S1.