Interpretation of statistics of lake ice time series for climate variability

Matti Leppäranta

ABSTRACT

Lake ice time series consist of the dates of freezing and ice break-up for the ice phenology, and ice coverage and ice thicknesses for the severity of ice seasons. This work analyses the physical interpretation of the statistics of lake ice time series illustrated by available data. The results provide tools to understand variations in the past lake ice seasons and to make projections into the future. The freezing date is related to fall air temperature with correlation time scale depending on the lake depth. Ice growth is primarily determined by the air temperature and snow accumulation. Ice break-up depends on the seasonal maximum ice and snow thicknesses, solar radiation and air temperature; the onset of melting is the primary question. Relationships are derived for ice season characteristics and climate changes. Warmer climate also brings qualitative changes to lake ice seasons by moving boundaries of ice climate zones.

Key words | climatology, freezing of lakes, ice break-up, ice thickness, interpretation

Matti Leppäranta
Department of Physics,
University of Helsinki,
Box 48 (Erik Palménin aukio 1),
FI-00014 Helsinki,
Finland
E-mail: *matti.lepparanta@helsinki.fi*

INTRODUCTION

Lake ice phenology time series have been collected to a larger extent since the 1800s (e.g. Wing 1943; Simojoki 1959; Magnuson *et al.* 2000). They consist of the dates of freezing and ice break-up and the number of ice days. In addition to the phenology, the thicknesses of ice and snow have been recorded in many lakes, and their annual maxima quantify the severity of ice seasons. In large lakes, the evolution of ice coverage has been monitored with its annual maximum as another characteristic of the season severity. The formation, growth and decay of lake ice are strongly related to air temperature, and therefore lake ice time series serve as good climatic indicators. Conversely, climate variations have a major impact on the ice season characteristics.

The evolution of ice conditions also depends on lake characteristics, primarily the depth distribution. The thermal memory of lakes is short to show any significant correlation between consecutive ice seasons (see Leppäranta 2009). The freezing date is independent of the previous ice break-up date, but there is a weak connection between the freezing date and the following break-up date. A stronger connection

over the ice season is seen in the deep-water temperature and oxygen content. In a windy autumn, forced deep convection may continue until water temperature is well beyond the temperature of maximum density, and then cold and oxygen-rich water results, influencing the conditions until spring turnover (Salonen *et al.* 2009).

Freezing and break-up of ice bring major changes to the physics and ecology of lakes and to the human living conditions in lake districts. Therefore, lake ice time series information has long been collected, and their statistical properties have been examined for variations in climate and environment (e.g. Livingstone 2000; Magnuson *et al.* 2000; Korhonen 2006). The importance of the ice season to the annual cycle of lakes has been recently emphasized (Salonen *et al.* 2009; Kirillin *et al.* 2012), and discussions of impacts of possible climate changes have clearly increased lake ice time series research. In general, most of the lake ice time series show trends toward a milder climate in the last 100 years superposed on aperiodic variations and noise. The trend comes from the general climatological warming

during the 20th century, especially in Eurasia and North America, where most of the lake ice investigations have been made.

A statistical approach has normally been taken in analyses of lake ice time series (e.g. Bernhardt *et al.* 2011; Efremova & Palshin 2011; Mishra *et al.* 2011). However, the connection of the statistical characteristics of these time series to the physics of lakes is desirable to understand, in particular to evaluate the impact of future climate variations to the annual cycle of temperature and ice conditions in lakes. It has been known that the ice season strongly reflects the evolution of air temperature at the lake during the ice season. The freezing date is related to autumn temperature, ice thickness and ice coverage are related to the freezing-degree-days, and ice break-up date is related to the positive-degree-days (Karetnikov & Naumenko 2008; Leppäranta 2009; Lei *et al.* 2012).

Not much research work has been carried out on the physical bases to qualify and quantify the relationships between lake ice conditions and climate. Since consequent ice seasons are independent, variations between ice seasons are externally caused. In very deep lakes, such as Lake Baikal, this is not necessarily true (Granin *et al.* 1999; see also Kirillin *et al.* 2012). Ice break-up is the most difficult case to predict. It depends on the maximum ice and snow thicknesses as the initial conditions, and it is driven primarily by solar radiation. Air temperature is correlated with solar radiation and, consequently, with the ice melting. However, the onset of melting is the primary question to be solved. Modelling experiments in the Baltic Sea have shown that simple analytic models for climate change impact provide results in rather good agreement with the outcome of advanced numerical ice–ocean models (Haapala & Leppäranta 1997). This is likely due to the fact that the ice season response is linear to small climate changes.

This paper attempts to work through the basic physics to provide an understanding of the relationships between climate and lake ice season. The analysis builds on an earlier work (Leppäranta 2009) with more advanced analytical models derived. The sensitivity of ice seasons to geographic co-ordinates can also be examined by using a similar approach. A set of selected time series publications is used to illustrate the results. The results provide tools to interpret the statistics of lake ice time series and to make projections into the future. The evolution of ice conditions also depends on lake characteristics, primarily the depth distribution, and the sensitivity to climate depends on the quality of the climate zone where the lake is located.

MATERIALS AND METHODS

Ice phenology is defined by the following quantities: ice occurrence, freezing date, ice break-up date, and the number of ice days. Consequent 'ice seasons' can be cut by the date of maximum summer heat storage of the lake, occurring normally in July–August. It is convenient to index the ice season with the year of its autumn, i.e. ice season 2012 refers to the period between the summer maxima of 2012 and 2013 (and it is called 'ice season' irrespective of whether ice actually occurs then or not). Other ice phenology characteristics would be the occurrence of perennial ice and full freezing of the water body to the bottom but these are not further worked here.

Ice occurrence is a binary variable $I = I(n)$, where n is ice season number, and $I(n) = 0$ or 1 whether the season is ice free or not, respectively (Jevrejeva *et al.* 2004). The *freezing* and *break-up dates*, t_F and t_B, respectively, are defined as the first and last days of ice in a season. Freezing date refers to the vicinity of the observation site. The term *freeze-up* or *ice-on* refers to the first date of complete ice cover in a lake, which was difficult to track in large lakes before the time of satellite remote sensing. Freezing and break-up dates can be defined only in seasons when the ice forms and decays. There may be several times of freezing and break-up during one ice season, especially close to the climatological margin of seasonally freezing lakes, and also then the first and last days of ice are taken to represent the freezing and break-up dates of the given ice season. The *length of an ice season* is the time between freezing and break-up dates, while the *number of ice days*, M, leaves out the possible open water periods during the ice season. Then the ratio $M/(t_B - t_F)$ describes the seasonal stability of the ice cover.

The evolution of ice cover is characterized by *ice and snow thickness* and *ice coverage*, their seasonal

characteristics given by the maxima. Snow and ice thicknesses are not independent, since snow accumulation influences ice thickness by thermal insulation and as the source of snow-ice growth (Leppäranta 1983). Ice coverage is the ice area divided by the total area of the lake. The seasonal maxima are not able to tell the differences between winters when the lake freezes over, and therefore ice coverage is more used to describe the whole seasonal evolution. Ice volume is the product of ice thickness and coverage, but there is not much research carried out on this, due largely because in large lakes mapping of the thickness distribution is difficult.

The present methods to connect the statistical properties of lake ice time series to physics are based on a linear air–lake heat transfer approximation (see Leppäranta & Myrberg 2009):

$$Q_a = k_0 + k_1(T_a - T) \tag{1}$$

where T_a and T are air and lake surface temperature, respectively, and k_0 and k_1 are coefficients, which do not explicitly depend on the surface temperature. Positive flux Q_a is directed toward the lake. The coefficient k_0 largely reflects the radiation balance and therefore depends on time, positive in summer and negative in winter, but k_1 is more stable representing primarily the turbulent exchange coefficient. When parameterized from routine weather station data, these coefficients depend on air temperature, humidity, wind speed and cloudiness. In fresh water lakes, the freezing point is $0\,°C$, and it is convenient to use the terms 'zero downcrossing' and 'zero upcrossing' for the times when the air temperature ($°C$) changes from positive to negative and vice versa, respectively.

Earlier investigations of ice time series are used to illustrate the results of the theoretical analyses. They concern mainly the connection between seasonal ice statistics and air temperature. Also the ice time series of Kallavesi, eastern Finland is worked on for examples (available at Hertta database of the Finnish Environment Institute, www.syke.fi/en-US/Services/Environmental_information_data_systems). It is one of the longest ones, starting at 1830 (Korhonen 2005). Meteorological data have been taken from the e-publication of the Finnish Meteorological Institute (Pirinen et al. 2012).

PROPERTIES OF ICE TIME SERIES

Ice occurrence

The time series of ice occurrence is a binary series $I(n)$. It was used first to analyse Baltic Sea ice time series where the northern coast freezes every year but in the south, ice-free seasons occur (Jevrejeva et al. 2004). The event 'ice occurs' is a binary variable, and the probability of ice occurrence p can be estimated as:

$$p = \frac{1}{N}\sum_{n=1}^{N} I(n), \ s = \sqrt{\frac{p(1-p)}{N}} \tag{2}$$

where N is the total number of seasons in the data, and s is the standard deviation of the estimator. The trend of the ice occurrence, which is equal to the trend of the probability p, tells whether ice occurrence is happening less or more frequently. The results of Bernhardt et al. (2011) for Berlin–Brandenburg lakes clearly illustrate how the occurrence of ice has become more rare in recent decades, but in Finland 1930 is the only year known when all lakes have not frozen over (then deep basins of Lake Päijänne remained ice-free).

In the cooling of cold region lakes, as soon as the autumn overturning has been reached, the lake surface may soon freeze. Lakes lose heat by terrestrial radiation and turbulent heat transfer, and the surface water temperature lags behind the air temperature. The lag depends on the depth of the lake (see Leppäranta 2009). Thus the governing factors are the mixed layer depth (or the lake depth in shallow lakes) H and the heat exchange between the lake and the atmosphere (e.g. Leppäranta 2009). Taking a linear air–lake heat transfer approximation (Equation (1)), we have the cooling model:

$$\frac{dT}{dt} = \lambda(T_a - T) + F; \ F = \frac{k_0}{\rho_w c_w H}, \ \lambda = \frac{k_1}{\rho_w c_w H} \tag{3}$$

where T represents the mixed layer temperature, λ and F are the model parameters, ρ_w is water density, c_w is specific heat of water, and H is the mixed layer depth. The parameter λ is the lake response time, and F can be taken as the radiational gain/loss factor. They vary slowly in the course of the year but can be considered as seasonal constants. Then

Equation (3) can be directly integrated. In autumn, $k_0 \sim -20 \, \text{W m}^{-2}$ and $k_1 \sim 20 \, \text{W m}^{-2} {}^{\circ}\text{C}^{-1}$ for the conditions in southern Finland. Then, for $H \sim 10 \, \text{m}$, we have $F \sim -0.041 \, {}^{\circ}\text{C} \cdot \text{day}^{-1}$ and $\lambda \sim 0.041 \, \text{day}^{-1}$.

The first question is whether the lake freezes or not. It is convenient to take a parabolic form for the air temperature evolution with $T_{a,min} < 0 \, {}^{\circ}\text{C}$ as the minimum air temperature and t_c as the length of the cold period ($T_a < 0 \, {}^{\circ}\text{C}$). Then, the lake freezes, if:

$$t_c > 2\lambda^{-1} \sqrt{\frac{1}{1 + F/(\lambda T_{a,min})}}, \quad F < -\lambda T_{a,min} \tag{4}$$

Thus, very shallow lakes freeze if air temperature falls below zero, but deep lakes may survive unfrozen throughout the winter (Figure 1). When turbulent heat losses dominate, the lake freezes if the cold period is at least twice the lake response time, $2\lambda^{-1}$; when radiational losses dominate, the condition is asymptotically $t_c > 2\lambda^{-1}\sqrt{\lambda T_{a,min}/F}$. In fact, when radiational losses are large, Equation (4) allows freezing with a slightly positive air temperature minimum: the condition is $F < -\lambda T_{a,min}$. However, here we simply consider the cold season defined by $T_a < 0 \, {}^{\circ}\text{C}$.

Freezing date

In case freezing takes place, we can obtain the delay of the freezing date from the date air temperature downcrosses
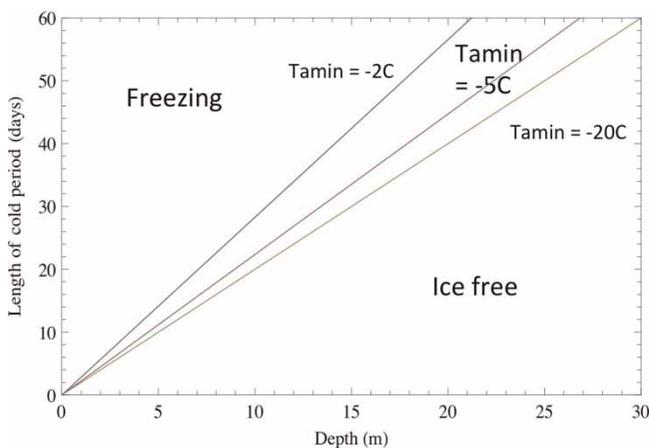
$0 \, {}^{\circ}\text{C}$. Assuming linear air temperature decrease in the cooling period, $T_a = -\gamma \cdot t$, Equation (3) gives the delay as:

$$t_F = \lambda^{-1}(1 + \gamma^{-1}F) \tag{5}$$

Via the response time λ^{-1}, the delay is proportional to mixed layer depth. Simojoki (1940) compared the delay to physical depths of lakes and showed that the proportionality of lake depths approximately holds until $10 \, \text{m}$ with $T[\text{days}] \sim 3 \, H[\text{m}]$ (Figure 2). In deeper lakes, the heat of the lower layer is not all used, and the delay increases slower than linearly with depth. Then a proper estimate for the mixed layer depth is required. In a very mild case, the parabolic course of the air temperature can be applied, and then the delay in freezing approaches the minimum length of the cold period given by Equation (4).

The connection between the air temperature and the freezing date has been examined in many time series analyses (see Kirillin et al. 2012), and here the relation is more widely explained in terms of the autumn weather conditions.

Ice thickness and compactness

Lake ice cover consists of four layers: snow, superimposed ice, primary ice and congelation ice. Primary ice is the initial ice layer, which it is very thin (<1 mm) congelation ice or thicker frazil ice. Congelation ice grows down and superimposed ice grows up from the lower and upper surfaces,
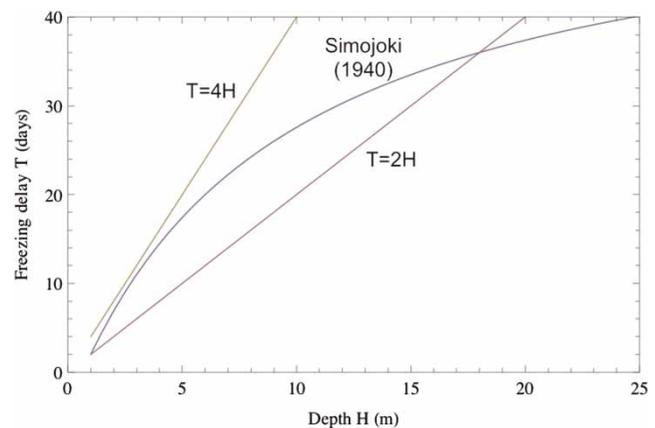

Figure 1 | Freezing conditions for lakes as a function of the lake depth and length of the cold period ($T_a < 0 \, {}^{\circ}\text{C}$).

Figure 2 | Delay of the freezing date from the air temperature downcrossing of $0 \, {}^{\circ}\text{C}$ as a function of the mixed layer depth (H) of the lake. The empirical curve of Simojoki (1940) is shown together with lines $T[\text{days}] = 2 \, H[\text{m}]$ and $4 \, H[\text{m}]$.

respectively, of primary ice. The total thickness of snow-free ice h is related to the freezing-degree-days S by the growth law derived in Barnes (1928):

$$h(t) = \sqrt{a^2 S(t) + b^2} - b, \quad S = \int_{t_0}^{t} \max(T_R - T_a, 0{°}C)dt' \quad (6)$$

where a and $b \sim 10$ cm are model parameters, t is time, t_0 is the initial freezing time, and $T_R = -k_0/k_1$ (e.g. Lepparänta 2009). In the presence of snow cover, the parameter a is empirically modified and it lies in the range of 1.5–3 $({°}C \cdot day)^{-1/2}$. Freezing-degree-days do not usually include T_R in the definition. The correction is based on a proper representation of the surface heat flux (Equation (1)), and since $T_R \sim 1\,{°}C$ the correction is significant near the climatological ice margin where the winter temperature is not much below the freezing point.

Bare ice grows fastest and then we have $a = 3$ cm $({°}C \cdot day)^{-1/2}$, but in the presence of snow cover the growth is reduced and a may be down to 50% of the bare ice value. In lakes the heat flux from the water body is usually small due to weak (or absence of) turbulence and is neglected here. However, in shallow lakes the bottom can store heat in summer to reduce the winter growth of ice, and in the presence of geothermal heating the lake can stay open in the cold period. However, when the heat flux from the deeper water or lake bottom is not large, snow accumulation dominates the variations of the maximum annual ice thickness.

Figure 3 shows results from model calculations of ice thickness when a fixed rate of snowfall has been assumed for the whole winter. If the rate is less than 0.5 mm snow water equivalent (SWE) per day, no snow-ice forms but increasing snowfall rate first gives a stronger insulation and slower ice growth. If the snowfall is much more, flooding takes place when the weight of snow has pushed the ice beneath the water surface level. Then the slush formed of snow and lake water can freeze into snow-ice. With snowfall more than 1.5 mm SWE per day, most of the ice that forms is snow-ice. The mean snowfall rate is about 1.0 mm SWE per day in Oulu (Lepparänta & Myrberg 2009).

The freezing-degree-days can also be expressed as the product of average positive value of $T_R - T_a$ and the length of the cold season: $S = \overline{(T_R - T_a)}_+ t_c$. A fraction of the cold
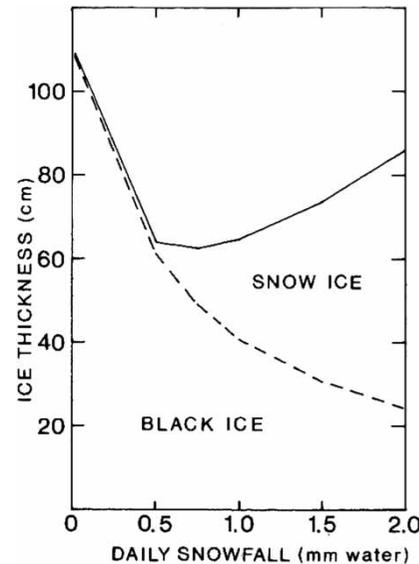

Figure 3 | Thickness of ice calculated with climatological air temperature (Oulu, Finland; 65.0°N, 25.5°E) as a function of daily snowfall (Lepparänta 1983). The solid line presents the total ice thickness, and the dashed line gives the proportions of congelation ice (black ice) and snow-ice.

season (S_0) is lost, however, for cooling of the water. In lakes, where the mixed layer is not deep in the cooling season, this fraction is not much. Cooling a 20-meter water layer from 4 to 0 ${°}C$ needs $1.6 \cdot 10^7\,J\,m^{-2}$ of heat to be taken out, and this corresponds to the heat release to grow 5 cm of ice. Formally, we can write $S_F = S - S_0$ as the effective freezing-degree days for ice growth.

The compactness of ice depends on the lateral cooling of the lake water body, and as the freezing date it is primarily connected to the depth distribution of the lake. As shown above (Equation (5)), the freezing date depends on the lake depth and air–lake heat exchange parameters. The hypsographic curve is defined as $D(z)$ equal to the area of lake with depth more than z. Then we have for the ice coverage A:

$$\frac{dA}{dt} = -\frac{dD}{dz} \cdot \left(\frac{dt_F}{dz}\right)^{-1}, \quad z \leq H \quad (7)$$

where z is depth. Ice coverage grows with time, and as the depth of the freezing zone reaches the mixed layer depth, the whole lake freezes over. If the freezing date is linearly related to the depth of a lake and the hypsographic curve is linear, then the ice coverage grows linearly in time.

Break-up date

The melt rate of ice is 1–3 cm per day. In spite of the non-linear progress of ice melting, simple positive degree-day formulae, where the melt rate is proportional to $\max(T_a, 0\,^\circ\text{C})$ are useful as a first approximation. However, to use the present linearized surface heat flux model (Equation (1)), the parameters need consideration. In the melting period, $T = 0\,^\circ\text{C}$ and $k_0 > 0$. When $Q_a > 0$, the heat gain is used for melting and ice thickness decreases according to:

$$h(t) = h_0 - \frac{1}{\rho L} \int_{t_m}^{t} \max(k_0 + k_1 T_a, 0)\,dt' \tag{8}$$

where t_m is the starting time of the melting period. Assuming that $k_0 + k_1 T_a > 0$, Equation (8) integrates to:

$$h_0 - h(t) = \frac{k_0}{\rho L}(t - t_m) + \frac{k_1}{\rho L} R(t; t_m) \tag{9}$$

where $R(t; t_m)$ is the sum of degree-days from the beginning of the melting period.

The parameter k_0 contains the net solar radiation, and to have it constant in the melting period is a limiting assumption. Incoming radiation increases and albedo decreases, and as a consequence the net solar radiation increases strongly in the melting period. In fact Equation (8) is valid more generally for $k_0 = k_0(t)$, and then in Equation (9) there should be an integral of $k_0(t)$ instead of $k_0(t-t_m)$. The radiation balance is written as:

$$Q_R = (1 - \alpha)Q_s + Q_{nL} \tag{10}$$

where Q_s is the incoming solar radiation and Q_{nL} is the net terrestrial radiation. The latter term is $\sim -50\,\text{W m}^{-2}$ and does not vary much, while the solar radiation is sensitive to time and albedo. If $Q_s \sim 500\,\text{W m}^{-2}$ and $\alpha = 0.9$ (dry snow cover on ice), we have $Q_R \sim 0$; but for bare ice, we have $\alpha \approx 0.5$ and $Q_R \sim 200\,\text{W m}^{-2}$. This level of incoming radiation can be reached in springtime, when the solar elevation is more than about 30° and the sky is clear. However, to reach $Q_R > 0$ (significantly) for snow-covered ice, the albedo must decrease. It is known that in the Antarctica,

the snow albedo is high and stable and the snow does not melt; snow is lost only due to sublimation (e.g. Leppäranta *et al.* 2013).

A possible mechanism to decrease the albedo is that advection of warm and moist air initiates melting at the surface in daytime, and recrystallization of snow results in day-and-night cycles. Liquid water and recrystallization, which increases the grain size, both tend to decrease the albedo. Once this has started, decrease of the albedo continues and melting is accelerated. In the beginning of the melting period, the surface of lake ice becomes patchy with snow and bare ice spots and meltponds, and open water first appears near the shoreline (Figure 4).

The key question is thus the time of zero upcrossing of the radiation balance. This date defines the beginning of the melting period, $t = t_m$. Thereafter the solar radiation increases by the rate μ, and it is assumed that μ and the net terrestrial radiation are constants. The albedo can be here taken as $\alpha = \alpha_0 - ct$ ($\alpha > 0.1$), where α_0 is the initial value, and c is the rate of decrease. Then the radiation energy absorption integrates into:

$$E = \int_{t_m}^{t} Q_R\,dt' = \left[\frac{1}{2}(1 - \alpha_0) + \frac{1}{3}c(t - t_m)\right]\mu(t - t_m)^2 \tag{11}$$

Figure 4 | Melting of lake ice begins from the shallow near-shore zone. Photograph from Lake Pääjärvi, southern Finland on April 23, 2004, about 1 week before the final ice break-up.

e.g. taking $\alpha_0 = 0.9$, $c = 0.015$ day^{-1}, and $\mu = 3$ W m^{-2} day^{-1} gives $E = 190$ MJ m^{-2} at $t - t_m = 50$ days, and this can melt 64 cm of ice. Since the first term in the brackets is much smaller than the second when $t > 1$ month, we have an approximate solution for the period of melting of ice with the initial thickness h_0:

$$t_b - t_m = \left[\frac{3}{\mu c}(\rho L h_0 - k_1 R)\right]^{\frac{1}{3}} \tag{12}$$

Thus, because of the accelerated process of melting, the break-up date is not highly sensitive to ice thickness. It is, however, possible that thinner ice has a faster albedo decrease (c) that would speed up its melting and add sensitivity of break-up date to ice thickness. However, in all, the timing of the start of the melting period is critical. The connection between the positive degree-days and ice break-up has been shown in several papers (e.g. Karetnikov & Naumenko 2008; Lei *et al.* 2012). However, the relationship is not very strong, as seen in Equation (12), but it is likely that the positive degree-days are correlated with the beginning of the melting period (t_m) that strengthens the correlation.

Sunlight is able to penetrate into the ice and water (see Jakkila *et al.* 2009). It is partly absorbed in the ice sheet and used for internal melting, which makes the ice more porous, and partly it goes into the water, where the water warms and the stored heat does not escape. As a result, except for a very thin layer under ice, the water temperature becomes close to 4 °C. As long as there is an ice cover, diffusion of heat is slow, but when the ice breaks, the warm water is mixed by the movement and quickly used for ice melting. This is why the last stage of ice decay is so fast, known well by fishermen from practice. When the internal melting has progressed far enough, to the porosity of ice at 30–50%, the ice breaks under its own weight, and the rest of the ice then disappears very fast, in a time of 1 day or so.

The freezing and break-up dates show different features (Figure 5). Freezing date varies much, since it is connected to autumn variable weather. In particular, variations arise due to North Atlantic Oscillation (e.g. Blenckner *et al.*

2007; George 2007). Break-up date is strongly tied to the calendar by solar radiation and therefore varies less. The time series of Lake Kallavesi contains the following characteristics:

	Freezing date	Break-up date
Mean	November 30	May 8
Median	November 28	May 8
Minimum	November 8	April 18
Maximum	January 27	June 3
St.dev.	13.6 days	7.7 days
Maximum–minimum	84 days	36 days
Trend	7.9 days/100 years	−5.3 days/100 years
Time correlation	0.30	−0.38

CLIMATE CHANGE

The results from the previous section can be examined for the impact of climate change on the ice season. The expected climate change, as we understand it now from the IPCC scenario, is predicted to influence the air temperature by $\Delta T_a > 0$ and change precipitation (in North-Europe, winter precipitation is expected to increase with more in the liquid phase). From the point of view of lake ice, both the amount and phase of precipitation are very critical. The temperature scenario varies from place to place, and therefore we consider a general case of the impact of an air temperature change. To analyse the impact of a change in precipitation is a more difficult task and is not examined here.

Analytic models were derived for the relation between temperature and ice phenology, thickness and coverage. The climate change by ΔT_a (this may have either sign in the calculations that follow) goes to the temperature forcing and the impact is then seen in the ice season characteristics. It is clear that the analytic models are first-order approximations, but their applicability to study the sensitivity to climate is stronger, since many simplifications made cancel out when changes are looked for and the system is fairly linear for small changes.

Whether a lake freezes or not, the condition for the length of the cold period was given by Equation (4). The climate change would shift the minimum temperature up
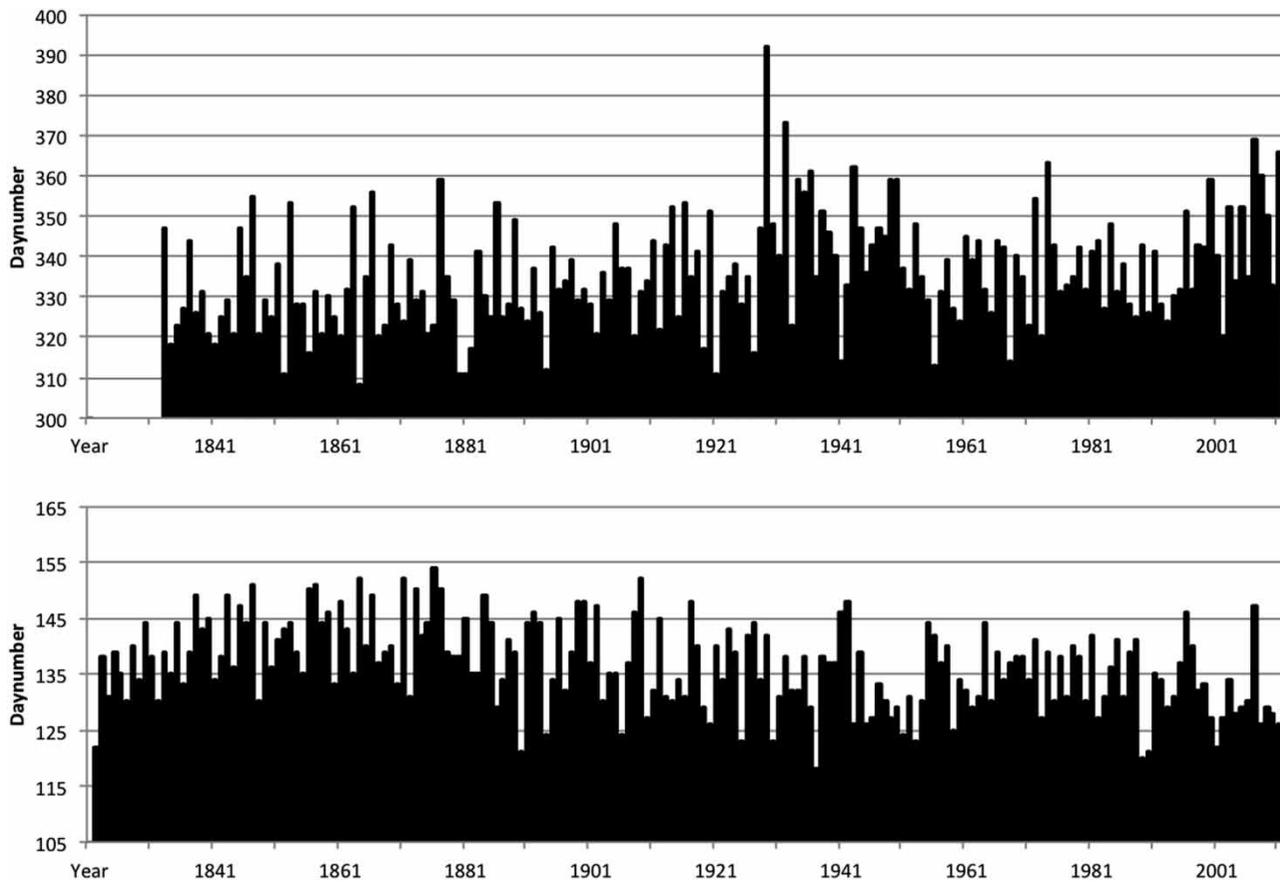
Figure 5 | Freezing date (top) and break-up date (bottom) of Lake Kallavesi, Finland (1830–2011). Year is shown in the horizontal axis and day of year in the vertical axis; in the freezing date plot, numbers from 365 or 366 to 395 refer to January in the following year.

by ΔT_a and then the length of the cold season would be reduced to:

$$t_{c2} = t_c \sqrt{1 + \frac{\Delta T_a}{T_{a,min}}} \tag{13}$$

What would happen can be evaluated from Equation (4).

The case with the freezing date is straightforward. With linear cooling, the temperature scenario is $T_a = bt + \Delta T_a$, where $b = \dot{T}_a = $ constant is the atmospheric fall cooling rate. It is directly seen that the freezing date would be shifted by:

$$\Delta t_F = \frac{\Delta T_a}{\dot{T}_a} \tag{14}$$

Thus the freezing day shift not only depends on the air temperature change but also on the rate of atmospheric cooling. In continental tundra the cooling is faster than in maritime climate and the climate change impact consequently weaker. The cooling rate must be proportional to the amplitude of the annual air temperature cycle. For example, in conditions in Finland, the atmospheric cooling rate is about $1\,^{\circ}C/5$ days, and therefore the freezing date would shift by 5 days of each temperature change of $1\,^{\circ}C$.

The maximum annual ice thickness is expected to become reduced since the freezing-degree-days are reduced. Using the parabolic model again and the growth law shown by Equation (6), and assuming $aS >> b^2$, the reduced ice thickness is:

$$\frac{h_2}{h_1} = \frac{a_2}{a_1} \cdot \left(1 + \frac{\Delta T_a}{T_{a,min}}\right)^{\frac{3}{4}} \tag{15}$$

where h_1 and h_2 are the thicknesses of ice in the present and future climate, respectively, and a_1 and a_2 are the corresponding coefficients. If $a_1 = a_2$, this makes about 10 cm for a

change of $1\,^{\circ}$C in air temperature. However, the parameter a may change due to changing snow conditions; it is typically in the range of 2–3 cm/$(^{\circ}$C $\cdot$ day$)^{-1/2}$, and therefore a small change in snow conditions may even compensate for the change in the air temperature. Evolution of ice coverage follows directly from Equation (7), with the difference that the initial time has shifted as given by Equation (14).

The question regarding ice break-up is interesting. Since the melting of ice is tied to the radiation balance, climate change impact is not directly clear. However, the start of the melting depends on the atmospheric conditions and state of the surface, and it is anticipated that melting becomes earlier if air temperature increases. There is another effect in that if the ice thickness has decreased there is less ice to melt but the result is not sensitive to this as discussed above (see Equation (12)). Consequently, the shift in the break-up date $\Delta t_b \sim$ shift in the zero upcrossing time of the radiation balance.

Equation (9) shows that the climate change impact on ice break-up date comes in three factors: (1) how much ice there is to melt, (2) the positive degree-days, and (3) the timing of the beginning of the melting period. The first factor is given by Equation (15). For the second factor a linear model $T_a = \kappa t + \Delta T_a$ can be taken resulting with $\Delta R = \frac{1}{2}\,\kappa^{-1}(\Delta T_a)^2 + \Delta T_a t$. It is plausible that the third factor is somewhere between zero and the shift in the zero upcrossing time of air temperature, $0 \le \Delta t_m \le \kappa^{-1}\,\Delta T_a$. Then, Equation (12) gives us the ice break-up date after a climate change, t_{b2}:

$$t_{b2} - (t_m + \Delta t_m) = \left\{ \frac{3}{\mu c}[\rho L(h_0 + \Delta h) - k_1(R + \Delta R)] \right\}^{\frac{1}{3}} \quad (16)$$

The role of Δt_m is the strongest, since ice thickness change and positive degree-days change appear under the cube root. The cube root was obtained from a simple model for the increase of solar radiation and decrease of albedo during the melting period, and it could be changed into a general power law with power v, $0 < v < 1$.

CONCLUSIONS

This paper attempted to work through the basic physics of the relationships between the climate and lake ice season.

A set of published time series papers was used to illustrate the questions. The results provide tools to interpret the variations in the past lake ice seasons and to make projections into the future. The ice season can be divided into three phases (Figure 6). Unstable early winter, when the ice is still thin and breaks easily; mid-winter with a stable ice cover, and spring, again with unstable ice in the melting stage. If the climate warms, the unstable parts are not much changed except that they get closer to each other in time, and therefore the stable period becomes shorter. The sensitivity of ice seasons to geographic co-ordinates can also be examined by using a similar approach.

Ice temperature follows the air temperature fall with a delay time equal to the response time of the lake, which is proportional to the mixed layer depth (each meter adds 2–3 days to the response time). To freeze a lake needs a cold period ($T_a < 0\,^{\circ}$C) longer than twice the response time of the lake. Ice growth follows the freezing-degree-days, and the key question to start the melt season is the timing of zero upcrossing of radiation balance in spring. Then melting takes place by 1–3 cm day^{-1}, the rate increasing with time due to the increase in solar radiation and decrease in albedo. Snow cover is the major question to bring uncertainty to evaluate future ice growth and break-up. Climate warming would bring milder ice climate as a response of lakes to a rising air temperature. For an increase of $1\,^{\circ}$C in air temperature level, the freezing date would delay by ~ 5 days and ice thickness would decrease by 10 cm. The change in the ice break-up date depends on the radiation balance rather than on air temperature directly. The
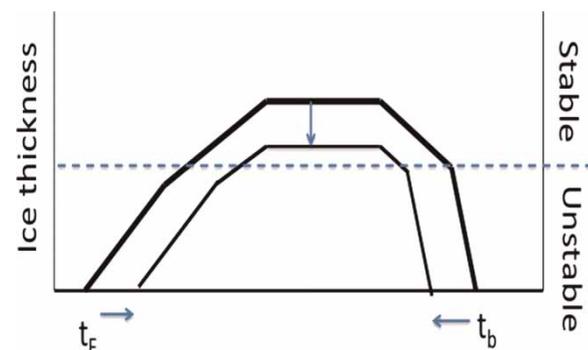

Figure 6 | The influence of climate warming on an ice season. Freezing and break-up dates t_F and t_b come closer to each other. Ice thickness decreases and that reduces the length of the stable ice cover period.

sensitivity to climate also depends on the quality of the climate zone where the lake is located.

The implications of a milder ice climate could be positive for the biota, first of all since shorter ice season means less oxygen problems. Lake ice is not considered as an important habitat for life, however, freshwater seals living in the Eastern Lake District of Finland and Lake Ladoga make their nests on ice and give birth to new puppies in February. Less snow on the ice would mean that primary production could continue over the winter instead of resting several months as in the present conditions. Lake ice is transparent and in snow-free conditions the euphotic depth would be about half of that in summer. In all, the ecology of freezing lakes would need to adapt to a new kind of ice seasons and as such is always a risk factor with yet unseen consequences. For society, milder ice seasons bring both positive and negative consequences. The traffic conditions will change. Thinner ice means more risks for on-ice traffic, and if the ice were too thin, then the whole ice season would be hostile in preventing both on-ice and boat traffic. In addition to traffic, lake ice cover provides recreation possibilities such as skiing, skating, ice sailing, and ice fishing, which all need a safe ice thickness. A shorter ice season, on the other hand, also means a longer boating season.

In the present climatic conditions in Finland, a warmer climate would bring significant qualitative changes to lake ice seasons. In particular, southern Finland could become a part of the ephemeral lake ice zone where lakes do not freeze over every year and also open and frozen phases could appear several times in a winter. In large lakes and in lake districts, local weather conditions would change with climate warming. Open water surfaces are warmer and more humid than ice or snow surfaces. Change from dry and cold to mild and moist is not preferable to all people. Also, persistent open water areas could generate more frazil ice and add frazil ice problems. One life style not affected is ice-water bathing, popular in northeast Europe, at least as long as there is any kind of ice left. However, as an overall, final conclusion, if the climate changes, not only the length of ice season and the thickness of ice changes, but also the quality of physics, ecology and practical life will be different.

In the continuation of this work, empirical studies of the derived first-order models will be made using ice and atmospheric time series data. It would be also interesting to look at two more ice phenology characteristics: the occurrences of perennial ice and full freezing of the water body to the bottom. Analytical methods can be easily derived from the results of the present paper but much less real data are available for these phenomena.

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