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Conference Report

Future Trends in Optimization

The purpose of this article is to provide a permanent record of the major ideas and questions raised during the panel session entitled "Future Trends in Optimization" at the 1980 Design Engineering Technical Conference in Beverly Hills, California. The panel members were Professors D. J. Wilde of Stanford University, E. J. Haug of the University of Iowa, K. M. Ragsdell of Purdue University, J. N. Siddall of McMaster University, and F. Freudenstein of Columbia University. They spoke, respectively, on Optimal Design Under Uncertainty; Computer-aided Design Sensitivity Analysis and Optimization of Dynamic Systems; Optimization: The Future of Design, Integration of Optimization with the Design Process; and Optimization in Mechanisms: Past, Present, and Future. It is hoped that the article will prove useful in guiding future efforts in the area of optimal mechanical design.

Introduction

On September 29, 1980, at the Design Engineering Technical Conference in Beverly Hills, California, the Design Automation Committee sponsored a panel session entitled "Future Trends in Optimization." The panel members were Professors D. J. Wilde of Stanford University, E. J. Haug of the University of Iowa, K. M. Ragsdell of Purdue University, J. N. Siddall of McMaster University, and F. Freudenstein of Columbia University. The goal of the session was to identify several promising directions for future work in the field of optimization.

Professor Wilde chose to speak on optimal design under uncertainty. Professor Haug discussed computer-aided design sensitivity analysis and optimization of dynamic systems. Optimization and the future of design was considered by Professor Ragsdell. Professor Siddall looked at the problem of integrating optimization with the design process, and Professor Freudenstein addressed the subject of optimization in mechanisms—past, present, and future.

The consensus of the Design Automation Committee is that the ideas expressed during this panel session are of general interest to a large audience in the design community. Accordingly, the purpose of this article is to provide a permanent record of the major ideas and questions raised in the session. It is hoped that the article will prove useful in guiding future efforts in the area of optimal mechanical design. In the following paragraphs, the salient points made by each panelist are presented.

Optimal Design Under Uncertainty

Professor D. J. Wilde, Stanford University

The recent and continuing development of analytical, noniterative, global optimization methodologies (see e.g., [1–5]) will prove to offer significant benefits to practicing designers in the years to come. Several advantages of such approaches (when they are applicable), compared to conventional general purpose nonlinear programming techniques, include the following.

(*a*) Insight into the nature of the design space and the interaction of the constraints is gained by the designer.

(*b*) A finite procedure is developed to identify the global optimum for any specified design requirements or parameters.

(*c*) Such procedures are inherently devoid of the arcane "algorithm tuning parameters" that are attendant to (and affect the performance of) virtually all nonlinear programming codes.

A major benefit of the foregoing from the perspective of the designer is that confidence in the results of the optimization process is no longer dependent on the designer's confidence in a potentially intractable mathematical programming algorithm. We can now turn our attention to questions of confidence in the system model and design parameters.

In the past, most optimal design studies have utilized a deterministic model to represent the objective function and constraint set. In actuality, design variables and requirements (constraints, parameters) behave stochastically. It is desirable to develop rational optimization strategies that can effectively deal with the stochastic nature of real design problems and this will be a very important area for future research. The global, noniterative, analytical optimization techniques appear to offer a straightforward avenue on which to approach this problem.

Computer-Aided Design Sensitivity Analysis and Optimization of Dynamic Systems

Professor E. J. Haug, University of Iowa

Potential exists for implementation of numerical methods for predicting the effect of design variations on the dynamic performance of large-scale mechanisms and machines. Real systems involve many degrees of freedom and numerous design variables, hence requiring computer-aided analysis methods to both construct and solve equations of motion and equations associated with design variations.

An extensive literature exists on dynamic system modeling and on iterative optimization methods. The missing link in obtaining a realistic optimization technique for design is computation of derivatives of response measures with respect to design variables. The literature on this subject is minimal, but basic techniques employed in optimal control and structural optimization offer potential for calculation of design derivatives.

Since complex systems must be dealt with, computer-generated dynamic modeling techniques such as IMP [6], ADAMS [7], and DADS [8] must be used for computer generation of the system equations of motion. The latter two methods employ a systematic technique for constructing equations of motion that can account for design dependence in the equations of motion. A general optimal design formulation that has been used for optimal control and structural and machine optimization [9, 10] has demonstrated capability for treatment of large-scale problems. Recent applications of computer-generated dynamic models and this optimal design formulation [11] demonstrate that it is feasible to computer generate not only the equations of motion, but also the equations required to obtain design derivatives. It is shown in reference [11] that derivative calculations that are required in constructing and solving these equations require only moderate additional computation, as compared to simply formulating and solving the equations of motion of the system. Therefore, it is possible to obtain design derivatives, or design sensitivity information, that tell the designer what the effect of a design variation will be. The computational cost of obtaining this information, along with dynamic response information, may be as little as 50 percent higher than the basic cost of the dynamic analysis. It is suggested that providing such information to a designer, along with dynamic response, is of much greater advantage to him than a purely dynamic response prediction with no indication of the direction in which the design should be changed to influence dynamic response.

Once the design derivatives are computed, they can be used as input to any of the well-known iterative optimization algorithms that are available to the designer. Since effectiveness of most optimization algorithms depends on control exercised by the designer (e.g., step-size, active constraint strategy, push-off factors, etc.), it is important that the designer select an optimization algorithm with which he is comfortable. This suggests that the algorithm and computer code for calculation of design derivatives can and should be separated from the optimization algorithm. The designer can then add or call the optimization package of his choice, using the design derivatives calculated by the computer code to carry out iterative design optimization.

While feasibility of the basic method appears clear, it is suggested that the mechanical design community could profitably devote effort to refinement and application of design sensitivity analysis techniques for interactive, computer-aided design optimization.

Optimization: The Future of Design

Professor K. M. Ragsdell, Purdue University

In this discussion, I shall assume the broadest possible meaning for the terms optimization and design. By "optimization," I mean the art and science of allocating scarce resources. This, of course, implies a choice between a number of alternatives or policies which are usually great in number. "Design" is a planning activity wherein man's needs and resources are merged. I do not confine myself here to the design of mechanical systems or to the location of minima of constrained functions, but these emphases will be present. There is an underlying structure which will forever encourage the marriage of design and optimization, which we call "decision theory." Hopefully, the relationship is clear.

The Decline of Natural Resources. We are all painfully aware of the current world energy crisis. This crisis is the result of political and technological forces. I would like to avoid consideration of the various political issues and concentrate on the technological implications of such shortages. My thesis is that political actions will periodically bring about such shortages, that is, that they are inevitable and we should plan accordingly. Examples of such shortages which we (the U.S.) have experienced in the recent past are: oil, sugar, copper, gold, and coffee. These are external examples; but what about internal, or domestic, supplies? We certainly do not have a domestic shortage of energy; ours is a shortage of available energy. We are currently excessively dependent on foreign oil and this dependence creates other domestic shortages or scarcities, such as fertilizer and plastics, as well as the more direct distillates, such as fuel oil and gasoline. A major shortage looms in the near future: water. The shortage of this life-sustaining natural resource is already being felt in the western states, such as Arizona. For instance, the Great Colorado River, with average yearly flow of 3.02 trillion gallons at the Utah-Arizona border, is reduced to a trickle as the river crosses into Mexico. Even now, with the inflow less than required to meet already legally allocated outflows, a new major aquaduct system is being constructed to divert a major portion of the river's flow to southern Arizona, including Tucson, where, I might add, it is desperately needed. The various Indian tribes in Arizona feel they have prior rights to this water! As the resource decreases, so does the competition for it increase! There are no simple answers here.

The Need for "High Level" Technology. Please examine the application of technology (such as robots, etc.) in the U.S. auto industry and the auto industries of Japan and Germany. Take a similar look at the foreign and domestic television and precision consumer products industry (such as video recorders, stereos, cameras, etc.). Foreign manufacturing facilities are highly automated, and the products produced are setting standards for the world. Technology is serving the people of these lands. We are being outplayed at a game we invented. Curse upon curse! Shame on us! We have no excuse. We have rested too long on our bloated reputation and past distant leadership posture. We have the need as a country to exploit every tool at our disposal (and some yet to be devised) in order to bring our productivity to the highest possible level. High level technology is the key.

The Role of Design Engineering. Designers of today play a pivotal role in our future, for it is the very nature of design to plan. Designers' plans today are tomorrow's solutions. Design engineers have a unique opportunity today to influence our future course. This is the reason I feel it is important now to examine the nature of design and the role of optimization in this activity.

Optimization as a Design Discipline. Optimization brings discipline to the design activity, that is, the theory of optimization. Now remember, we are using the terms in the broadest sense, offering a degree of rigor which the design function simply cries out for. Optimization provides a framework in which design opportunities are very conveniently formulated, that is, the structure requires the formulation of a design objective, such as reliability, cost, weight, or other such functions of the available design variables, which must as well be defined. In addition, we must decide upon the constraints or functional limitations for the design in advance. Obviously, we must select the design variables, that is, those quantities which are at our disposal to "perfect" the design. Finally, we must have a model of the system which relates design variables to objective and constraint values. This model can take many forms; it might be a purely mathematical formulation, a simulation, or experimental in nature. It is not necessary that the model be deterministic, just "calculable."

A great deal of work has been done in this field, but I shall not dwell here except to mention major directions. In the mechanical sciences, mechanism synthesis has produced a significant stimulus for the development of optimization strategies. The work of Chebyshev is a good early example. In the modern era, tremendous progress is being made, but I will not elaborate or else I will speak too long, and probably embarrass several among us today.

The chemical engineers have advanced the frontier with their need for optimal process and unit designs, and civil engineers have for some time pursued the optimal design of structures. The contribution made by industrial engineers is simply too great to characterize briefly. On the other hand, let us set our sights on the horizon, and consider some of the major unresolved problems of optimal design.

(*a*) Discrete variables: How shall we handle problems when many of the design variables can only take on values from a list of finite length? Most problems in engineering are of this class; we simply pretend that they are not. As far as I know, no generally accepted algorithms currently exist, and the need is great for the reason just given. The work of Lin [12] and Lee and Freudenstein [13] shows promise here.

(*b*) Multiple objectives: It is very common in practice to have great difficulty in identifying a single objective. Instead, we sense that two or more functions of the design variables characterize desirable designs. These objectives may be complementary or competing, but we seldom know this in advance. How shall we weigh these multiple objectives, and what is the nature of efficient algorithms when $f(x)$ is vector valued rather than a scalar? There have been some recent very promising advances here, namely, game theory and the priority theory of Saaty [14]. Saaty has recently shown how we can automate the selection of multiple objective weights using pairwise comparisons. Lootsma [15] has very recently explored Saaty's priority theory as a mechanism for rating algorithms using multiple performance criteria. The procedure shows great promise, and should have extensive application in all areas of decision science.

(*c*) Large problems: What is the best, or even a feasible, strategy for attacking very large problems? We have to date essentially ignored the prevalence of very large and nonlinear optimal design problems. We have out of necessity abstracted them to a much smaller and more manageable size. Unfortunately, in so doing we often remove or drastically reduce the value of our results. Have no doubts that large problems are real. For instance, consider the design of everyday space trusses. It is relatively straightforward to generate NLP's with hundreds of design variables and thousands of nonlinear constraints. Recent advances which exploit the sparsity of these problems or employ decomposition strategies show real promise.

(*d*) Global solutions: Most current algorithms cannot hope to produce global solutions to practical nonconvex problems, or even to sense a global solution if it is encountered. Real progress is being made by Wilde [2], his students [3, 4], and others (e.g., [5]) in this area, but the area deserves much additional attention.

(*e*) Comparative data and software development: Finally, there is a crying need for data on the performance of general and special purpose optimization software. Comparative data of limited value are always better than no data at all. We need more generally available general purpose software.

Changes in Design Education. Design education must experience change if optimization is to assume the role I envision. Our students are currently too "analysis conditioned." We should examine closely the precepts of "guided design" as set forth by Charles Wales. That is, I feel we should provide a greater opportunity for all students to experience design throughout their formal education. More emphasis should be given in the freshman and sophomore years to the concepts of objectives, constraints, and feasibility. We should encourage and seek to ensure a higher level of "computational maturity" in our students. In most areas of engineering, students will profit from decreased emphasis upon modeling of physical phenomena and increased emphasis upon manipulation of same for the good of mankind via a unified decision science constructed upon the basic tenets of optimization theory.

Finally, design educators should address themselves to the needs of practitioners, possibly via short courses, advanced training courses, and the like. We have at least two generations of designers in practice who, for a variety of reasons, have not been exposed to the material we discuss here.

As you see, I am convinced that optimization is an integral part of the future of design, and that design should and will shape developments in optimization. The marriage will, I feel, benefit us all.

Integration of Optimization With the Design Process

Professor J. N. Siddall, McMaster University

An important goal in the development of optimization is to integrate it into the work of designers in low and medium technology. We are likely doing well in this respect in high technology where groups of highly sophisticated engineers are available to work out the modeling, the optimization formulation, and the execution problems. But most engineering work is not done by such groups.

As an aside, it is important not to make the mistake of assuming that what we commonly call high technology is more important, any more difficult, or necessarily any more complex than those technologies that are thought of as low or medium. The difference is often simply one of newness.

Traditional technologies have used (with great success) evolutionary design techniques, whereas newer, high technologies are in more of a hurry, and tend to use highly sophisticated modeling and testing to bypass the slow evolutionary procedure. We should bend all of our efforts to bring new design techniques and ideas to traditional technologies. The possibilities in them are immense.

However, it is not enough to simply present engineers in the traditional technologies with mathematical optimization techniques. We must make optimization simple, easy, convenient, and attractive to use. Optimization provides the designer much more than just a straightforward optimum design. It provides an insight into the design that is not otherwise easily obtained.

We hear a great deal about microcomputers these days. There is no question that they will have a central role in mechanical engineering and add a new dimension to its practice. This will occur in two ways. First, they will be an important tool in the design process, completing the computer revolution in design work. And second, every mechanical device will have an onboard microcomputer for control or information processing. Optimization is strongly relevant to both of these roles.

Requirements for Embedding Optimization in the Design Process. To achieve the goal of embedding formal optimization techniques in the design process, we must satisfy the following general requirements.

1 The designer must do the optimizing.
2 The designer must not be required to do sophisticated analysis beyond the modeling work he would normally do.
3 Formulation should be straightforward, with clear guidelines to avoid easily encountered troubles.
4 Easily used software should be available.
5 Clear guidelines should be available to facilitate execution.
6 The optimization software should not just provide the optimum, but additional information providing insight into the design. This includes items such as trade-off curves, sensitivity curves, specification decisions, etc.

We do not really know how to do all of this very well yet. Very likely a good many would-be optimizers, who are practicing designers, have become discouraged and have given up because of all these complications.

Practical Aspects of Optimization. The practical difficulties of formulation and execution are many, and it is perhaps useful to summarize them briefly.

1 The choice of design variables can be quite difficult.
2 Defining all of the required optimization criteria and constraint functions is not easy or straightforward. These functions can include quite subtle ones that are not at all obvious, and may not show up until runs are attempted. Also, it is important to confine iterative searches to the region where the modeling is valid.
3 There can be unanticipated blow-ups due to indeterminate forms such as the log or square root of a negative quantity.
4 The use of tables and charts (such as steam tables) with a program can sometimes stump the inexperienced engineer.
5 Diagnosing execution problems can be quite difficult.
   - Is it a formulation error?
   - Is it the optimization algorithm used?
   - Is it a bad starting point?
   - Is the problem overconstrained?
   - Has a true local optimum been reached or has the search gotten hung up in a tangle of constraints?

We have still much to learn about how to systematically handle these difficulties.

Interface with Microcomputers. The first question that arises in the interface with microcomputers is: Are microcomputers powerful enough to handle real life optimization problems? We must anticipate that microcomputers will rapidly take over as the tool for computer-aided design work in engineering design offices. Access to large, high speed, centralized mainframe computers will become more difficult. It is doubtful if 8-bit microcomputers, or even 16-bit ones, currently have sufficient precision or high speed memory capacity to handle optimization problems of sizes commonly encountered. High speed mass memory devices such as the bubble memory or rigid disks may be the answer to the problem, but their cost is still rather high. The 32-bit microcomputer, comparable in power to conventional mainframe computers, is apparently on the horizon—only a few years away. It may be the answer. The problem is certainly an interesting one.

The second aspect of the microcomputer question is related to their tremendous potential for on-board control of almost all mechanical devices. Until quite recently, computer control applications have been limited to process control. The smallness and cheapness of microcomputers have removed this limitation. The development of optimal control algorithms and strategies that can be used with these small computers adds a new and fascinating dimension to optimization work. The optimal control theory developed for process work is not adequate nor general enough for use with mechanical devices.

Conclusions. The rate of spread of optimization as a discipline both in engineering practice and in engineering education, although certainly not negligible, is at this time disappointing. Barriers to growth include the lack of integration of optimization into the design process, a failure to make it really a part of the discipline of design, and a failure to make it easy for the practicing designer to use.

It is to be hoped that we do not perpetuate this same failure in the interaction with microcomptuers.

Optimization in Mechanisms: Past, Present, and Future

Professor F. Freudenstein, Columbia University

The process of optimization is as old as history itself. In nature, the rules of adaptation and survival of the fittest may be viewed as optimization by evolution. The evolution of an optimal design is an extremely slow, but effective, process. If we had 100 million years and could test a comparable number of proposed designs, we could do very well. The difficulties arise when we try to speed the optimization along.

At the time of the Industrial Revolution, evolutionary and what might be called intuitive optimization methods were powerful, prevalent techniques. In recent years, optimization methods have become more systematic and a mathematically sound basis has been developed for many problems. The general approaches of optimization theory are not always applicable to mechanical systems, however. For this reason, there is a real need for an optimization strategy that capitalizes on the inherent abilities of the designer. The human mind does not operate only in numerical terms; it conceives, it evaluates, it judges, and it has experience. The human mind prefers clear pictures to abstract number crunching. If we could develop optimization strategies that exhibit such flexibilty, we would really have a powerful tool. It is doubtful that this goal will ever be fully realized, but much can be done along the way.

Looking back in the field of mechanisms, the earliest scientific approaches to optimal design were conceived by

Chebyshev [16] in the days of steam engines, engine-indicator motions, and straight-line motions (circa 1850-1870). The methods developed by Chebyshev remain among the most sophisticated available today. In 1929, Alt [17] considered the quality of force transmission in plane linkages with the goal of finding suitable proportions to maximize force transmission and minimize frictional losses.

Chebyshev and Alt worked long before the modern era of sophisticated computation. They used their insight to formulate solutions taking advantage of the characteristics of the specific problem involved. There were no generally applicable nonlinear codes or computing machines. Hence, they were forced to use their ingenuity and they were rewarded with rich results. Their problems were highly specialized, however. In most problems of realistic complexity, an unusual combination of difficulties confront the designer in the area of mechanisms. We must deal with gross motions. It is not possible to linearize to small displacements when a crank must rotate continuously. And we are faced with system models characterized by many parameters, large numbers of components, and highly nonlinear and complex interactions.

The modern engineer may utilize large-scale, general computer programs (such as DRAM [18], ADAMS [7], IMP [6], KINSYN [19], etc.) which are capable of analyzing a wide variety of mechanisms. Optimization may be accomplished by repeated application of such analysis-oriented codes.

Heuristic approaches to optimization have also been posed [12, 13]. As was observed by Svoboda, in many real design problems the globally optimal solution is not required. The engineer's interest is in identifying a solution that will perform acceptably well. How to define "acceptably well" in a computer program is an important question that a successful design effort must continue to address.

If optimization is to yield truly worthwhile results in the future, it will be necessary to utilize a multidisciplinary approach to the problems in mechanical systems design. Kinematics, dynamics, strength of materials, controls, cost, and production must all be considered. Team efforts will often very likely be the most fruitful. Effective communication among all parties will also be a key element in the success of the design.

Finally, optimal design techniques for the future must maintain a balance between exact and intuitive methods. Perhaps a goal that is achievable within the next 20-plus years would be the development of a computer-aided design methodology that combines the analytical capabilities of a code such as DRAM, for example, with a heuristically guided optimization capability and a well-defined facility for user interaction. Insofar as mechanical design is concerned, this may represent a beginning in tailoring the optimization process to the human mind.

The Discussion

An open discussion period followed the formal presentations. During the discussion, it was observed that the success of any design endeavor is measured by the actual performance of the end product. The role of experimentation in establishing a viable mathematical model and in verifying the performance of the "optimal" design should not be underemphasized. It was also noted that many algorithmic optimization strategies require that a feasible design (i.e., a design satisfying all constraints) be known at the outset. Identifying a feasible starting point can prove to be a formidable task in complex problems. It was suggested that the global, noniterative optimization strategies would be more attractive to potential users if the necessary algebraic manipulations could be systematized and automated.

The importance of focusing future efforts on industrial applications was brought up. Several problems must be resolved if we are to realize the full benefit of applied optimization in practical, industrial environments. Some of these problems were addressed by the panelists in their formal remarks (see e.g., Ragsdell and Siddall). It was suggested that there should be close cooperation between the academic and industrial communities. The design problems in industry are challenging the real. Both university and industry stand much to gain through joint research efforts. Contract research and summer work opportunities for faculty were mentioned as possible starting points to effect such cooperation.

Educators must also address the problem of integrating design synthesis and optimization into the undergraduate curriculum. As Professor Ragsdell pointed out in his prepared remarks, students tend to be conditioned for analysis rather than synthesis during their undergraduate studies. Today's students are tomorrow's engineers. The universities must provide industry with engineers who are aware of the potential benefits to be obtained by optimal design.

Conclusion

Great advances in optimization theory have occurred over the past three decades. In this article, we have taken a look at the future of optimization in general and as it relates to mechanical design. New areas for research continue to develop as we attempt to apply optimization theory to optimal mechanical design. There is much to do, but the future appears extremely bright.

Acknowledgments

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