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A Study of Soilwater and Groundwater Flow of Hillslopes – Using a Mathematical Model

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The soil and groundwater flow of a hillslope adjacent to a stream has been studied using a physically based mathematical model. Values of soil parameters have been taken from investigations of Swedish till soils. Model simulations have been made in order to calculate groundwater outflow during and after a rainfall event, and to study the response to infiltration of soil and groundwater flow in different parts of the slope. The effects of soil layering and slope configuration have also been studied.

The simulated peak of groundwater outflow, occurring just after cessation of infiltration, is of the same order of magnitude as a typical streamflow peak in a small Swedish watershed. The increase of groundwater outflow during and after a rainfall event is, according to model simulation, caused by infiltration close to the stream. Hence, the soil properties of this part of the slope are of great importance to runoff generation.

Introduction

In the generation of streamflow runoff, the properties of the hillslope leading down to the stream are of great importance. To study the runoff generation, a physically based mathematical model describing water flow of hillslopes may be used. Hillslope models have been developed by among others Freeze (1971), Beven (1977) and Yeh (1980). For practical reasons they are made for two dimensions, neglecting flow parallel to the stream. In these models the importance of soil parameters, slope configuration and rainfall intensities may be studied.

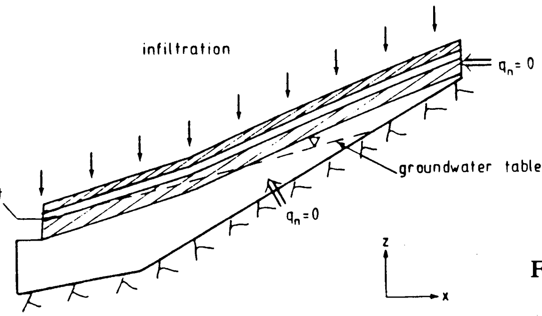

Fig. 1. Principal model structure.

Freeze (1971) and Beven (1977) have simulated many different conditions. In this paper model simulations for Swedish conditions are described, utilizing soil parameters measured by Lundin (1982). His investigations were carried out in a small watershed where the soil cover was composed of till soils.

Rodhe (1984) has, by using the isotope oxygen-18 as a tracer, found that in many Swedish watersheds the main part of streamflow runoff consists of groundwater outflow, also during flow peaks. Beven and Freeze found that if groundwater was to dominate streamflow runoff during storms, a certain threshold value of saturated hydraulic conductivity had to be exceeded. This threshold value was dependent on initial conditions and slope configurations. A question to answer when making the simulations for Swedish conditions was then: Do the measured hydraulic conductivities lead to a simulated outflow, having typical streamflow peak values?

The flowpaths may be studied using a two-dimensional model. For example the effect of soil layering may be investigated, as well as the origin of groundwater outflow during a flow peak. However, the effects of the inhomogeneity of the soil and the three-dimensional topography are not considered in the model.

The model structure is described by Fig. 1.

Theoretical Background

The equation used to describe the two-dimensional soil and groundwater flow is obtained by combining Darcy's law and the mass balance equation (see e.g. Bear 1979).

$$\frac{\partial}{\partial x}\left(K(\psi,x,z)\frac{\partial\psi}{\partial x}\right) + \frac{\partial}{\partial z}\left(K(\psi,x,z)\left(\frac{\partial\psi}{\partial z}+1\right)\right) = C(\psi,x,z)\frac{\partial\psi}{\partial t} \qquad (1)$$

where

$K(\psi, x, z)$ – hydraulic conductivity
ψ – pressure potential
$C(\psi, x, z)$ – specific moisture capacity

The following assumptions have been made when using this equation: 1) the density of water is constant, 2) the flow is independent of the temperature and chemical composition of the water, 3) Darcy's law is also valid in the unsaturated zone, and 4) the soil is isotropic.

In this paper it is assumed that the unsaturated hydraulic can be calculated with formulas derived by Mualem (1976)

$$\begin{aligned} K(\psi,x,z) &= K_s(x,z)\left(\frac{\psi_a}{\psi}\right)^{2+(2+n)\lambda} && \psi \le \psi_a \\ K(\psi,x,z) &= K_s(x,z) && \psi > \psi_a \end{aligned} \qquad (2)$$

where

K_s (x,z) – saturated hydraulic conductivity
ψ_a (x,z) – air entry tension
λ (x,z) – poor size distribution factor
n – a parameter set to unity in the present paper

To be able to use the same parameters when calculating specific moisture capacity ($d\theta/d\psi$), the following expressions were used (Brooks and Corey 1969)

$$\begin{aligned} C(\psi,x,z) &= -\frac{\lambda}{\psi_a}(\theta_s-\theta_r)\left(\frac{\psi}{\psi_a}\right)^{-(\lambda+1)} && \psi \le \psi_a \\ C(\psi,x,z) &= -\frac{\lambda}{\psi_a}(\theta_s-\theta_r)\left(\frac{\psi_a}{\psi}\right)^{-(\lambda+1)} && \psi < \psi_a \le 0 \\ C(\psi) &= 0 && \psi > 0 \end{aligned} \qquad (3)$$

where

θ_s (x,z) – porosity
θ_r (x,z) – residual water content

When using this formula to calculate specific moisture capacity it is assumed that the relationship $\theta = \theta(\psi)$ is a unique one. Eqs. (2) and (3) have previously been used for Swedish conditions in the SOIL-model (Jansson and Halldin 1979).

In the model used in this study, the flow Eq. (1) is solved by a finite element method, using a general program package, TWODEPEP (1981). For the hypothetical hillslopes in the simulations, with a length of 40 m and a thickness of 1 m, about 400 elements were used. The element net was made denser close to the stream and the soil surface. The mean time step was 10 minutes, but varied during the simulations from about 2 minutes at the beginning to 25 minutes at the end. Boundary conditions were given either as constant pressure potential or as flow rates. Due to the number of elements needed and the non-linearity of the equations, the required computer capacity was large.

Soil Parameters

In this study the model is applied for Swedish conditions. As great parts of Sweden are covered by till soils, soil parameters for this type of soils are used. Few measurements have been made in till soils, but one thorough investigation has been carried out by Lundin (1982). He investigated a small watershed (0.19 km^2) situated in central Sweden. Soil properties as well as soil and groundwater conditions were carefully studied.

The investigated area is covered by coniferous forest and the soil cover is composed of silty till soil to sandy till soil. A subdivision of the watershed was made, with respect to topography and soil moisture status. For each subarea a representative spot was chosen where the soil profile was studied in detail. Both saturated hydraulic conductivity and porosity were found to decrease with increasing soil depth. In the model simulations described in this paper measurements of the saturated hydraulic conductivity and the water retention curves are used.

Model Simulations

Results of model simulations from four hypothetical hillslopes will be briefly described. The hillslopes have a length of 40 m, the soil thickness is 1 m and the overall slope is 5 %. The stream level is held constant during the simulations, 0.6 m above the the bottom of the slope. Three of the hillslopes are straight – one homogeneous and two layered. Of the layered hillslopes one has layers parallel to soil surface, and the soil parameter values represent recharge areas in the watershed investigated by Lundin (1982). The mean saturated hydraulic conductivity and the mean porosity of this slope are used as values for the homogeneous hillslope. In order to study the effects of varying soil properties along the hillside a hillslope with layers having less slope than the ground surface was chosen. Simulations were also made for a layered concave hillslope, using the same soil parameters as for the straight slope with layers parallel to ground surface.

As an initial state for the non-stationary simulations, stationary conditions with a low rate of infiltration where chosen. Results are presented as lines of constant pressure potential (*m*) and as flow vectors. Due to great variations in the calculated rates of soil and groundwater flow (they vary from about 10^{-9}m/s to 10^{-6}m/s) the smallest rates can not be illustrated. When the flow rates are very low, an arrow without a tail is drawn, just to indicate the flow direction. It should also be noted that the flow vectors are drawn with the same scale in horizontal and vertical directions, while the hillslope is drawn with a vertical exaggeration of four.

Straight Slope – Stationary Conditions

The modeled situation in a homogeneous hillslope during stationary conditions, with an infiltration rate of 0.07 mm/h, is shown in Fig. 2. Soil parameters are listed

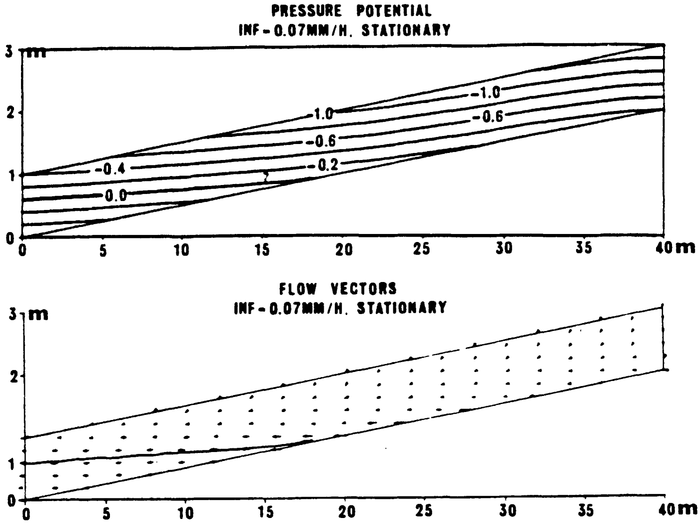

Fig. 2a-b.
Stationary conditions.
Homogeneous slope.

in Table 1.

It is seen in the figure that the vertical change of pressure potential in the unsaturated zone is close to 1 m/m. This means that the total hydraulic head, i.e. the sum of the pressure potential and the gravitational potential, has a very small vertical gradient. The size of this vertical potential gradient is of the same order of magnitude as the horizontal potential gradient, which is caused by the slope of the hillside. Thus, the flow vectors in the unsaturated zone in most parts of the slope have a sizeable horizontal component (Fig. 2b).)If the infiltration rate increases the horizontal gradient remains, but the vertical potential gradient will dominate.

Just above the groundwater table the flow rates are of the same size and direction as below. The pressure potential is here very close to zero which, according to Eq. (2), means that the unsaturated hydraulic conductivity is the same as the saturated.

Fig. 3 shows simulated conditions for a layered slope with layers parallel to soil surface (see Table 2 for soil parameters). Typically for Swedish till soils, deeper soil layers are less permeable then those near soil surface. When comparing Figs. 2 and 3 it is seen that with the same infiltration rate, the groundwater zone is more

Table 1 – Soil Parameters, homogeneous slope.

K_s (10^{-5}m/s)	6.9
θ_s	0.36
ψ_a (m)	−0.17
λ	0.24
n	1
θ_r	0.002

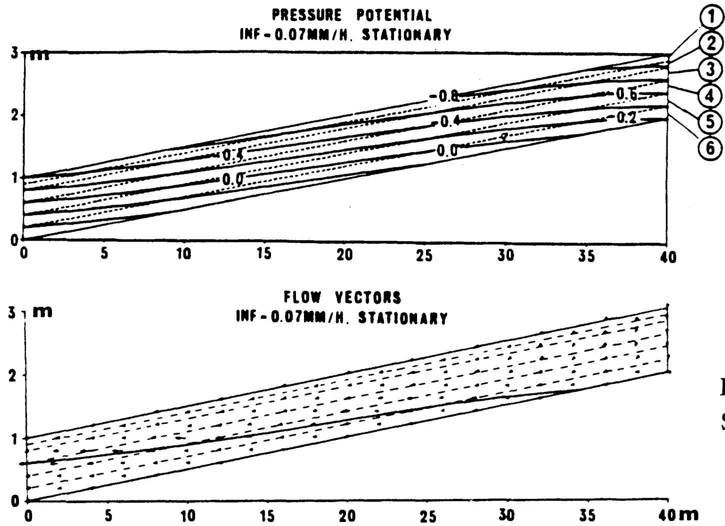

Fig. 3a-b.
Stationary conditions.
Layered slope.

Table 2 – Soil parameters, layered slope.

Layer	1	2	3	4	5	6
K_s (10^{-5}m/s)	25	13	7	5.5	1.8	1.3
θ_s	0.69	0.38	0.33	0.45	0.30	0.20
ψ_a (m)	−0.15	−0.17	−0.12	−0.12	−0.20	−0.25
λ	0.21	0.24	0.24	0.24	0.24	0.24
n	1	1	1	1	1	1
θ_r	0.002	0.002	0.002	0.002	0.002	0.002

extended and the slope of the groundwater table is steeper for the layered hillslope. The reason for this is the lower hydraulic conductivity in the deeper layers of the layered slope. The same amount of water needs to be transported down the slope, and with a lower hydraulic conductivity a thicker groundwater zone and a steeper groundwater table is required.

Straight Slope – Non-Stationary Conditions

Starting with stationary conditions (infiltration rate = 0.07 mm/h) simulations were made for an infiltration rate of 5 mm/h during 6 hours for the layered slope (Fig. 4). This represents quite a large rainfall with medium intensity in Sweden. As the infiltration rate starts to increase, the groundwater table close to the stream rises, causing the groundwater flow here to increase due to growing thickness of the groundwater zone and increasing slope of the groundwater table. By studying the flow rates in different parts of the slope it is seen that the flow coming from the upper parts increases much less than the flow at the base. This means

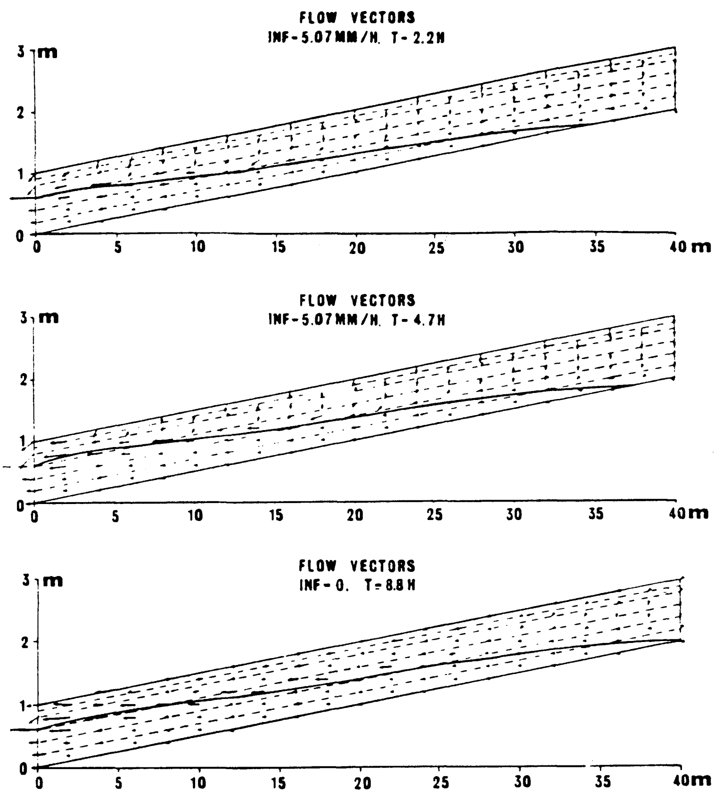

Fig. 4a-c. Non-stationary conditions. Layered slope.

that the quick response of groundwater outflow to infiltration is caused by local infiltration close to the stream. After cessation of infiltration, the groundwater level soon decreases, as the contribution from the upper parts of the slope is not large enough to maintain the high level. Fig. 5 shows the variations of the groundwater table at different distances from the stream.

The outflow hydrograph is shown in Fig. 6. As can be seen the groundwater outflow after 2.2 hours of infiltration has increased three times. The maximum value is 0.40 mm/h, reached just after infiltration has ceased, whereupon the outflow to the stream soon decreases.

One of the aims of this study has been to investigate if it is possible, using measured values of soil parameters, to simulate groundwater outflow hydrographs of shapes and peaks similar to streamflow hydrographs. When comparing observed and simulated hydrographs it however has to be considered that the values of soil parameters in the model have been taken from point measurements. As the variations of soil properties are great in till soils, the parameters values can only by their order of magnitude be said to represent the integrated value of a hillslope. Therefore there is no sense in making detailed comparisons between

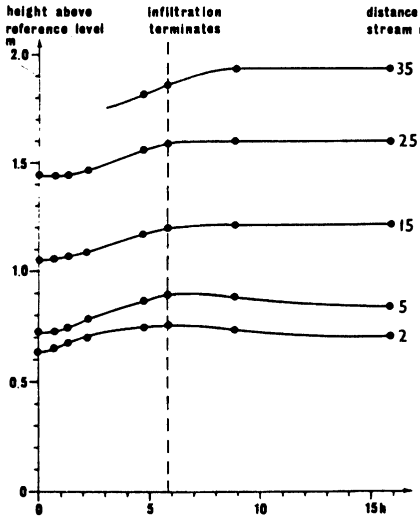

Fig. 5. Variations of groundwater table at different distances from stream. Infiltration ≡ 5.07 mm/h. Layered slope.

observed and simulated hydrographs. Due to difficulties in measuring the outflow from a single "two-dimensional hillslope", comparisons have to be made with hydrographs from a watershed. The hydrograph most similar to the simulated one should be found in a very small watershed with short transit times. One example of an observed streamflow hydrograph from a small watershed (0.04 km^2) in the west part of Sweden is shown in Fig. 7 (Rodhe 1984). This hydrograph was caused by a rainfall of totally 56 mm, lasting for about 36 hours. A separation of the hydrograph by means of the isotope oxygen-18 showed that the groundwater fraction of the total runoff volume was 73 %. The measured values of saturated hydraulic conductivity in this area are of the same order of magnitude as those used in the model. Comparing the observed hydrograph with the simulated outflow from the straight layered hillslope, it can be seen that the measured values of

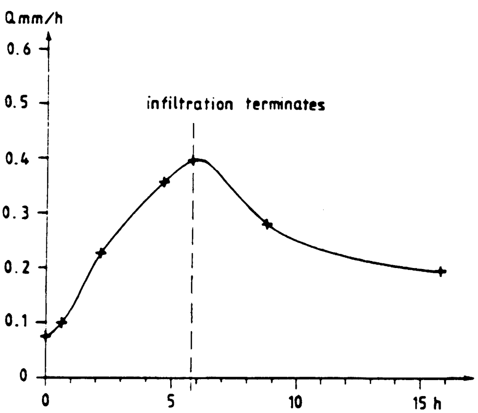

Fig. 6.
Groundwater outflow hydrograph. Infiltration = 5.07 mm/h. Layered slope.

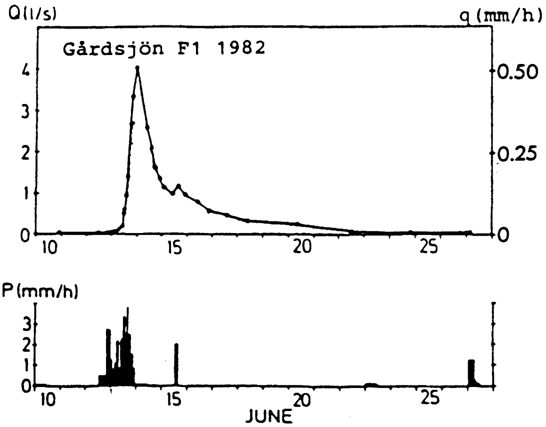

Fig. 7.
Observed streamflow hydrograph from a small watershed. (Rodhe 1984).

saturated hydraulic conductivity in this case lead to a simulated groundwater outflow hydrograph of shape and peak similar to the streamflow hydrograph. According to the simulations it can consequently be concluded that it is possible that a runoff peak mainly is produced by groundwater outflow.

Stream level is kept constant during the simulations. Since one reason for the quick response of groundwater outflow to infiltration is the increasing slope of the groundwater table close to the stream, this may seem to be a serious restriction of the model. But since the rise of the groundwater table is much greater than what can be expected of stream level, the assumption of a constant stream level may be an acceptable approximation.

The reaction of a homogeneous hillslope to infiltration is principally the same as that of the layered one, except that the changes of the groundwater table are smaller. The explanation for this is the higher saturated hydraulic conductivity in the bottom layers of the homogeneous slope. Therefore a given increase of the groundwater level causes a larger increase in the groundwater flow in the homogeneous slope than in the layered slope.

Straight Slope – Varying Soil Properties along the Slope

In the area investigated by Lundin (1982) the saturated hydraulic conductivity was found to be lower in discharge areas than in recharge areas. Such conditions are simulated using a layered slope, with the layers having less slope than the ground surface (Fig. 8, Table 3). The initial state had to be slightly changed from earlier simulations to avoid a too high groundwater table. The stream level is also 0.1 m higher than in the other simulations. Starting with 0.005 mm/h, an infiltration of 5 mm/h is applied for 6 hours. After less than 5 hours simulated time the groundwater table at the base of the slope reaches the soil surface and saturation overland flow occurs. The low hydraulic conductivity at the base restricts the groundwater outflow. Since the initial groundwater table is near the ground surface, the storage

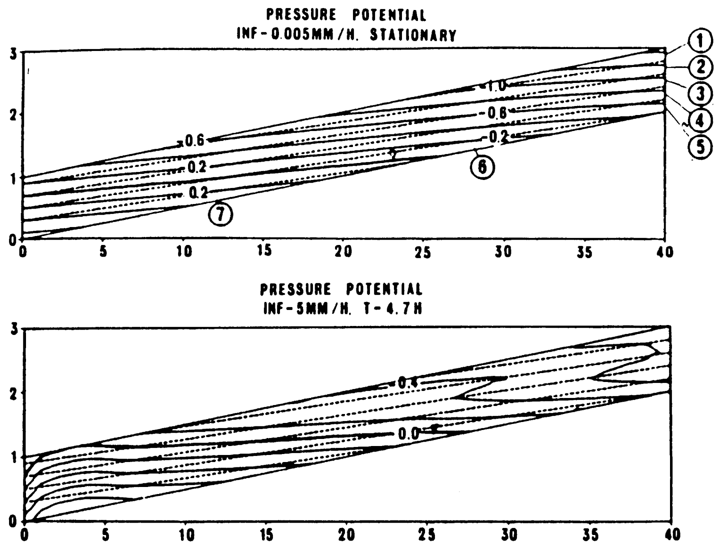

Fig. 8a-b.
Non-stationary conditions.
Layered slope.

Table 3 – Soil parameters, layered slope.

Layer	1	2	3	4	5	6	7
K_s (10^{-5}m/s)	20	7	2	0.8	0.3	0.07	0.02
θ_s	0.69	0.33	0.30	0.30	0.20	0.20	0.20
ψ_a (m)	−0.15	−0.12	−0.12	−0.20	−0.25	−0.25	−0.25
λ	0.21	0.24	0.24	0.24	0.24	0.24	0.24
n	1	1	1	1	1	1	1
θ_r	0.002	0.002	0.002	0.002	0.002	0.002	0.002

capacity of the unsaturated zone is small and the whole soil profile soon becomes saturated. As soon as saturation overland flow occurs, even if only from a small part of the slope, this flow will, according to model simulations, dominate the outflow to the stream. For this simulation the peak of saturation overland flow was about ten times as high as the groundwater outflow peak.

Concave Hillslope

In order to study the influence of topography upon runoff generation, simulations are made for a concave hillslope (Fig. 9). The soil parameters are the same as for the straight slope of Fig. 3 (see Table 2). As for the straight slope, an initial state of stationary conditions with a rate of infiltration of 0.07 mm/h is used. A simulated infiltration of 5 mm/h for 6 hours gave no conclusive difference between the straight and the concave hillslope. But when infiltration was increased to 15 mm/h for 6 hours some differences could be noted. With such a high intensity, for such a

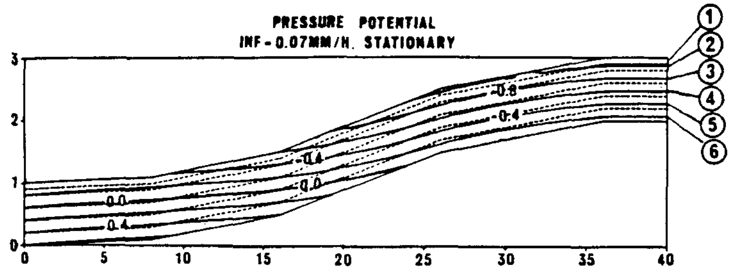

Fig. 9.
Stationary conditions.
Concave, layered slope.

long time, the soil profile at the base of the slope becomes fully saturated (Fig. 10). When this occurs the groundwater flow is determined by the slope of the ground surface. Consequently the flow at the base is smaller for the concave than for the straight hillslope. Less of the infiltrating water is then transported to the stream and hence, the area where saturation overland flow occurs will grow faster. Consequently total outflow from the concave slope will be greater than from the straight slope due to higher overland flow.

Due to the importance of the initial state a difficulty occurs when comparing slope configurations. When choosing the same initial rate of infiltration for the slopes, the groundwater table is initially nearer to ground surface at the base of the concave slope. The storage capacity of the unsaturated zone is then smaller, why the soil profile becomes fully saturated more quickly. Similarly, the conditions at the start of a rainfall in nature, will be different for slopes with different topography. The important effect of the initial conditions on calculated groundwater outflow are more fully demonstrated by Beven (1977).

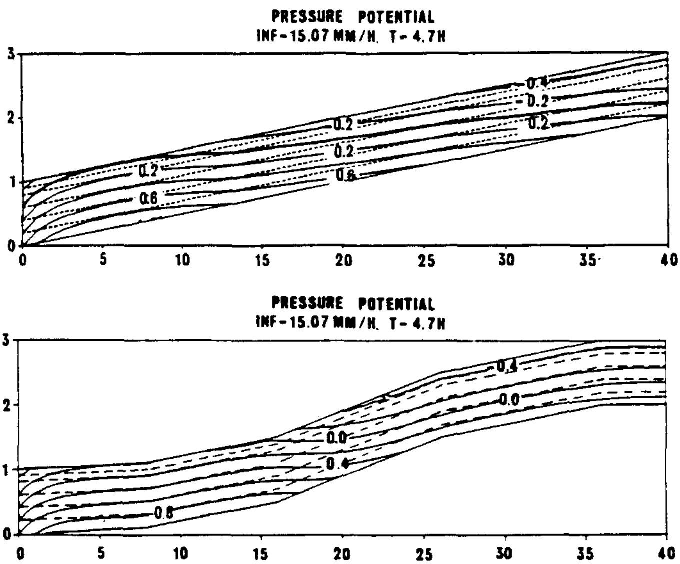

Fig. 10a-b.
Non-stationary conditions, straight (a) and concave (b) layered slope.

Conclusions

Simulations have been made for a hillslope utilizing soil parameters for Swedish till soils. It has been possible to simulate a groundwater outflow, that produces streamflow hydrographs of peak and shape corresponding to observations from small Swedish watersheds. The quick response of groundwater outflow to rainfall is caused by infiltration at the base of the hillslope. Hence the soil properties of this part of the slope are of great importance to runoff generation.

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