

# On runoff generation and the distribution of storage deficits

A. Wood and K. J. Beven

## ABSTRACT

A number of hydrological models use a distribution function to develop the non-linear rainfall–runoff catchment response. In this study the beta function is applied to represent a distribution of soil moisture storages in conjunction with a fast and slow pathway routing. The BETA3 and BETA4 modules, presented in this paper, have a distribution of discrete storage elements that have variable and redistributed water levels at each timestep. The PDM-BETA5 is an analytical solution with a similar structure to the commonly used probability distribution model (PDM). Model testing was performed on three catchments in the Northern Pennine region in England. The performances of the BETA models were compared with a commonly used formulation of the PDM. The BETA models performed marginally better than the PDM in calibration and parameter estimation was better with the BETA models than for the PDM. The BETA models had a small advantage in validation on the hydrologically fast responding test catchments.

**Key words** | beta function, PDM, parameter estimation, uncertainty

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## INTRODUCTION

Much of the non-linearity in the storm response of catchments is associated with patterns of surface and subsurface runoff generation as conditioned on antecedent patterns of wetness or storage deficits, whether that runoff be generated by infiltration excess, saturation excess or purely subsurface stormflow processes (e.g., [Beven 2001, 2006](#)). Representations of such variability have been explicitly included in hydrological models since the very first implementations on digital computers. The Stanford Watershed Model had a runoff generation function that included a distribution of infiltration capacities in a catchment ([Crawford & Linsley 1966](#)); Topmodel based the generation of saturation excess overland flow on the pattern of a topographic index ([Beven & Kirkby 1979; Beven 1997](#)); while the Xinanjiang/ARNO/VIC (variable infiltration capacity) model ([Wood \*et al.\* 1992; Zhao & Liu 1995; Todini 1996](#)) and the probability distribution model (PDM) ([Moore & Clarke 1981; Moore 1985, 2007](#)) both provided functional representations of a changing fast runoff coefficient in terms of distributions of storage deficits in a catchment. It has also been suggested

that the Soil Conservation Service (SCS) curve number approach can be interpreted in terms of either spatially variable infiltration capacities ([Yu 1998](#)) or saturation excess runoff generation ([Steenhuis \*et al.\* 1995](#)).

The principle is therefore well established. The difficulty is in applying it to catchments where the subsurface characteristics that are fundamental in controlling the patterns of storage deficits and runoff generation are poorly known. Thus, in most cases, the parameters of these models have been mostly inferred by calibration. Indeed, one of the original reasons for developing the PDM model was to make the calibration process better posed by reducing the effects of thresholds common in earlier conceptual models ([Moore & Clarke 1981](#)). This is not a problem when there are adequate data to perform a calibration, but is much more problematic when the application is to an ungauged catchment. It is known that regionalisation of calibrated parameter values to ungauged catchments is fraught with difficulty (see discussion in [Lamb & Kay \(2004\), Calver \*et al.\* \(2005\)](#) and [Buytaert & Beven \(2009\)](#)): hence the

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International Association of Hydrological Sciences (IAHS) Prediction of Ungauged Basins (PUB) initiative (Sivapalan *et al.* 2003).

This general type of distribution function model can also be interpreted in terms of the physical characteristics of the catchment soils. Effectively, in all of these models, given some initial antecedent state of the catchment, the runoff generation depends on filling a pattern of storage deficits (whatever runoff mechanism might be involved). Thus, the requirements in any application to an ungauged catchment are estimates of minimum and maximum storage deficits for runoff generation and representation of the way in which the pattern of deficits changes with antecedent state in the structure of the model. This will then depend on the representation of the distributional form of potential deficits. Thus the Stanford Watershed Model used a uniform distribution, Topmodel related deficits to topographic index (including the gamma distributed form of Sivapalan *et al.* (1990)), the Xinanjiang/ARNO/VIC model used a power law distribution and the PDM provided a choice of distributional forms. In the Grid to Grid (G2G) version of the PDM, this is then taken further in relating the model parameters to soil and slope characteristics for landscape grid elements, allowing the G2G model to be applied to ungauged catchments and, indeed, the whole UK (Bell & Moore 1998; Bell *et al.* 2004). There are also ways of identifying appropriate forms of non-linearity and transfer functions directly from the observations (e.g., Young & Beven 1994; Young 2003; Kirchner 2009; Beven *et al.* 2011).

The aims of this study are similar: to provide a simple representation of the non-linearity of runoff generation that might be related to maximum and minimum potential deficits in a catchment. A flexible distribution to represent the pattern of deficits within these limits is the beta distribution. The beta function has been used for the development of unit hydrographs (Bhunya *et al.* 2004) and it has been used in the lumped version of the TOPKAPI (Topographic Kinematic Approximation and Integration) model to partition the precipitation into direct runoff and infiltration (Liu & Todini 2002).

The resulting model presented here (Figure 1) is completed by a generalised subsurface drainage component based directly on the measurable recession characteristics of the catchment (Lamb & Beven 1997), a simple

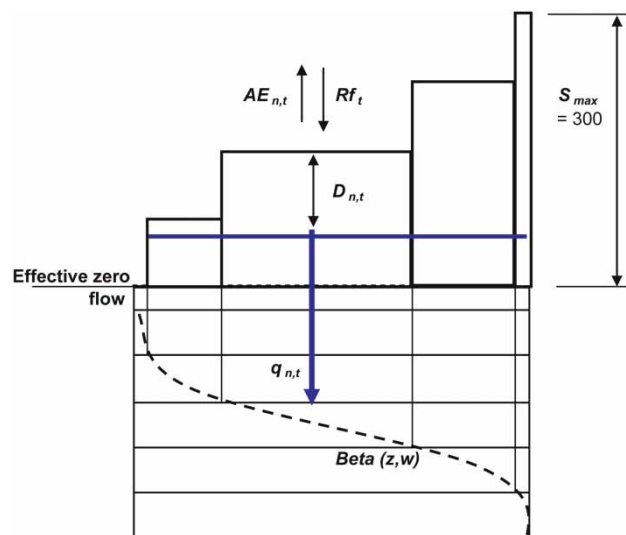


Figure 1 | BETA3 module concept.

representation of evapotranspiration based on storage deficits, and a runoff routing function based on a Nash Cascade (Nash 1957). A relatively small number of direct measurements might be adequate to constrain the nature of these relationships in an ungauged catchment (Seibert & Beven 2009).

In this paper we outline the implementation of the beta distribution model in a number of different forms. These are then treated as hypotheses about catchment response to be tested given a set of calibration data (see also Piñol *et al.* (1997), Clark *et al.* (2009), Beven (2010), Buytaert & Beven (2011) and the rainfall-runoff modelling toolbox in Wagener *et al.* (2004) and Fenicia *et al.* (2008)).

## DEVELOPMENT OF THE BETA MODELS

### The flexibility of the beta function

The advantage of the beta function is its flexibility and that only two parameters,  $z$  and  $w$ , are required to vary the shape of the probability distribution which is given by

$$Beta(z, w) = \int_0^1 x^{z-1} (1-x)^{w-1} dx \quad (1)$$

Variation of the  $z$  and  $w$  parameters within the beta function results in a number of possibilities for the shape of the probability density function. The beta function was applied

to a soil moisture module to represent the distribution of storages which is termed the BETA module. A number of prototypes were tested using the beta function to represent a distribution of storage depths (the BETA1 version) and a distribution of storage areas (the BETA2 version). Other versions investigated a beta distribution of infiltration capacities. These versions performed less well in early trials. In this paper only the model versions that are parsimonious and yet maintained a satisfactory performance are presented. They are as follows.

- BETA3. A model with a beta distribution of discrete independent tubes, each with its own threshold and own variable water level that is adjusted at each timestep.
- BETA4. A model that is similar to the BETA3 but with a water level that is redistributed at each timestep across all tubes.
- PDM-BETA5. A model based on the PDM using the beta function in an analytical solution with a redistributed water level at each timestep. The formulation for this model is presented in Appendix A (available online at <http://www.iwaponline.com/nh/044/119.pdf>).

The final BETA model comprises the BETA module together with the routing components. All the models are run over discrete timesteps.

## BETA3 model

### BETA3 module – soil moisture store

The BETA3 module develops the concept of the catchment as an arbitrary number of discrete tubes that increase in depth linearly between a fixed minimum ( $S_{\min}$ ) equal to zero and a fixed maximum value ( $S_{\max}$ ).  $S_{\min}$  and  $S_{\max}$  may be set as additional free parameters in the model but this was found to offer only a minor benefit to performance but a greater disadvantage for parameter estimation.

In the BETA3 the beta function is used to determine the areas associated with the tubes, which fill during rainfall events and also drain and evaporate independently at each timestep. No water redistribution takes place between tubes within the BETA3 module. The proportion of the soil moisture store (or number of tubes) that overflows in a

rainfall event generates rainfall–runoff through a fast pathway. Drainage from the tubes generates a flow of water through a slow-pathway.

For the catchments modelled in this study a fixed maximum tube depth ( $S_{\max}$ ) of 300 mm was found to be adequate based on early model trials. The module has not been tested on groundwater-dominated catchments and for this case the fixed upper threshold may need to be raised. A fixed maximum tube depth that is suitable for all catchments is advantageous in estimation of the parameters for the module.

The BETA3 module is shown schematically in Figure 1. In this figure, for reasons of clarity, only five tubes are shown ( $N_s = 5$ ) and the cumulative beta distribution is drawn at 90 degrees to more usual graphical presentations of the function in order to illustrate the graph intercepts and the variability of tube areas.

Since the tube depth capacities are distributed linearly from this the available depth capacity,  $S_{r,n}$  of the  $n$ th tube is calculated by the formula

$$S_{r,n} = S_{\max} \cdot (n - 1)/(N_s - 1) \quad (2)$$

The fractional areas of each tube are calculated using the beta probability density function and this is given by Equation (1). The partial beta function at  $x$  is sometimes called the incomplete beta function (for example, the *betainc* Matlab routine). It is given by the normalised form of the beta function so that

$$\text{betainc}(x, y, z) = \frac{1}{\text{Beta}(z, w)} \int_0^x x^{z-1} (1-x)^{w-1} dx$$

for  $0 \leq x \leq 1$  and  $z, w > 0$  (3)

In the BETA3 soil moisture module, for the case of  $N_s$  tubes, the incomplete beta function is calculated for  $N_s + 1$  steps between 0 and 1 where the step size is  $1/N_s$ . If the step is  $x_n$  ( $0 < x_n < 1$ ) then the fractional area  $A_n$  of the  $n$ th tube is then given by

$$A_n = (\text{betainc}(x_n + 1, z, w) - \text{betainc}(x_n, z, w)) \cdot A_c \quad (4)$$

where  $A_c$  is the area of the catchment. The total capacity of

each tube in the soil moisture module is therefore established by its depth  $S_{r,n}$  and its area  $A_n$ .

Drainage: At the start of each timestep the drainage from each tube in the soil moisture module,  $q_{n,t}$  is calculated as a function of the water level  $W_{n,t}$  in the store where

$$W_{n,t} = S_{r,n} - D_{n,t} \quad (5)$$

where  $D_{n,t}$  is the deficit in the tube at time  $t$ , and so that

$$q_{n,t} = f(W_{n,t}) \quad (6)$$

The summation of the flow from each tube is the total slow-pathway flow from the module. There are linear or non-linear possibilities for the form of this drainage function and these are discussed below.

Actual evaporation: Across a river basin, actual evaporation (AE) from the soil is related to the potential evaporation (PE) and is dependent on the type of vegetation and the local soil moisture deficit. Some areas of the catchment may be at field capacity which may evaporate close to the potential rate, for example, low lying areas close to the watercourse. Other areas, with a larger soil moisture deficit, may evaporate at a rate much lower than the potential rate or not at all. The BETA3 module applies a simple evaporation curve for the evaluation of actual evaporation from each tube.

A straightforward and tested approach is to develop the potential evaporation using a sine curve regulated with a climatological mean in conjunction with a root constant (RC) model (Penman 1949; Calder *et al.* 1983). In the BETA3 a root constant is applied to each tube based on the predominant land use for the subject catchment (Shaw 1994). With this assumption, three approaches were considered for the regulating function in each tube; a Penman constant, a linear and a non-linear power law method. The latter function produced a similar drying curve to that used in the PDM.

Initial tests with BETA3 on the River Eden catchment at gauging station 76014 showed that the Penman constant (Method 1) resulted in high rates of evaporation and zero flow from the model in dry periods. Between the linear (Method 2) and non-linear (Method 3) regulating functions

there did not seem to be a strongly preferred approach for the high-flow analysis in the study, and the linear approach (Method 2) was finally chosen as more parsimonious. The linear drying curve can be written as

$$\frac{AE}{PE} = 1.0 - (SMD/RC) \quad \text{for } SMD < RC$$

$$\frac{AE}{PE} = 0 \quad \text{for } SMD \geq RC \quad (7)$$

Values for RC have been suggested for different vegetation cover (Shaw 1994). The BETA3 model testing for the study was limited to catchments with predominantly grassland land use but the results suggested that the 75-mm value for permanent grassland suggested in Shaw (1994) was a reasonable value for the BETA3 module. The use of these root constants avoided the need to optimise an additional parameter in the models.

Wetting and drying water balance: In the BETA3 the tubes are considered to be open topped and for each tube at each timestep a new water level  $W_{2,n,t}$  is calculated. Rainfall at time  $t$  is denoted by  $Rf_t$  and if  $Rf_t$  is zero this becomes a drying situation. If the water level in the tube at the start of the timestep is given by  $W_{1,n,t}$  then the new water level  $W_{2,n,t}$  is calculated from

$$W_{2,n,t} = W_{1,n,t} - q_{n,t} + Rf_t - AE_{n,t} \quad (8)$$

where  $q_{n,t}$  is the drainage from the tube given by Equation (6). The water balance can be shown diagrammatically by Figure 2 for a single tube. The new water levels and the overflow from the tube  $W_{o,n,t}$  are then determined by one of three conditions as follows.

**Condition 1:**  $W_{2,n,t}$  is less than zero in which case the tube has dried out. In this case the overflow depth from the  $n$ th tube,  $W_{o,n,t}$  is zero and the depth for the next timestep is set as  $W_{1,n,t+1} = 0$ .

**Condition 2:** This is the case where  $W_{2,n,t}$  is less than the tube capacity depth but greater than zero. In this case, although rainfall may have occurred, the new tube water depth is below the top of the tube and therefore the depth for the next timestep is  $W_{1,n,t+1} = W_{2,n,t}$ . In this case there is no tube overflow and  $W_{o,n,t} = 0$ .

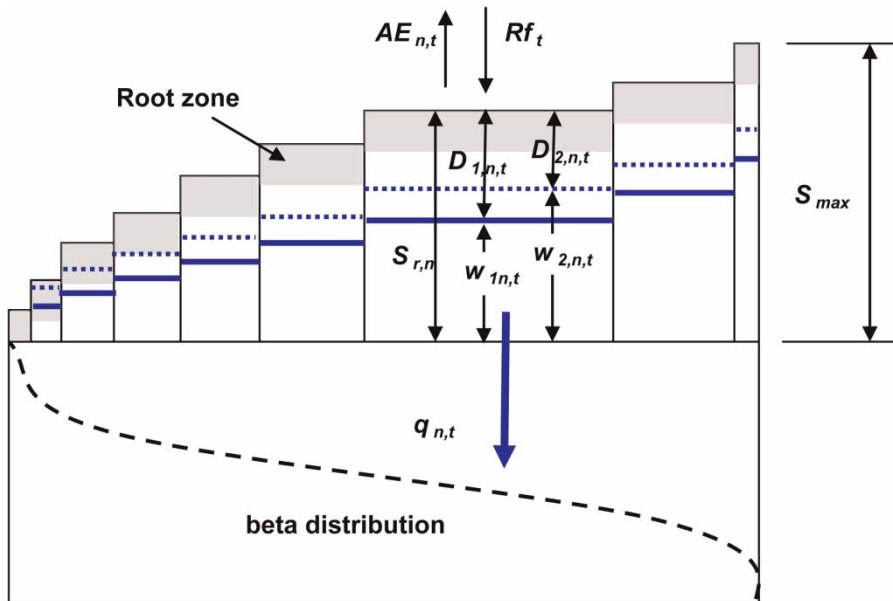


Figure 2 | Water balance in the BETA3 soil moisture module after a storm event.

**Condition 3:** This is the tube overflowing case where  $W_{2,n,t}$  is greater than the tube capacity depth. The overflow depth is given by

$$W_{o,n,t} = W_{2,n,t} - S_{r,n} \quad (9)$$

The new depth of the tube for the start of the next time-step is  $W_{1,n,t+1} = S_{r,n}$ . In this case the volume of the runoff to the fast pathway,  $V_{o,n,t}$  at time  $t$  is calculated from

$$V_{o,n,t} = A_n \cdot W_{o,n,t} \quad (10)$$

The overflow volumes from the individual tubes are then summed to give the total volume of overflow  $V_o$  for the time-step

$$V_{o,t} = \sum_1^{N_s} (V_{o,n,t}) \quad (11)$$

The overflow volume is passed to the fast pathway routing and the formulation for this is described below.

**Number of tubes:** The application of individual tubes in the BETA3 module, rather than a continuous distribution as

in the PDM, raises a question of calibration accuracy for the higher flows caused by spill thresholds (Kavetski *et al.* 2006; Clark & Kavetski 2010). This problem was overcome in the BETA3 by using a sufficient number of tubes, selected by the beta function, to generate smooth fast runoff response during rainfall events.

To test the sensitivity of the BETA3 to the number of tubes, simulations in calibration were undertaken with a variation in the number of tubes ( $N_s$ ) from 1 (i.e. a single tank) up to 1,000 by comparing the runoff coefficients to the fast pathway routing and model hydrographs. Very little difference in the plots and model hydrographs were observed once the module had more than approximately 10 tubes. The benefit of a greater number of tubes was observed to be a small improvement in smoothness of response for the lower runoff coefficients, i.e., a greater closeness to an integral solution of the soil moisture module. However, since computational time increases with a greater number of tubes there is benefit in minimising  $N_s$  commensurate with model performance. For this study 30 tubes were generally used on all the subject catchments and this had a marginal increase on model run times compared to earlier trials with 10 tubes.

### Slow-pathway drainage

In Equation (6) the drainage flow from each of the tubes was given by simple function of the depth in the tube with one of three possibilities for the relationship: linear, non-linear or exponential being preferred for a particular catchment. The preferred form of the slow-pathway routing was determined, wherever possible, by recession curve analysis (Lamb & Beven 1997).

**Linear (L):** Drainage using a linear function was the simplest method which is a basic building block synonymous with the use of a single order transfer function (Beven 2012, pp. 107–111). The slow-pathway flow in a tube,  $q_{n,t}$  in mm/hr is given by

$$q_{n,t} = W_{n,t}/T_s \quad (12)$$

where  $T_s$  is a parameter for the slow-pathway in hours and  $W_{n,t}$  is the depth in mm in the  $n$ th tube at time  $t$ . To compare the parameter for the linear store with the non-linear approach described below a small modification was made so that

$$k_L = 1/T_s \quad (13)$$

$$\text{so that } q_{n,t} = k_L \cdot W_{n,t} \quad (14)$$

**Power law (N):** For the non-linear power law function, flow from the tube  $q_{n,t}$  at time  $t$  is given by

$$q_{n,t} = k_N \cdot W_{n,t}^\alpha \quad (15)$$

where  $k_N$  is a parameter for the slow-pathway,  $\alpha$  is a power coefficient and  $W_n$  is the depth in tube  $n$ .  $k_N$  and  $\alpha$  are the same for all tubes. This non-linear form approaches the linear function when  $\alpha$  is close to 1.

Using the power law approach there is an additional parameter which has to be optimised, which is a small disadvantage. The experience of the use of some models such as the Environment Agency (EA) Thames Region in-house catchment rainfall-runoff model (Catchmod) (Wilby *et al.* 1994) have been run with fixed values of  $\alpha$  in the non-linear store. This may be considered as a suitable compromise to reduce the number of free parameters.

**Exponential (E):** The exponential form of decay curve has been used in Topmodel (Beven & Kirkby 1979). For the BETA3 model the expression takes the form

$$q_{n,t} = q_o e^{W_{n,t}/m} \quad (16)$$

where  $q_o$  is described as the baseflow.  $m$  has been described as a parameter controlling the rate of decline of transmissivity in the soil profile.

The total flow in the slow-pathway  $q_{s,t}$  (in  $\text{m}^3/\text{hr}$ ) is evaluated by summing all the flows  $q_{n,t}$  (in mm/hr) from the individual tubes multiplied by the fractional area of the tube

$$q_{s,t} = \sum_1^{N_s} (q_{n,t} A_n) \quad (17)$$

The complete BETA3 module comprises the BETA3 soil moisture store with the slow-pathway drainage, which are solved explicitly. The additional component that is still required is the fast-pathway routing.

The PDM model often uses a linear reservoir routing component which takes water from the groundwater store (Senbeta *et al.* 1999). For the BETA3 this additional routing component was not found to be necessary on the faster responding upland catchments that were modelled and the parsimonious form of the module was preferred.

### Fast-pathway flow routing

The development of unit hydrograph theory for transforming rainfall into runoff has developed considerably since the introduction of the concept by Sherman (1932). Nash (1957) introduced the concept of modelling runoff through a series of linear reservoirs. This basic concept was adopted for the fast-pathway routing for the BETA3 model with the use of a discrete number of tanks.

In its simplest form of the Nash Cascade, a single linear reservoir has already been described in an equivalent form for the slow-pathway routing Equation (12). For the fast pathway the volume of runoff is given by Equation (11), where  $T_f$  is a parameter in hours and the flow  $q_{f,t}$  is

$$q_{f,t} = V_{o,t}/T_f \quad (18)$$

In the more complex Nash Cascade, using an explicit solution, the flow is simply passed to the next tank at each timestep which has the same  $T_f$  parameter and so on. The problem in calibration is to determine the number of linear reservoirs that are needed to attain the representative shape of hydrograph for the catchment.

The total flow of the catchment is calculated by summing the flow from the slow and fast pathways

$$q_t = q_{f,t} + q_{s,t} \quad (19)$$

**Parameters:** The parameters that are required for the BETA3 model are shown in Table 1. The table shows the options for the slow-pathway routing. It can be seen that the BETA3 has four parameters in its simplest form with the linear drainage component. In addition to this the number of tanks in the Nash Cascade of the fast pathway must be determined. Typically, on the test catchments, the BETA3 was a five-parameter model since non-linear slow drainage components were used.

**Table 1** | Parameters required for the BETA3 model with options for slow-pathway routing

	Parameter	Units	Description
<b>BETA3 module</b>			
Beta	$z$	–	Beta function shape parameter
Beta	$w$	–	Beta function shape parameter
<b>Drainage option</b>			
1. Linear	$T_s (1/k_L)$	hr	Linear slow-pathway time constant
2. Power law	$k_N$	–	Non-linear parameter
	$\alpha$	–	Non-linear power coefficient
3. Exponential	$q_o$	mm/hr	Minimum flow in tube
	$m$	mm	Transmissivity constant
<b>Fast-pathway routing</b>			
Nash Cascade	$T_f$	hr	Linear tank fast-pathway time constant
Tanks	$N_f$	–	No. of tanks in fast pathway

The BETA3 model notation is denoted by BETA3\_N\_XF. The first term, BETA3, refers to the soil moisture module. The second term refers to the form of slow drainage, L, N or E. The third term refers to the fast-pathway routing where X is the number of tanks in the fast pathway.

### The BETA4 and PDM-BETA5 modules

In the PDM model a redistribution of the soil moisture is applied at each timestep within the storage element with an analytical solution. The analytical solution develops smooth runoff generation during periods of continuous rainfall and it avoids threshold spills and consequent errors that are characteristic of many bucket-type models (Kavetski *et al.* 2006). A possible criticism of the BETA3 is that it is a model with a distribution of buckets and threshold spills.

For a progression towards an analytical solution for the BETA module two other modules were developed. The first of these, a module which formed an intermediate step, is called the BETA4. The full analytical module is the PDM-BETA5.

### BETA4 module – water redistribution

Senbeta *et al.* (1999) showed how a variation in water level distribution in the PDM model affected the performance of the model. The BETA4 module varies from the BETA3 module only in the water balance computation. In the BETA4 the discrete tubes are maintained but a water redistribution is applied at each timestep so that the water level is the same across all tubes and the water balance is retained. The tube filling, evaporation and drainage is otherwise identical to the BETA3.

If a very large number of tubes are used in this model the form of the model tends towards an analytical solution with a continuous distribution.

### PDM-BETA5 module – an analytical solution

The analytical computation in the PDM-BETA5 module is similar to the algorithm used in the PDM (see Appendix A – available online at <http://www.iwaponline.com/nh/044/119.pdf> – for development of the analytical solution). The

PDM-BETA5 module uses a single store with a continuous beta distribution. The solution is tractable because, as in the PDM, a water level redistribution is applied at each time-step which allows the soil moisture and the runoff generation to be analysed.

## The PDM

The BETA models were compared with the PDM on test catchments. The PDM model used the typically applied Pareto function with shape parameter  $b$  to represent the distribution of the soil moisture store which has a maximum depth capacity  $C_{\max}$ . A groundwater drainage coefficient  $G$  is applied to determine the drainage from the store and the groundwater was passed to a routing module comprising a slow-pathway linear reservoir with parameter  $T_s$ .

The fast-pathway routing for the PDM was identical to the Nash Cascade used in the BETA models as described above. Table 2 shows the parameters that are required for the PDM as used in the study.

## TEST CATCHMENTS

### Catchment description and data

Three test catchments were used to develop and test the models using hourly rainfall data. For each catchment a 3-year period was selected for the calibration. A 2-year period was used for validation. For all the catchments

**Table 2** | Parameters required for the PDM

Parameter	Units	Description
<b>Soil moisture store</b>		
$B$	–	The shape parameter of the reflected power (Pareto) distribution
$C_{\max}$	mm	The maximum storage depth of the store element
$G$	–	The groundwater runoff coefficient
<b>Slow-pathway routing</b>		
$T_s$	hr	Linear slow-pathway time constant
<b>Fast-pathway routing</b>		
$T_f$	hr	Linear tank fast-pathway time constant
$7N_f$	–	No. of tanks in fast pathway

runoff is natural to within 10% of the flow value which is exceeded 95% of the time.

The River Eden catchment at Station 76014 is a high relief watershed catchment with an area of 67.9 km<sup>2</sup>. The solid geology is Carboniferous limestone and the middle reaches of the catchment are floored by Permian sandstone and the drift geology is variable boulder clay. Peat and moorland is the predominant land cover.

The geology for the 71006 Ribble catchment at Henthorn Farms is mainly Carboniferous limestone overlain by boulder clay in the valleys. The catchment area is 456 km<sup>2</sup>. On the south-east side of the catchment is millstone grit. The hill tops are mainly moorland. There is mixed farming over most of the catchment with several small towns.

The geology for the 23006 Tyne site is mainly Carboniferous limestone with some superficial deposits of peat and boulder clay. The catchment area is 322 km<sup>2</sup>. Vegetation is predominantly moorland and the land is used for rough grazing.

The calibration and validation modelling periods that were chosen for the sites are summarised in Table 3. The table also shows the warm-up periods for the models to allow the stores and tanks to reach preferred antecedent conditions before commencing model performance evaluation. The 23006 Tyne catchment, in particular, had a problem of missing rainfall and flow data for a number of years. Years were omitted from the analysis if the percentage of missing data was greater than 5% for either rainfall or flow. Where possible rainfall data were filled in from neighbouring rain gauges but where this was not possible the missing rainfall periods were flagged and they were discounted in the model performance analysis. For each of the catchments a catchment average hourly rainfall was developed. Missing flow data were not filled and any missing periods were flagged for exclusion in the performance evaluation.

### Preferred model structure

#### Procedure

The preferred model structure was determined for each catchment by a three-step procedure on the calibration data. The first step was to use the data to carry out a



**Table 3** | Calibration and validation model periods for the test sites

		76014 Eden	71006 Ribble	23006 Tyne
Calibration	Model warmup (hr)	1 July 1995 to 30 September 1995 (2,208 hr)	1 July 1990 to 30 September 1990 (2,208 hr)	1 July 1992 to 30 September 1992 (2,208 hr)
	Calibration (hr)	1 October 1995 to 30 September 1998 (26,304 hr)	1 October 1990 to 30 September 1993 (26,304 hr)	1 October 1992 to 30 September 1995 (26,280 hr)
Validation	Model warmup (hr)	1 July 2001 to 30 September 2001 (2,208 hr)	1 July 1991 to 30 September 1991 (2,208 hr)	1 July 1998 to 30 September 1998 (2,208 hr)
	Validation (hr)	1 October 2001 to 30 September 2003 (17,520 hr)	1 October 1991 to 30 September 1993 (17,544 hr)	1 October 1998 to 30 September 2000 (17,544 hr)

recession curve analysis. It was intended that this would provide the preferred function for the slow-pathway routing (Equations (14), (15) or (16) above). The second step was to apply a Monte Carlo routine for the BETA3 model, each with a different number of tanks on the fast pathway to determine the preferred Nash Cascade. The third step was to apply the different soil moisture modules with the preferred Nash Cascade.

### Recession curve analysis

The analysis of recession curves has been applied for a long time in hydrology and extensive reviews have been provided by Hall (1968) and Tallaksen (1995). In spite of the well-known difficulties of recession variability and partitioning the flow sources, an analysis of recession curves may give some general indications of subsurface drainage and therefore reduce the number of assumptions required in selecting a routing module for a catchment (Lamb & Beven 1997). Secondly, the parameter values thus obtained may be used as a guide for setting limits for the parameter distributions for the subsequent Monte Carlo analysis.

Ideally, true flow recessions should only be selected where there is no recharge during the flow recession period. This is an ideal which is difficult to achieve unless very long recessions are available and the early, steeper, post-storm recession is omitted. A practical compromise is to place a limit. The amount of rainfall in the selection of the recession curves and a 5% limit on total catchment rainfall/outflow, taken between the peak of a storm event and the end of the subsequent recession curve, was used to filter out recession curves where excessive recharge was taking place after the storm peak. After inspection of the

remaining curves a further limit on the minimum length of curve was also applied depending on the length of the recessions of the particular catchment.

Lamb & Beven (1997) describe a typical procedure for developing the master recession curve (MRC) which was pieced together from individual shorter recession curves. Owing to the variability in individual recession curves the use of the MRC approach is a compromise for any catchment (Tallaksen 1995). Some of the variability was explored by constructing MRCs for different winter periods from October to March inclusive and by the development of the MRC for the all the winter periods together for the available period of data. The lower part of the MRC curve was extracted using a log transform to identify for some catchments a break point between fast and slow pathways. This break point is termed the low flow limit (as noted in Table 4). Each of the three drainage functions (Equations (14), (15) or (16) above) was fitted in turn to the MRC using a least squares optimisation routine. The function that offered the preferred fit by least squares and inspection was adopted for the slow drainage pathway. Owing to a variability of recession curves the preferred drainage function from the recession curve analysis was not always clear-cut for the catchments using individual winter seasons. Therefore, the parameters that were identified from this routine were used as indicative benchmark values to set the parameter range in the subsequent Monte Carlo analysis.

### Monte Carlo method

In the Monte Carlo method for each model 1,000 simulations were carried out to check that the parameter range

**Table 4** | Results of recession curve analysis and preferred model structure in calibration

Catchment		76014 Eden			71006 Ribble			23006 Tyne		
<b>Step 1 – Master recession curve</b>										
Winter periods		1995–1997			1990–1992			1991–1994		
Total no. recession curves		898			1,146			1,571		
<b>Filter criteria</b>										
Rainfall/flow limit (%)		10			5			5		
Curves after filter 1		540			798			1,227		
Min. curve length (hr)		48			72			48		
Curves after filter 2		8			5			4		
Low flow limit (mm/hr)		0.21			0.10			0.13		
<b>Preferred fit and parameters</b>										
Preferred fit		Pow			Exp			Pow		
Linear ( <i>L</i> )	$T_s$ (hr)	–			–			–		
Power Law ( <i>N</i> )	$k_N$	0.00092			$2.37 \times 10^{-15}$			0.00018		
	$\alpha$	1.71			7.89			2.35		
Exponential ( <i>E</i> )	$q_0$ (m <sup>3</sup> /s)	–			0.0048			–		
	$m$ ( $=1/K_E$ )	–			6.52			–		
<b>Step 2 – Preferred structure identification for BETA3</b>										
		<b>Min</b>	<b>Max</b>	<b>Opt</b>	<b>Min</b>	<b>Max</b>	<b>Opt</b>	<b>Min</b>	<b>Max</b>	<b>Opt</b>
<b>BETA3</b>										
Beta <i>z</i>		0.1	50	3.4	0.1	50	11.1	0.1	50	0.9
Beta <i>w</i>		0.1	50	28.3	0.1	50	26.3	0.1	50	3.5
$T_f$ (hr)		1	10	1.7	1	40	7.3	1	20	3.3
No. tanks fast pathway		1	5	2	1	5	2	1	5	2
Power Law ( <i>N</i> )	$k_N$	$1.0 \times 10^{-5}$	$1.0 \times 10^{-2}$	$6.0 \times 10^{-4}$	$2.4 \times 10^{-17}$	$2.4 \times 10^{-13}$	$4.4 \times 10^{-14}$	$1.0 \times 10^{-8}$	$1.0 \times 10^{-3}$	$9.0 \times 10^{-4}$
	$\alpha$	1.0	4.5	2.01	1.0	10.0	6.4	1.0	3.5	1.5
BETA3_N_1F (NSE <sub>MAX</sub> )		–	–	0.849	–	–	0.821	–	–	0.728
BETA3_N_2F (NSE <sub>MAX</sub> )		–	–	0.852	–	–	0.843	–	–	0.732
BETA3_N_3F (NSE <sub>MAX</sub> )		–	–	0.812	–	–	0.839	–	–	0.690
BETA3_N_4F (NSE <sub>MAX</sub> )		–	–	0.801	–	–	0.839	–	–	0.691
BETA3_N_5F (NSE <sub>MAX</sub> )		–	–	0.794	–	–	0.819	–	–	0.697

Note: The grey highlighted cells show the preferred form of the recession curves for each catchment.

was sufficiently wide for each of the parameters that are shown in Table 4, Step 2. Lacking any strong knowledge about prior distributions of the parameters, values were selected using a uniform distribution and a random number generator. The parameter realisations were applied to each BETA3, BETA4, PDM-BETA5 and PDM model with a single tank for the fast pathway. The models were compared using the Nash–Sutcliffe Efficiency (NSE) objective function (Nash & Sutcliffe 1970). If required, adjustments to the parameter ranges were made at this step from an inspection of dotty plots of NSE performance against parameters. For the BETA3 models, for example, this initial set of 1,000 runs takes less than 10 minutes using 3 years' worth of hourly data on a local PC with a 2.61 GHz dual core processor and 2.0 GB of RAM.

The second step in structure identification was to identify the preferred model structure for the fast-pathway routing for each catchment. The Monte Carlo routine was run for 20,000 simulations on the Lancaster University high performance computer with combinations of 1–5 linear tanks in series for the fast pathway routing with the BETA3 module. Each BETA3 model was run separately and the NSE results for each of the models with the different number of fast tanks were evaluated.

The third step applied the preferred Nash Cascade for the catchment to each of the other BETA4, PDM-BETA5 and PDM modules in the same Monte Carlo framework with 20,000 runs.

For evaluation the simulations for each of the respective models were ranked according to their NSE performance measure. The maximum NSE was recorded which is termed  $NSE_{MAX}$ . In recognition of the concept of equifinality and the consequent uncertainty the generalised likelihood uncertainty estimation (GLUE) technique (Beven & Binley 1992) was used to evaluate the models in terms of the performance of a behavioural set of simulations for a model and the validation of the respective uncertainty limits. GLUE has been described and applied in numerous texts and further elaboration will not be given here (see examples in Beven & Freer (2001) and Beven (2006)). GLUE is often applied with a cutoff value of the performance measure for the selection of behavioural simulations. This has been noted as a weakness of the method (Blasone *et al.* 2008) and a limits of acceptability

approach has been introduced to make the procedure more rigorous (Beven 2006). However, a difficulty encountered here in the comparison of models is that the  $NSE_{MAX}$  results suggested a large degree of variability across models and to ensure a reasonable comparison of model performance it was preferable to choose the same number of simulations for each model. From the NSE ranking the top 100 simulations were arbitrarily selected which represents 0.5% of the Monte Carlo simulations for each model run. Using this method there was, therefore, an implied threshold  $NSE_T$  for behavioural simulation selection which is the NSE performance of the 100th ranked simulation.

In a set of behavioural simulations, when the likelihood values have been calculated and the cumulative distribution frequency (cdf) curve has been drawn, the GLUE 50th percentile is typically chosen to represent the best estimate prediction of the model behavioural set which was also applied here. To evaluate performance, from a model with an ensemble of simulations  $M$  with parameter sets  $\Theta$ , and the median of the ensemble represented by  $\tilde{M}_\Theta$ , the NSE ( $\tilde{M}_\Theta$ ) can be obtained for the assessment of prediction performance from the median of the behavioural simulations, defined as

$$NSE(\tilde{M}_\Theta) = 1 - \frac{\sum_{t=1}^T (\varepsilon_t(\tilde{M}_\Theta))^2}{\sum_{t=1}^T (Q_t - \bar{Q})^2} \quad (20)$$

where the simulation error is  $\varepsilon_t$  at time  $t$ ,  $Q_t$  is the gauged flow at time  $t$  and  $\bar{Q}$  is the mean gauged flow.  $NSE(\tilde{M}_\Theta)$  is 1 for a perfect fit from the simulation set.  $T$  is the length of the time series.

The uncertainty limits were determined from the GLUE 2.5th and 97.5th percentile values on the cdf curve which are termed the  $M_{\Theta,2.5}$  and  $M_{\Theta,97.5}$ . The uncertainty limits should ideally exactly embrace (or overlap) the observed flows to some target level, for example 95% in terms of a traditional statistical confidence limit, however, this may not be possible to achieve for a model owing to input and model errors. And so to evaluate the  $M_{\Theta,2.5}$  and  $M_{\Theta,97.5}$  uncertainty limits a second performance measure termed the *Reliability (Rel)* was used. *Reliability*,

expressed as a percentage, is calculated from

$$C_t = 1 \quad \text{if } M_{\Theta,97.5,t} \geq Q_{Q,t} \quad \text{and } M_{\Theta,2.5,t} \leq Q_{Q,t}$$

$$C_t = 0 \quad \text{if } M_{\Theta,97.5,t} < Q_{Q,t} \quad \text{or } M_{\Theta,2.5,t} > Q_{Q,t}$$

and

$$Rel = \frac{\sum_{t=1}^T C_t \cdot 100}{T} \quad (21)$$

Here, *Rel* of 100% represents all the observed flows embraced by the uncertainty limits from the model. A target figure for *Reliability* may be based on previous knowledge of input data error and experience of modelling at a particular site if, for example, it is to be used for predictive purposes on the same site.

The fourth and final step was to apply the behavioural set of parameters for each model to a separate period of validation data for each of the sites. For this new data period the NSE, NSE<sub>MAX</sub>, NSE( $\tilde{M}_\Theta$ ) and *Rel* values were recalculated.

## Results

Since the Monte Carlo method relies on repeated runs of the models, high computing power is an obvious advantage. The BETA3, BETA4, PDM-BETA5 and PDM models were calibrated on each of the catchments. The initial trials on the 76014 Eden catchment showed a large difference in run times. The PDM-BETA5 analytical algorithm was slow to run which was a serious disadvantage of this model. The problem was identified as the computational time of the minimisation function or lookup function for  $D_t$  with the incomplete beta function (*betainc* in Matlab software) (Appendix A, available online at <http://www.iwaponline.com/nh/044/119.pdf>). Although there are more computational steps in the discrete tubes BETA3 and BETA4 modules the run times were much quicker owing to the simplicity of calculation. The PDM-BETA5 is a very slow model to run by comparison and is considered to be too inefficient for practical use.

Table 4 shows the results of the two recession curve analyses (Step 1) and Monte Carlo analysis (Step 2) required to identify a preferred fast pathway routing for each catchment.

The winter periods for evaluation of the MRC for each catchment are shown in Table 4. Since the preferred drainage function from the recession curve analysis was not always clear-cut for the catchments using individual winter seasons, the preference finally adopted was to use the form of recession function derived from the whole period of calibration data for the catchment. Table 4 lists the total number of available curves for the calibration period and the numbers of curves after the successive filtering steps. On the River Eden the MRC was less robust owing to frequent storms and short recessions for this catchment. On this catchment the rainfall/flow limit had to be increased to 10% to get a sufficient number of recession curves over the 3-year period. On the River Ribble the preference for the exponential function was not particularly helpful since the adoption of this form in the BETA models results in a tendency for all the flow to be routed through the slow-pathway and with no overland fast-pathway flow. The power law was therefore the final preference for this catchment. The grey highlighted cells in Table 4 show the preferred form of the recession curves for each catchment. Overall, the non-linear power law drainage function was preferred and it gave an adequate fit to the MRC for all catchments. The parameters that are given in Table 4 for each of the catchments show similar results for the Eden and the Tyne. For the Ribble, values were more extreme. This presents a challenge for the estimation of slow drainage parameters based on catchment characteristics.

For Step 2, Table 4 also shows the ranges of parameters that were applied in the Monte Carlo analysis between the minimum and maximum values. Using the BETA3, for each of the catchments, the preferred structure was the BETA3\_N\_2F i.e. there were two tanks in series for the fast pathway. This model is highlighted in the table. For each catchment this was the structure that gave a noticeable peak in the curve of maximum NSE performance against the number of tanks. The grey highlighted cells in Table 4 show the preferred model structure for each river catchment. The parameter values for the best performing simulation are also shown in the table.

For the BETA3 (also for the BETA4 and PDM-BETA5) model it was generally observed that the beta  $z$  parameter is generally lower than the  $w$  parameter on faster responding catchments using the fixed maximum tube depth of 300 mm.

Of course, this may not necessarily be the case if the maximum tube depth value is altered. The optimum parameters for the slow pathway did differ from those obtained from the recession curve analysis. On faster responding catchments plots of beta  $z$  and  $w$  parameters against NSE performance showed a ridge pattern with a noticeable peak for the preferred parameters as shown in Figure 3.

The BETA4 and PDM-BETA5 models were run with the same parameter ranges and the same number of tanks on the fast pathway that showed a preference in Table 4. For the PDM, the parameter ranges are not shown in Table 4 but the model was also run with two tanks on the fast pathway. Table 5 shows the results in calibration for all the models.

In calibration using the behavioural simulation sets the BETA3\_N\_2F was the preferred model for the Eden catchment, and the BETA3\_N\_2F and PDM-BETA5\_N\_2F showed equal performance on the Ribble catchment. On both catchments the  $NSE(\tilde{M}_\Theta)$  performance of the behavioural set median was higher than the best performing individual simulation as shown in Table 5. The PDM showed a slightly better  $NSE(\tilde{M}_\Theta)$  for the Tyne catchment. The performances of all models on the Tyne catchment were lower which was caused by poorer data. The preferred models for each catchment are denoted by the highlighted cells in Table 5. In all cases in calibration the BETA3 outperformed the BETA4 in calibration by a small margin.

The performance of the BETA3, BETA4, PDM-BETA5 and PDM models on the validation data are shown in Table 6. On the Eden the BETA3\_N\_2F was the preferred model and on the Ribble the PDM-BETA5\_N\_2F showed a slight improvement over the BETA3\_N\_2F. On both

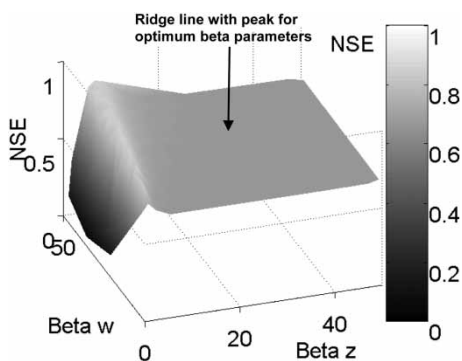


Figure 3 | Typical distribution of beta  $z$  and  $w$  parameters versus NSE performance for fast responding catchments in the BETA3 module.

Table 5 | Results of models in Monte Carlo analysis in calibration

Step 3 – Monte Carlo calibration for BETA3, BETA4, PDM-BETA5 and PDM				
Catchment		76014 Eden	71006 Ribble	23006 Tyne
BETA3_N_2F	$NSE_{MAX}$	0.852	0.843	0.732
	$NSE_T$	0.772	0.800	0.652
	$NSE\tilde{M}_\Theta$	0.852	0.850	0.673
	Rel	76.6	82.2	60.7
BETA4_N_2F	$NSE_{MAX}$	0.827	0.843	0.700
	$NSE_T$	0.752	0.800	0.629
	$NSE\tilde{M}_\Theta$	0.827	0.829	0.702
	Rel	59.6	74.0	60.8
PDM- BETA5_N_2F	$NSE_{MAX}$	0.850	0.844	0.688
	$NSE_T$	0.782	0.817	0.617
	$NSE\tilde{M}_\Theta$	0.840	0.850	0.645
	Rel	70.9	48.5	50.5
PDM_L_2F	$NSE_{MAX}$	0.826	0.834	0.736
	$NSE_T$	0.808	0.824	0.721
	$NSE\tilde{M}_\Theta$	0.778	0.680	0.706
	Rel	27.8	36.9	25.3

Note: The grey highlighted cells show the preferred model structure for each catchment.

Table 6 | Results of models in Monte Carlo analysis in validation

Step 4 – Models in validation for BETA3, BETA4, PDM-BETA5 and PDM				
Catchment		76014 Eden	71006 Ribble	23006 Tyne
BETA3_N_2F	$NSE_{MAX}$	0.741	0.832	0.730
	$NSE\tilde{M}_\Theta$	0.751	0.836	0.708
	Rel	70.0	81.2	64.7
BETA4_N_2F	$NSE_{MAX}$	0.728	0.835	0.739
	$NSE\tilde{M}_\Theta$	0.726	0.816	0.723
	Rel	57.4	72.3	53.5
PDM- BETA5_N_2F	$NSE_{MAX}$	0.779	0.833	0.706
	$NSE\tilde{M}_\Theta$	0.738	0.841	0.683
	Rel	67.4	50.9	54.3
PDM_L_2F	$NSE_{MAX}$	0.770	0.835	0.742
	$NSE\tilde{M}_\Theta$	0.673	0.680	0.713
	Rel	26.0	36.9	20.7

Note: The grey highlighted cells show the preferred model structure for each catchment.

catchments the behavioural set showed an improved performance over the single best performing model. The BETA4\_N\_2F performed best on the 23006 Tyne catchment but, again, it should be noted that the data were poorer than for the other two catchments. Overall, the results show that the type of catchment as well as the selected data period influence the preferred model structure.

Figure 4 shows an example of the simulation hydrographs for the models in validation for a short period of data representing the highest storm event on the River Eden catchment. The median of the behavioural set (the black line) is underpredicting for all the models. However, the uncertainty limits for the BETA3 and BETA4 are sufficiently wide to embrace the observed flows (the dots) for this particular storm event. For the PDM-BETA5 and the PDM the uncertainty limits are of insufficient width derived from the GLUE and the 100 best performing simulations. For these models it is necessary to choose a greater number of simulations, i.e. a lower implied threshold  $NSE_T$  to achieve wider limits in the GLUE computation. In the case of the PDM the uncertainty limits are particularly narrow which suggests that the selected simulations have a similar signal response. Table 5 shows that there is only a small difference between the  $NSE_{MAX}$  and the  $NSE_T$  for the PDM.

Figure 5 shows the BETA3 model and same set of behavioural simulations as were shown for the BETA3 in

Figure 4 on a succeeding series of storm events on the River Eden. This is a sample storm period within the whole validation period. The median and the uncertainty limits derived in GLUE are similarly shown. In this case the first two peaks are overpredicted and the third storm from time 5,750 hr is underpredicted by the model. Between 5,700 and 5,750 hr there are some obvious errors between observed data and the simulations. The observed flow shows a recession that is not well represented by the model. Here underprediction of the recession would appear to be related to the underprediction of the first two peaks. This could be a result of limitations of the model formulation but could also be derived from epistemic error in the areal rainfall calculation. There is no guarantee that such errors will be similar in both calibration and validation periods (see Beven (2010) and Beven & Westerberg (2011)). Uncertainty limits are then insufficiently wide to embrace the observed flows. At time 5,830 hr there is a model response that is not seen within the data even though the preceding

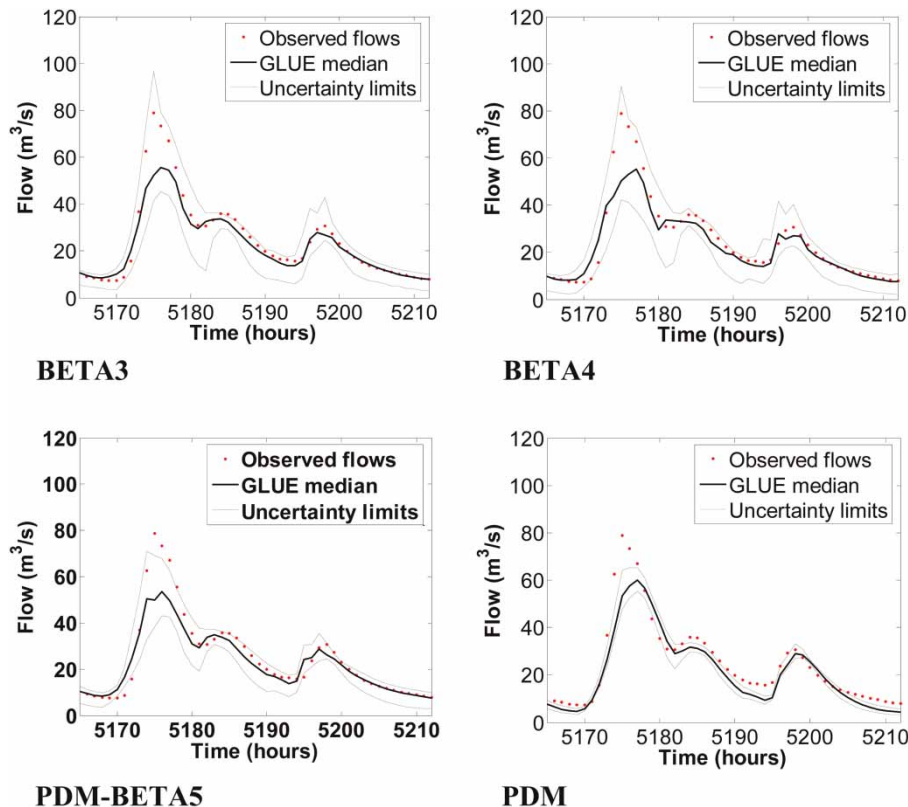


Figure 4 | Comparison of the BETA2, BETA3, BETA4, PDM-BETA5 and PDM for the highest storm event in validation on the Eden catchment.

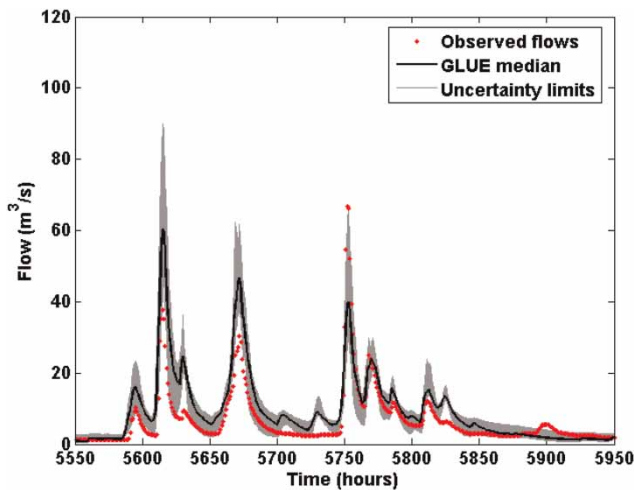


Figure 5 | The BETA3 for a series of storm events in validation on the Eden catchment.

storm at 5,810 hr is simulated. In this case it is possible that there is too much calculated rainfall.

## SUMMARY AND CONCLUSIONS

The study has looked at the process of runoff generation using the beta function to represent a distribution of storages between a minimum and a maximum storage depth in three different model forms. The models have been compared with a commonly used form of the PDM model using the Pareto function for the distribution of storages.

Testing of the models was on three fast responding catchments in the Northern Pennine region in England. An extension of the study would be to apply the BETA models to groundwater-dominated catchments. For this type of catchment it is expected that some modification of the BETA models may be needed, for example, the addition of a slow-pathway routing tank to represent bulk storage of groundwater in addition to the drainage function used here.

The use of recession curve analysis to select the form of function for the slow pathway with indicative parameters showed mixed results. This is likely to be the case on faster responding upland catchments with higher rainfalls for which the recessions tend to be short between storm events in the winter periods. Summer recessions are inevitably longer but for this case evaporation effects must be accounted for before fitting drainage functions. The non-linear power

law was the most useful function for representing the slow drainage from the BETA models. Also, the study does demonstrate that there is a benefit in optimising the fast pathway flow routing for the runoff generation. The three test catchments all showed a marked preference for a Nash Cascade with two linear reservoirs in series on the calibration data.

The performances of all models were better on the Eden and Ribble catchments. The Tyne catchment had poorer data and this was reflected in the generally lower NSE values in calibration and validation. In calibration the BETA behavioural model set generally showed some improvement in performance over the PDM though there was no BETA model that was preferred for all catchments. However, the BETA models performed slightly better in validation on the three test sites for the chosen periods of data. A wider study would be needed to see whether this is generally the case on catchments which have different hydrological responses. The results suggest that the choice of model is a factor of the type of catchment and the choice of the data period.

In the BETA models, with the flexibility of the beta function, the minimum and maximum storage depths do not necessarily need to be specified which makes for a reduced number of parameters. The beta function is therefore used to control the distribution of storage depths within the range zero to  $S_{\max} = 300$  mm via the function  $z$  and  $w$  parameters. Although there is a very small loss of performance with fewer parameters there is a benefit in parameter identifiability for the BETA models. Faster responding catchments do seem to show a preference within parameter space for the beta  $z$  and  $w$  parameters. BETA models with five parameters were sufficient to simulate flows adequately on the test catchments. The study does suggest that the flexibility of the beta function may demonstrate model performance improvement on some catchments over the commonly applied Pareto function in the PDM.

The BETA3 and BETA4 models were the fastest models to run. This is an important factor in Monte Carlo computations that are demanding on process power. However, there was a disadvantage of slow computation for the PDM-BETA5 analytical solution using the beta function. The much slower computation is a hindrance for practical use of the model and the possible small advantage of an analytical runoff response does not seem to outweigh the additional computation time. The PDM, which typically

uses the simpler Pareto function, does not suffer from this problem. The models were tested using Matlab software and run times may be improved on other software packages.

For the BETA models, the use of the beta function for the PDM-BETA5 analytical solution does not seem to have an obvious advantage for runoff generation compared to the discrete tube BETA3 and BETA4 versions. This will be the case when a sufficient number of tubes (say  $\geq 30$ ) are used to generate a smooth fast pathway response during storm events. Also, the BETA4 with water level redistribution does not seem to offer an advantage compared to the BETA3 which has a simpler water balance computation in each tube and allows tube water levels to vary. The additional computational time for water redistribution in the BETA4 therefore seems unnecessary.

The BETA4 showed preferred fit in validation only on the Tyne catchment but this improvement was on a catchment with generally poorer quality data. For all the catchments there was some evidence of epistemic errors in the rainfall and flow data that will have had an effect on both the calibration and validation results and model preference. Further work will explore ways of allowing for these types of errors within a limits of acceptability approach to the choice of the behavioural model set (see Beven (2006, 2010) and Liu *et al.* (2009)).

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